A New
Joint Confidence Region for a Ranking

Tommy Wright

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2024 Joint Statistical Meetings
Portland, Oregon
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Disclaimer: Any views expressed are those of the speaker and not the U.S. Census Bureau.

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Measure and show UNCERTAINTY in ESTIMATED Ranking.

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difference of parameters $\bar{Y}_k - \bar{Y}_{k'}$

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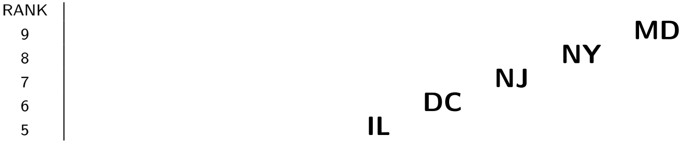

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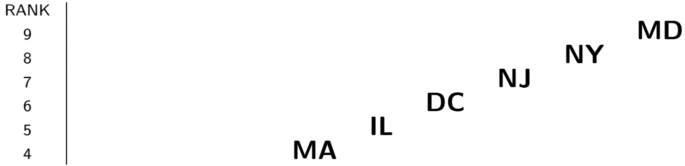

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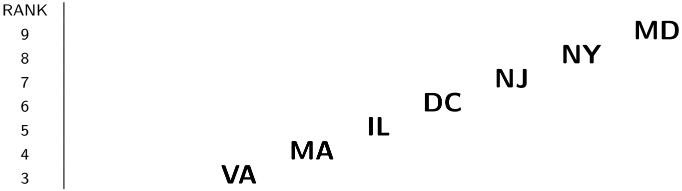

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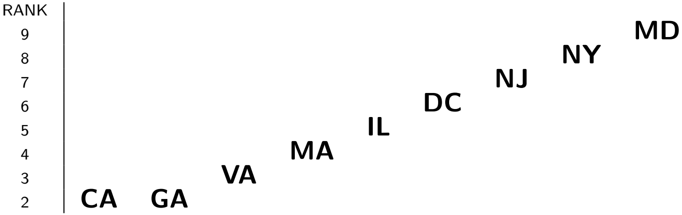

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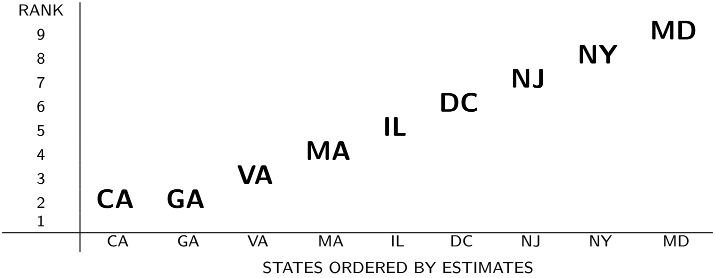

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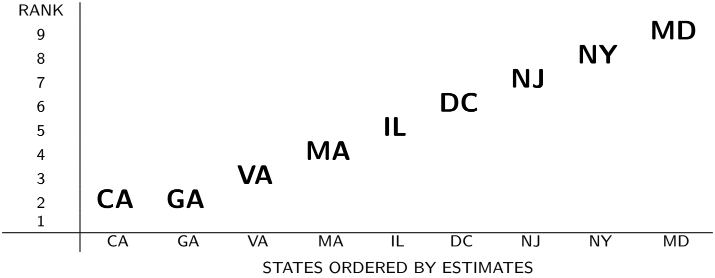

ESTIMATED Ranking based on probability sampling & has *Uncertainty*.

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Lower estimates get Lower estimated ranks.

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$$\Big(32.2 - (2.523)(0.1), 32.2 + (2.523)(0.1)\Big)$$

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Table 1a: Computing Details for $K = 9$ INDI 90% Joint Confidence Intervals for $\bar{Y}_k$

$\hat{r}_k$	State (k)	ESTIMATE $\bar{y}_k$	STANDARD ERROR	INDI 90% Joint Confidence Intervals for $\bar{Y}_k$'s

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5-IL	{GA, CA}	{MA, VA}	$\{3, 4, 5\}$
4-MA	{GA, CA}	{IL, VA}	$\{3, 4, 5\}$
3-VA	{CA}	{IL, MA, GA}	$\{2, 3, 4, 5\}$
2-GA	$\emptyset$	{VA, CA}	$\{1, 2, 3\}$
2-CA	$\emptyset$	{GA}	$\{1, 2\}$

INDI 90% JOINT CONFIDENCE REGION FOR TRUE RANKING...

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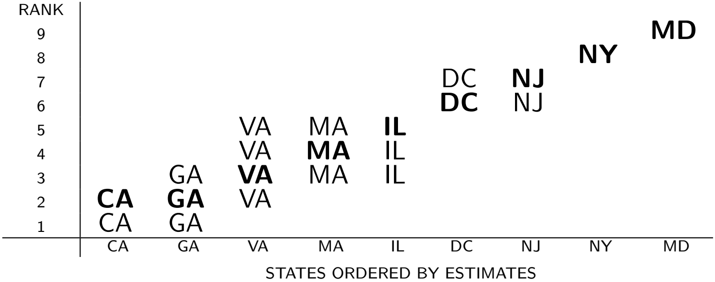
... is a collection of possible TRUE Rankings.



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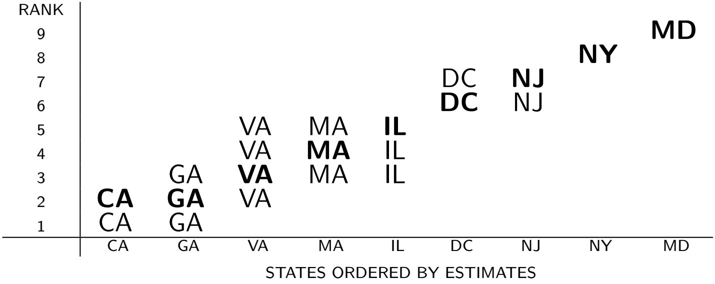

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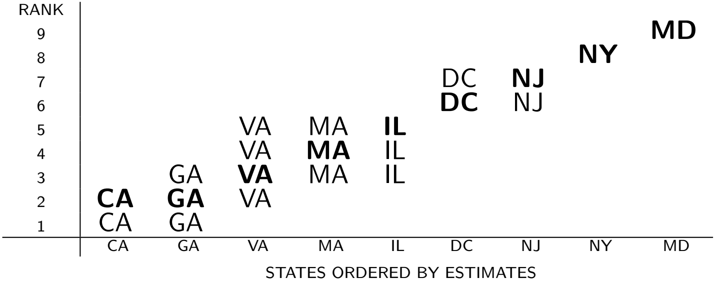

The 90% joint confidence region:

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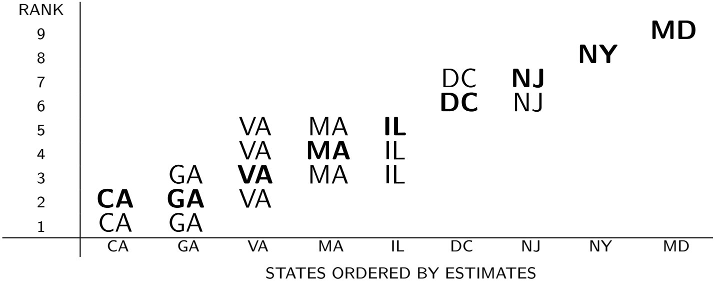

The 90% joint confidence region:

- measures uncertainty in ESTIMATED Ranking.

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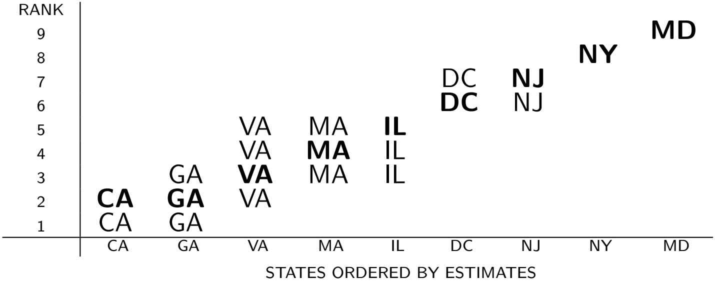


The 90% joint confidence region:

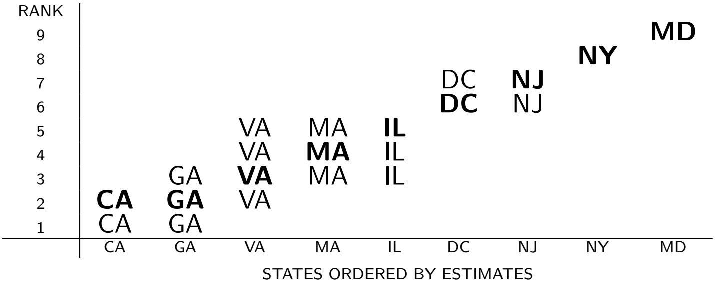
- measures uncertainty in ESTIMATED Ranking.

- contains ESTIMATED Ranking (**bold**), possibly TRUE Ranking.

Another possible TRUE Ranking is shown in last column.

RANK	CA	GA	VA	MA	IL	DC	NJ	NY	MD	*A Second Possible TRUE Ranking*
9									**MD**	**MD**
8								**NY**		**NY**
7						DC	**NJ**			DC
6						**DC**	NJ			NJ
5			VA	MA	**IL**					**IL**
4			VA	**MA**	IL					**MA**
3		GA	**VA**	MA	IL					**VA**
2	**CA**	**GA**	VA							**CA**
1	CA	GA								GA

STATES ORDERED BY ESTIMATES


RANK
9
8
7
6
5
4
3
2
1
MD
NY
DC
NJ
VA
MA
IL
GA
CA
CA GA VA MA IL DC NJ NY MD
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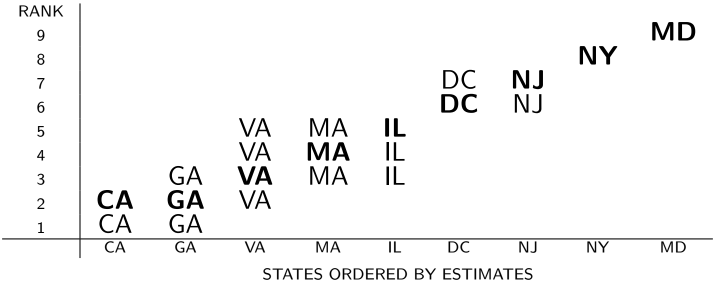
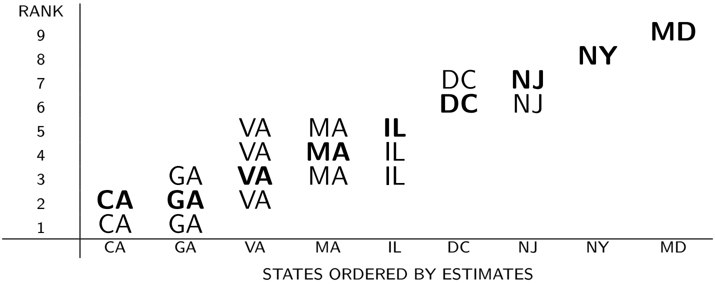
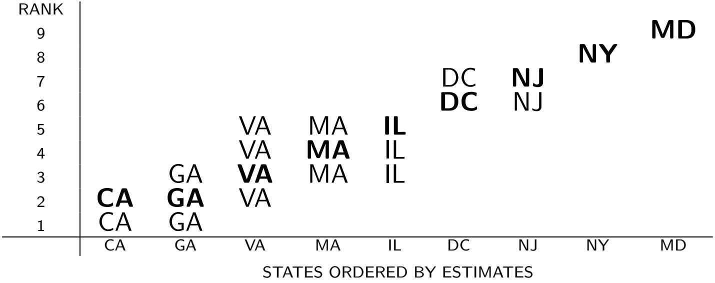


90% joint confidence region



90% joint confidence region reveals four things:



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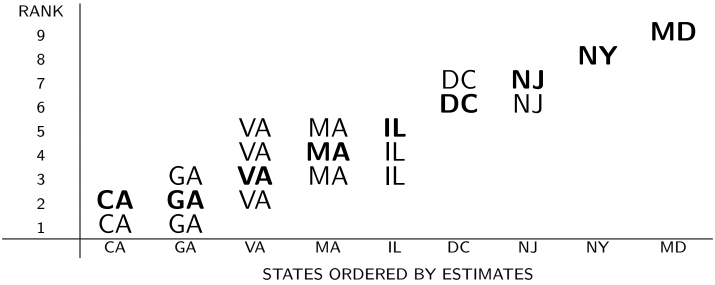
- uncertainty in ESTIMATED Ranking;



90% joint confidence region reveals four things:

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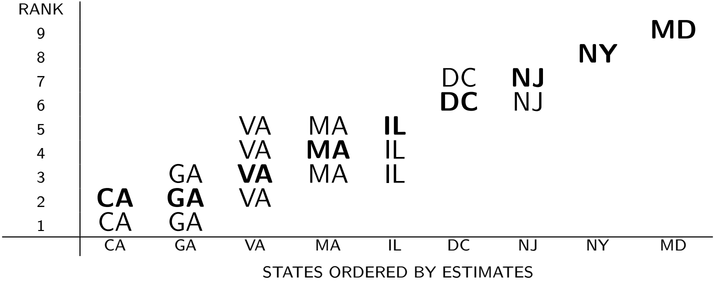

90% joint confidence region reveals four things:

- uncertainty in ESTIMATED Ranking;

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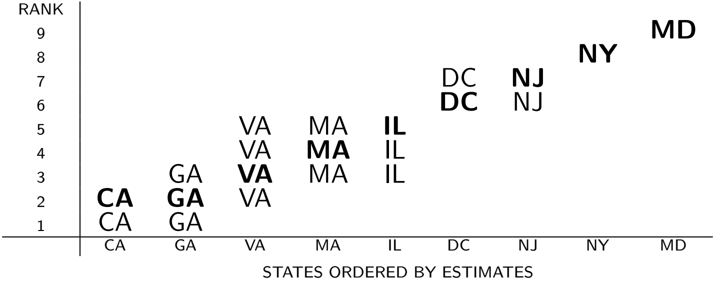
- MA's TRUE Rank could range from 3 to 5;



90% joint confidence region reveals four things:

- uncertainty in ESTIMATED Ranking;
- other possible TRUE Rankings beyond ESTIMATED Ranking;
- MA's TRUE Rank could range from 3 to 5;
- rank 4 could be given to VA, MA, or IL.

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$$\left((\bar{y}_k - \bar{y}_{k'}) - z_{\frac{\gamma*}{2}} SE_{kk'}, \quad (\bar{y}_k - \bar{y}_{k'}) + z_{\frac{\gamma*}{2}} SE_{kk'} \right),$$

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Call adjustment from α to $\gamma*$ a Bonferroni Correction.

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$$\gamma * = \frac{\alpha}{\binom{K}{2}} = \frac{0.10}{[9(8)]/2} = 0.002777$$

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$$\left((30.5-30.1)-(2.991316)\sqrt{[0.1]^2+[0.3]^2}, (30.5-30.1)+(2.991316)\sqrt{[0.1]^2+[0.3]^2}\right)$$

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$$\left((27.7-27.1)-(2.991316)\sqrt{[0.1]^2+[0.1]^2}, (27.2-27.1)+(2.991316)\sqrt{[0.1]^2+[0.1]^2}\right)$$

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Thus, Bonferroni corrected joint confidence interval

for $\bar{Y}_{NewJersey} - \bar{Y}_{DistrictofColumbia}$ is

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Table 2a: Computing Details for $\binom{9}{2}$ DIFF 90% Joint Confidence Intervals $\bar{Y}_k - \bar{Y}_{k'}$

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k	CA	GA	VA	MA	IL	DC	NJ	NY

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k	*CA*	*GA*	*VA*	*MA*	*IL*	*DC*	*NJ*	*NY*
MD	(4.68, 5.52)	(4.43, 5.77)	(4.08, 4.92)	(3.78, 4.62)	(3.58, 4.42)	(1.15, 3.05)	(1.28, 2.12)	(0.28, 1.12)
NY	(3.98, 4.82)	(3.73, 5.07)	(3.38, 4.22)	(3.08, 3.92)	(2.88, 3.72)	(0.45, 2.35)	(0.58, 1.42)	

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MD	(4.68, 5.52)	(4.43, 5.77)	(4.08, 4.92)	(3.78, 4.62)	(3.58, 4.42)	(1.15, 3.05)	(1.28, 2.12)	(0.28, 1.12)
NY	(3.98, 4.82)	(3.73, 5.07)	(3.38, 4.22)	(3.08, 3.92)	(2.88, 3.72)	(0.45, 2.35)	(0.58, 1.42)	
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DC	$(2.05, 3.95)$	$(1.92, 4.08)$	$(1.45, 3.35)$	$(1.15, 3.05)$	$(0.95, 2.85)$			
IL	$(0.68, 1.52)$	$(0.43, 1.77)$	$(0.08, 0.92)$	$(-.22, 0.62)$				
MA	$(0.48, 1.32)$	$(0.23, 1.57)$	$(-.12, 0.72)$					
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k'								

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Table 2b: State k Additional Detailed DIFF Computations for
$r_k \in \left\{|A_{Rk}|+1, |A_{Rk}|+2, |A_{Rk}|+3, ..., |A_{Rk}|+|A_{Ok}|+1\right\}$

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k	A_{Rk}	A_{Ok}	DIFF 90% Joint Confidence Region for True Ranking

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k	A_{Rk}	A_{Ok}	DIFF 90% Joint Confidence Region for True Ranking
MD	$\{CA, GA, VA, MA, IL, DC, NJ, NY\}$	$\emptyset$	$\{9\}$
NY	$\{CA, GA, VA, MA, IL, DC, NJ\}$	$\emptyset$	$\{8\}$

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k	CA	GA	VA	MA	IL	DC	NJ	NY
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Table 2b: State k Additional Detailed DIFF Computations for $r_k \in \{|A_{Rk}| + 1, |A_{Rk}| + 2, |A_{Rk}| + 3, ..., |A_{Rk}| + |A_{Ok}| + 1\}$

k	A_{Rk}	A_{Ok}	DIFF 90% Joint Confidence Region for True Ranking
MD	$\{CA, GA, VA, MA, IL, DC, NJ, NY\}$	$\emptyset$	$\{9\}$
NY	$\{CA, GA, VA, MA, IL, DC, NJ\}$	$\emptyset$	$\{8\}$
NJ	$\{CA, GA, VA, MA, IL\}$		

Table 2a: Computing Details for $\binom{9}{2}$ DIFF 90% Joint Confidence Intervals $\bar{Y}_k - \bar{Y}_{k'}$

k	CA	GA	VA	MA	IL	DC	NJ	NY
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k	A_{Rk}	A_{Ok}	DIFF 90% Joint Confidence Region for True Ranking
MD	$\{CA, GA, VA, MA, IL, DC, NJ, NY\}$	$\emptyset$	$\{9\}$
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NJ	$\{CA, GA, VA, MA, IL\}$	$\{DC\}$	

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k	CA	GA	VA	MA	IL	DC	NJ	NY
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IL	(0.68, 1.52)	(0.43, 1.77)	(0.08, 0.92)	(−.22, 0.62)				
MA	(0.48, 1.32)	(0.23, 1.57)	(−.12, 0.72)					
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Table 2b: State k Additional Detailed DIFF Computations for $r_k \in \{|A_{Rk}|+1, |A_{Rk}|+2, |A_{Rk}|+3, ..., |A_{Rk}|+|A_{Ok}|+1\}$

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MD	$\{CA, GA, VA, MA, IL, DC, NJ, NY\}$	$\emptyset$	$\{9\}$
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NJ	$\{CA, GA, VA, MA, IL\}$	$\{DC\}$	$\{\mathbf{6}, \mathbf{7}\}$

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k	CA	GA	VA	MA	IL	DC	NJ	NY
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k'								

Table 2b: State k Additional Detailed DIFF Computations for $r_k \in \{|A_{Rk}| + 1, |A_{Rk}| + 2, |A_{Rk}| + 3, ..., |A_{Rk}| + |A_{Ok}| + 1\}$

k	A_{Rk}	A_{Ok}	DIFF 90% Joint Confidence Region for True Ranking
MD	$\{CA, GA, VA, MA, IL, DC, NJ, NY\}$	$\emptyset$	$\{9\}$
NY	$\{CA, GA, VA, MA, IL, DC, NJ\}$	$\emptyset$	$\{8\}$
NJ	$\{CA, GA, VA, MA, IL\}$	$\{DC\}$	$\{\mathbf{6}, \mathbf{7}\}$
DC			

Table 2a: Computing Details for $\binom{9}{2}$ DIFF 90% Joint Confidence Intervals $\bar{Y}_k - \bar{Y}_{k'}$

k	CA	GA	VA	MA	IL	DC	NJ	NY
MD	$(4.68, 5.52)$	$(4.43, 5.77)$	$(4.08, 4.92)$	$(3.78, 4.62)$	$(3.58, 4.42)$	$(1.15, 3.05)$	$(1.28, 2.12)$	$(0.28, 1.12)$
NY	$(3.98, 4.82)$	$(3.73, 5.07)$	$(3.38, 4.22)$	$(3.08, 3.92)$	$(2.88, 3.72)$	$(0.45, 2.35)$	$(0.58, 1.42)$	
NJ	$(2.98, 3.82)$	$(2.73, 4.07)$	$(2.38, 3.22)$	$(2.08, 2.92)$	$(1.88, 2.72)$	$(-.55, 1.35)$		
DC	$(2.05, 3.95)$	$(1.92, 4.08)$	$(1.45, 3.35)$	$(1.15, 3.05)$	$(0.95, 2.85)$			
IL	$(0.68, 1.52)$	$(0.43, 1.77)$	$(0.08, 0.92)$	$(-.22, 0.62)$				
MA	$(0.48, 1.32)$	$(0.23, 1.57)$	$(-.12, 0.72)$					
VA	$(0.18, 1.02)$	$(-.07, 1.27)$						
GA	$(-.67, 0.67)$							
k'								

Table 2b: State k Additional Detailed DIFF Computations for $r_k \in \{|A_{Rk}|+1, |A_{Rk}|+2, |A_{Rk}|+3, ..., |A_{Rk}|+|A_{Ok}|+1\}$

k	A_{Rk}	A_{Ok}	DIFF 90% Joint Confidence Region for True Ranking
MD	$\{CA, GA, VA, MA, IL, DC, NJ, NY\}$	$\emptyset$	$\{9\}$
NY	$\{CA, GA, VA, MA, IL, DC, NJ\}$	$\emptyset$	$\{8\}$
NJ	$\{CA, GA, VA, MA, IL\}$	$\{DC\}$	$\{\mathbf{6}, \mathbf{7}\}$
DC	$\{CA, GA, VA, MA, IL\}$		

Table 2a: Computing Details for $\binom{9}{2}$ DIFF 90% Joint Confidence Intervals $\bar{Y}_k - \bar{Y}_{k'}$

k	CA	GA	VA	MA	IL	DC	NJ	NY
MD	$(4.68, 5.52)$	$(4.43, 5.77)$	$(4.08, 4.92)$	$(3.78, 4.62)$	$(3.58, 4.42)$	$(1.15, 3.05)$	$(1.28, 2.12)$	$(0.28, 1.12)$
NY	$(3.98, 4.82)$	$(3.73, 5.07)$	$(3.38, 4.22)$	$(3.08, 3.92)$	$(2.88, 3.72)$	$(0.45, 2.35)$	$(0.58, 1.42)$	
NJ	$(2.98, 3.82)$	$(2.73, 4.07)$	$(2.38, 3.22)$	$(2.08, 2.92)$	$(1.88, 2.72)$	$(-.55, 1.35)$		
DC	$(2.05, 3.95)$	$(1.92, 4.08)$	$(1.45, 3.35)$	$(1.15, 3.05)$	$(0.95, 2.85)$			
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VA	$(0.18, 1.02)$	$(-.07, 1.27)$						
GA	$(-.67, 0.67)$							
k'								

Table 2b: State k Additional Detailed DIFF Computations for $r_k \in \{|A_{Rk}| + 1, |A_{Rk}| + 2, |A_{Rk}| + 3, ..., |A_{Rk}| + |A_{Ok}| + 1\}$

k	A_{Rk}	A_{Ok}	DIFF 90% Joint Confidence Region for True Ranking
MD	$\{CA, GA, VA, MA, IL, DC, NJ, NY\}$	$\emptyset$	$\{9\}$
NY	$\{CA, GA, VA, MA, IL, DC, NJ\}$	$\emptyset$	$\{8\}$
NJ	$\{CA, GA, VA, MA, IL\}$	$\{DC\}$	$\{\mathbf{6}, \mathbf{7}\}$
DC	$\{CA, GA, VA, MA, IL\}$	$\emptyset$	

Table 2a: Computing Details for $\binom{9}{2}$ DIFF 90% Joint Confidence Intervals $\bar{Y}_k - \bar{Y}_{k'}$

k	CA	GA	VA	MA	IL	DC	NJ	NY
MD	(4.68, 5.52)	(4.43, 5.77)	(4.08, 4.92)	(3.78, 4.62)	(3.58, 4.42)	(1.15, 3.05)	(1.28, 2.12)	(0.28, 1.12)
NY	(3.98, 4.82)	(3.73, 5.07)	(3.38, 4.22)	(3.08, 3.92)	(2.88, 3.72)	(0.45, 2.35)	(0.58, 1.42)	
NJ	(2.98, 3.82)	(2.73, 4.07)	(2.38, 3.22)	(2.08, 2.92)	(1.88, 2.72)	$(-.55, 1.35)$		
DC	(2.05, 3.95)	(1.92, 4.08)	(1.45, 3.35)	(1.15, 3.05)	(0.95, 2.85)			
IL	(0.68, 1.52)	(0.43, 1.77)	(0.08, 0.92)	$(-.22, 0.62)$				
MA	(0.48, 1.32)	(0.23, 1.57)	$(-.12, 0.72)$					
VA	(0.18, 1.02)	$(-.07, 1.27)$						
GA	$(-.67, 0.67)$							
k'								

Table 2b: State k Additional Detailed DIFF Computations for $r_k \in \{|A_{Rk}|+1, |A_{Rk}|+2, |A_{Rk}|+3, ..., |A_{Rk}|+|A_{Ok}|+1\}$

k	A_{Rk}	A_{Ok}	DIFF 90% Joint Confidence Region for True Ranking
MD	$\{CA, GA, VA, MA, IL, DC, NJ, NY\}$	$\emptyset$	$\{9\}$
NY	$\{CA, GA, VA, MA, IL, DC, NJ\}$	$\emptyset$	$\{8\}$
NJ	$\{CA, GA, VA, MA, IL\}$	$\{DC\}$	$\{\mathbf{6}, \mathbf{7}\}$
DC	$\{CA, GA, VA, MA, IL\}$	$\emptyset$	$\{\mathbf{6}\}$

Table 2a: Computing Details for $\binom{9}{2}$ DIFF 90% Joint Confidence Intervals $\bar{Y}_k - \bar{Y}_{k'}$

k	CA	GA	VA	MA	IL	DC	NJ	NY
MD	(4.68, 5.52)	(4.43, 5.77)	(4.08, 4.92)	(3.78, 4.62)	(3.58, 4.42)	(1.15, 3.05)	(1.28, 2.12)	(0.28, 1.12)
NY	(3.98, 4.82)	(3.73, 5.07)	(3.38, 4.22)	(3.08, 3.92)	(2.88, 3.72)	(0.45, 2.35)	(0.58, 1.42)	
NJ	(2.98, 3.82)	(2.73, 4.07)	(2.38, 3.22)	(2.08, 2.92)	(1.88, 2.72)	(−.55, 1.35)		
DC	(2.05, 3.95)	(1.92, 4.08)	(1.45, 3.35)	(1.15, 3.05)	(0.95, 2.85)			
IL	(0.68, 1.52)	(0.43, 1.77)	(0.08, 0.92)	(−.22, 0.62)				
MA	(0.48, 1.32)	(0.23, 1.57)	(−.12, 0.72)					
VA	(0.18, 1.02)	(−.07, 1.27)						
GA	(−.67, 0.67)							
k'								

Table 2b: State k Additional Detailed DIFF Computations for $r_k \in \{|A_{Rk}|+1, |A_{Rk}|+2, |A_{Rk}|+3, ..., |A_{Rk}|+|A_{Ok}|+1\}$

k	A_{Rk}	A_{Ok}	DIFF 90% Joint Confidence Region for True Ranking
MD	$\{CA, GA, VA, MA, IL, DC, NJ, NY\}$	$\emptyset$	$\{9\}$
NY	$\{CA, GA, VA, MA, IL, DC, NJ\}$	$\emptyset$	$\{8\}$
NJ	$\{CA, GA, VA, MA, IL\}$	$\{DC\}$	$\{\mathbf{6}, \mathbf{7}\}$
DC	$\{CA, GA, VA, MA, IL\}$	$\emptyset$	$\{\mathbf{6}\}$
IL	$\{CA, GA, VA\}$	$\{MA\}$	$\{4, 5\}$
MA	$\{CA, GA\}$	$\{VA\}$	$\{3, 4\}$
VA	$\{CA\}$	$\{GA\}$	$\{2, 3\}$
GA	$\emptyset$	$\{CA\}$	$\{1, 2\}$

Table 2a: Computing Details for $\binom{9}{2}$ DIFF 90% Joint Confidence Intervals $\bar{Y}_k - \bar{Y}_{k'}$

k	CA	GA	VA	MA	IL	DC	NJ	NY
MD	$(4.68, 5.52)$	$(4.43, 5.77)$	$(4.08, 4.92)$	$(3.78, 4.62)$	$(3.58, 4.42)$	$(1.15, 3.05)$	$(1.28, 2.12)$	$(0.28, 1.12)$
NY	$(3.98, 4.82)$	$(3.73, 5.07)$	$(3.38, 4.22)$	$(3.08, 3.92)$	$(2.88, 3.72)$	$(0.45, 2.35)$	$(0.58, 1.42)$	
NJ	$(2.98, 3.82)$	$(2.73, 4.07)$	$(2.38, 3.22)$	$(2.08, 2.92)$	$(1.88, 2.72)$	$(-.55, 1.35)$		
DC	$(2.05, 3.95)$	$(1.92, 4.08)$	$(1.45, 3.35)$	$(1.15, 3.05)$	$(0.95, 2.85)$			
IL	$(0.68, 1.52)$	$(0.43, 1.77)$	$(0.08, 0.92)$	$(-.22, 0.62)$				
MA	$(0.48, 1.32)$	$(0.23, 1.57)$	$(-.12, 0.72)$					
VA	$(0.18, 1.02)$	$(-.07, 1.27)$						
GA	$(-.67, 0.67)$							
k'								

Table 2b: State k Additional Detailed DIFF Computations for $r_k \in \{|A_{Rk}| + 1, |A_{Rk}| + 2, |A_{Rk}| + 3, ..., |A_{Rk}| + |A_{Ok}| + 1\}$

k	A_{Rk}	A_{Ok}	DIFF 90% Joint Confidence Region for True Ranking
MD	$\{CA, GA, VA, MA, IL, DC, NJ, NY\}$	$\emptyset$	$\{9\}$
NY	$\{CA, GA, VA, MA, IL, DC, NJ\}$	$\emptyset$	$\{8\}$
NJ	$\{CA, GA, VA, MA, IL\}$	$\{DC\}$	$\{\mathbf{6}, \mathbf{7}\}$
DC	$\{CA, GA, VA, MA, IL\}$	$\emptyset$	$\{\mathbf{6}\}$
IL	$\{CA, GA, VA\}$	$\{MA\}$	$\{4, 5\}$
MA	$\{CA, GA\}$	$\{VA\}$	$\{3, 4\}$
VA	$\{CA\}$	$\{GA\}$	$\{2, 3\}$
GA	$\emptyset$	$\{CA\}$	$\{1, 2\}$
CA	$\emptyset$	$\emptyset$	$\{1\}$

Table 2a: Computing Details for $\binom{9}{2}$ DIFF 90% Joint Confidence Intervals $\bar{Y}_k - \bar{Y}_{k'}$

k	CA	GA	VA	MA	IL	DC	NJ	NY
MD	$(4.68, 5.52)$	$(4.43, 5.77)$	$(4.08, 4.92)$	$(3.78, 4.62)$	$(3.58, 4.42)$	$(1.15, 3.05)$	$(1.28, 2.12)$	$(0.28, 1.12)$
NY	$(3.98, 4.82)$	$(3.73, 5.07)$	$(3.38, 4.22)$	$(3.08, 3.92)$	$(2.88, 3.72)$	$(0.45, 2.35)$	$(0.58, 1.42)$	
NJ	$(2.98, 3.82)$	$(2.73, 4.07)$	$(2.38, 3.22)$	$(2.08, 2.92)$	$(1.88, 2.72)$	$(-.55, 1.35)$		
DC	$(2.05, 3.95)$	$(1.92, 4.08)$	$(1.45, 3.35)$	$(1.15, 3.05)$	$(0.95, 2.85)$			
IL	$(0.68, 1.52)$	$(0.43, 1.77)$	$(0.08, 0.92)$	$(-.22, 0.62)$				
MA	$(0.48, 1.32)$	$(0.23, 1.57)$	$(-.12, 0.72)$					
VA	$(0.18, 1.02)$	$(-.07, 1.27)$						
GA	$(-.67, 0.67)$							
k'								

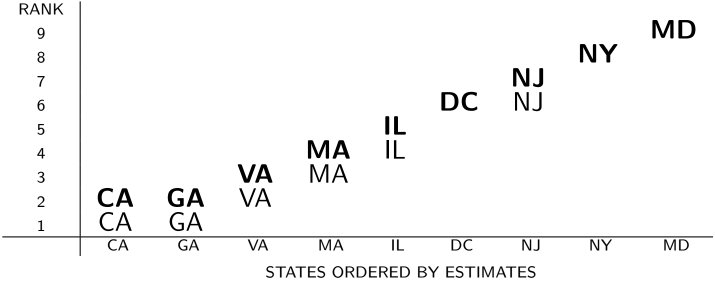
Table 2b: State k Additional Detailed DIFF Computations for $r_k \in \{|A_{Rk}| + 1, |A_{Rk}| + 2, |A_{Rk}| + 3, ..., |A_{Rk}| + |A_{Ok}| + 1\}$

k	A_{Rk}	A_{Ok}	DIFF 90% Joint Confidence Region for True Ranking
MD	$\{CA, GA, VA, MA, IL, DC, NJ, NY\}$	$\emptyset$	$\{9\}$
NY	$\{CA, GA, VA, MA, IL, DC, NJ\}$	$\emptyset$	$\{8\}$
NJ	$\{CA, GA, VA, MA, IL\}$	$\{DC\}$	$\{\mathbf{6}, \mathbf{7}\}$
DC	$\{CA, GA, VA, MA, IL\}$	$\emptyset$	$\{\mathbf{6}\}$
IL	$\{CA, GA, VA\}$	$\{MA\}$	$\{4, 5\}$
MA	$\{CA, GA\}$	$\{VA\}$	$\{3, 4\}$
VA	$\{CA\}$	$\{GA\}$	$\{2, 3\}$
GA	$\emptyset$	$\{CA\}$	$\{1, 2\}$
CA	$\emptyset$	$\emptyset$	$\{1\} \rightarrow \{1, 2\}$

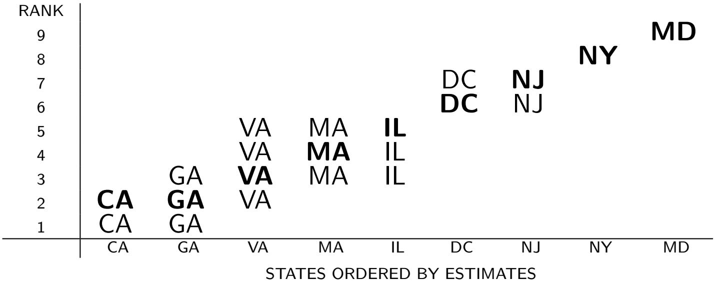
United States Census Bureau

DIFF 90% JOINT CONFIDENCE REGION FOR TRUE RANKING...

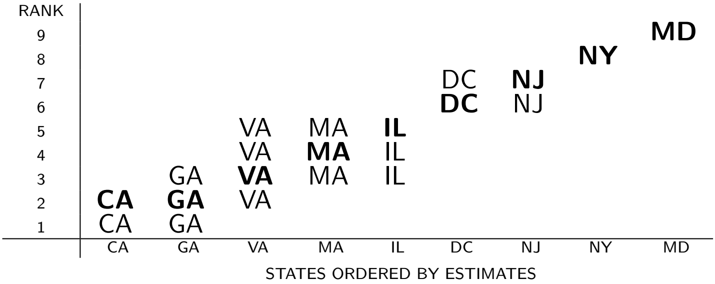
... is a collection of possible TRUE Rankings.



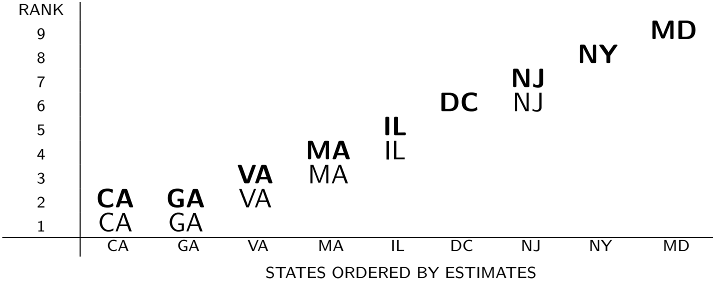
INDI 90% JOINT CONFIDENCE REGION FOR TRUE RANKING...



INDI 90% JOINT CONFIDENCE REGION FOR TRUE RANKING...



DIFF 90% JOINT CONFIDENCE REGION FOR TRUE RANKING...



INDI: $r_k \in \left\{|\Lambda_{Lk}| + 1, |\Lambda_{Lk}| + 2, |\Lambda_{Lk}| + 3, ..., |\Lambda_{Lk}| + |\Lambda_{Ok}| + 1\right\}$

INDI: $r_k \in \left\{ |\Lambda_{Lk}| + 1, |\Lambda_{Lk}| + 2, |\Lambda_{Lk}| + 3, ..., |\Lambda_{Lk}| + |\Lambda_{Ok}| + 1 \right\}$

DIFF: $r_k \in \left\{ |A_{Rk}| + 1, |A_{Rk}| + 2, |A_{Rk}| + 3, ..., |A_{Rk}| + |A_{Ok}| + 1 \right\}$

INDI: $r_k \in \left\{|\Lambda_{Lk}| + 1, |\Lambda_{Lk}| + 2, |\Lambda_{Lk}| + 3, ..., |\Lambda_{Lk}| + |\Lambda_{Ok}| + 1\right\}$

DIFF: $r_k \in \left\{|A_{Rk}| + 1, |A_{Rk}| + 2, |A_{Rk}| + 3, ..., |A_{Rk}| + |A_{Ok}| + 1\right\}$

$\hat{r}_k$	k	INDI Λ_{Ok}	DIFF A_{Ok}	INDI 90% Joint Confidence Region for Ranking	DIFF 90% Joint Confidence Region for Ranking

$$\textbf{INDI: } r_k \in \left\{|\Lambda_{Lk}|+1, |\Lambda_{Lk}|+2, |\Lambda_{Lk}|+3, ..., |\Lambda_{Lk}|+|\Lambda_{Ok}|+1\right\}$$

$$\textbf{DIFF: } r_k \in \left\{|A_{Rk}|+1, |A_{Rk}|+2, |A_{Rk}|+3, ..., |A_{Rk}|+|A_{Ok}|+1\right\}$$

$\hat{r}_k$	k	INDI Λ_{Ok}	DIFF A_{Ok}	INDI 90% Joint Confidence Region for Ranking	DIFF 90% Joint Confidence Region for Ranking
9	MD	$\emptyset$	$\emptyset$	$\{9\}$	$\{9\}$
8	NY	$\emptyset$	$\emptyset$	$\{8\}$	$\{8\}$
7	NJ	{DC}	{DC}	$\{6,7\}$	$\{6,7\}$
6	DC	{NJ}	$\emptyset$	$\{6,7\}$	$\{6\}$
5	IL	{MA, VA}	{MA}	$\{3,4,5\}$	$\{4,5\}$
4	MA	{IL, VA}	{ VA }	$\{3,4,5\}$	$\{3,4\}$
3	VA	{IL, MA, GA}	{ GA }	$\{2,3,4,5\}$	$\{2,3\}$
2	GA	{VA, CA}	{ CA }	$\{1,2,3\}$	$\{1,2\}$
2	CA	{GA}	$\emptyset$	$\{1,2\}$	$\{1,2\}^*$

INDI: $r_k \in \left\{|\Lambda_{Lk}| + 1, |\Lambda_{Lk}| + 2, |\Lambda_{Lk}| + 3, ..., |\Lambda_{Lk}| + |\Lambda_{Ok}| + 1\right\}$

DIFF: $r_k \in \left\{|A_{Rk}| + 1, |A_{Rk}| + 2, |A_{Rk}| + 3, ..., |A_{Rk}| + |A_{Ok}| + 1\right\}$

$\hat{r}_k$	k	INDI Λ_{Ok}	DIFF A_{Ok}	INDI 90% Joint Confidence Region for Ranking	DIFF 90% Joint Confidence Region for Ranking
9	MD	$\emptyset$	$\emptyset$	$\{9\}$	$\{9\}$
8	NY	$\emptyset$	$\emptyset$	$\{8\}$	$\{8\}$
7	NJ	{DC}	{DC}	$\{6, 7\}$	$\{6, 7\}$
6	DC	{NJ}	$\emptyset$	$\{6, 7\}$	$\{6\}$
5	IL	{MA, VA}	{MA}	$\{3, 4, 5\}$	$\{4, 5\}$
4	MA	{IL, VA}	{ VA }	$\{3, 4, 5\}$	$\{3, 4\}$
3	VA	{IL, MA, GA}	{ GA }	$\{2, 3, 4, 5\}$	$\{2, 3\}$
2	GA	{VA, CA}	{ CA }	$\{1, 2, 3\}$	$\{1, 2\}$
2	CA	{GA}	$\emptyset$	$\{1, 2\}$	$\{1, 2\}^*$

Sets in **DIFF** column

$$\textbf{INDI: } r_k \in \left\{|\Lambda_{Lk}|+1, |\Lambda_{Lk}|+2, |\Lambda_{Lk}|+3, ..., |\Lambda_{Lk}|+|\Lambda_{Ok}|+1\right\}$$

$$\textbf{DIFF: } r_k \in \left\{|A_{Rk}|+1, |A_{Rk}|+2, |A_{Rk}|+3, ..., |A_{Rk}|+|A_{Ok}|+1\right\}$$

$\hat{r}_k$	k	INDI Λ_{Ok}	DIFF A_{Ok}	INDI 90% Joint Confidence Region for Ranking	DIFF 90% Joint Confidence Region for Ranking
9	MD	$\emptyset$	$\emptyset$	$\{9\}$	$\{9\}$
8	NY	$\emptyset$	$\emptyset$	$\{8\}$	$\{8\}$
7	NJ	{DC}	{DC}	$\{6,7\}$	$\{6,7\}$
6	DC	{NJ}	$\emptyset$	$\{6,7\}$	$\{6\}$
5	IL	{MA, VA}	{MA}	$\{3,4,5\}$	$\{4,5\}$
4	MA	{IL, VA}	{ VA }	$\{3,4,5\}$	$\{3,4\}$
3	VA	{IL, MA, GA}	{ GA }	$\{2,3,4,5\}$	$\{2,3\}$
2	GA	{VA, CA}	{ CA }	$\{1,2,3\}$	$\{1,2\}$
2	CA	{GA}	$\emptyset$	$\{1,2\}$	$\{1,2\}^*$

Sets in **DIFF** column are subsets of sets in **INDI** column.

Theorem:

Theorem:

If $z_{\frac{\gamma*}{2}} \sqrt{[SE(\bar{Y}_k)]^2 + [SE(\bar{Y}_{k'})]^2}$

Theorem:

If $z_{\frac{\gamma *}{2}} \sqrt{[SE(\bar{Y}_k)]^2 + [SE(\bar{Y}_{k'})]^2} < z_{\frac{\gamma}{2}} \left[SE(\bar{Y}_k) + SE(\bar{Y}_{k'}) \right]$,

Theorem:

If $z_{\frac{\gamma *}{2}} \sqrt{[SE(\bar{Y}_k)]^2 + [SE(\bar{Y}_{k'})]^2} < z_{\frac{\gamma}{2}} \left[SE(\bar{Y}_k) + SE(\bar{Y}_{k'}) \right]$, then

Theorem:

If $z_{\frac{\gamma *}{2}} \sqrt{[SE(\bar{Y}_k)]^2 + [SE(\bar{Y}_{k'})]^2} < z_{\frac{\gamma}{2}} \left[SE(\bar{Y}_k) + SE(\bar{Y}_{k'}) \right]$, then

$$A_{Ok}$$

Theorem:

If $z_{\frac{\gamma *}{2}} \sqrt{[SE(\bar{Y}_k)]^2 + [SE(\bar{Y}_{k'})]^2} < z_{\frac{\gamma}{2}} \left[SE(\bar{Y}_k) + SE(\bar{Y}_{k'}) \right]$, then

$$A_{Ok} \subseteq \Lambda_{Ok}.$$

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