# Parametric Bootstrap Mean Square Error Estimates for Different Small Areas in the Annual Survey of Public Employment & Payroll<sup>1</sup>

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# Abstract

The Annual Survey of Public Employment & Payroll (ASPEP), conducted by the U.S. Census Bureau, provides statistics on the number of federal, state, and local government civilian employees and their gross payrolls. Empirical Best Linear Unbiased Predictor (EBLUP) offers a big improvement in the estimation of the ASPEP data with the use of auxiliary information from the preceding Census of Governments. In this paper, we use our EBLUP model to estimate the mean square error. In order to evaluate the method, we replicated 1000 samples from the frame using the production design. For each replicate, we generated 200 samples (parametric bootstrap samples) given by the EBLUP model. Then we compared the Relative Root Mean Square Error (RRMSE) produced by parametric bootstrap with the simulated true MSE.

Keywords: Government Units, Monte Carlo Simulation, Parametric Bootstrap, EBLUP

# 1. Introduction

Over the last few decades, the U.S. Census Bureau has pioneered in developing innovative small area methodologies in different programs. In one of the most cited papers in small area estimation (SAE) literature, Fay and Herriot (1979) developed a parametric empirical Bayes method to estimate per-capita income of small places with population less than 1,000 and demonstrated, using the Census data, that their method was superior to both direct design-based and synthetic methods. More recently, researchers at the U.S. Census Bureau implemented both empirical and hierarchical Bayes methodologies in the context of Small Area Income and Poverty Estimates (SAIPE) and Small Area Health Insurance Estimates (SAHIE) programs; see Bell et al. (2007) and Bauder et al. (2008).

Besides the Census Bureau's well-known SAIPE and SAHIE programs, researchers in the ESMD are actively pursuing state-of-the-art small area estimation techniques to improve the current estimation methodologies for small areas. Some results on the ASPEP estimation were presented at 2013 SAE conference in Thailand, and 2014 SAE in Poznan, Poland, and 2015 SAE in Chile. There is a large number of small area estimators available in the literature. These estimators typically use either implicit or explicit models to combine survey data with different administrative and Census records. The properties of such estimators are usually studied using the model used to derive the estimator. However, the design-based properties of small area estimators, which are most appealing to the survey practitioners, are largely unknown. In this paper, we show a method (parametric bootstrap) to estimate the mean square error of the estimates produced by the EBLUP estimator in different small areas (29 function codes). The model used the ASPEP data and auxiliary information from the preceding Census of Governments. The universe is the intersection of the two census data, 2007 and 2012. We developed a

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design-based Monte Carlo simulation experiment in which we draw repeated samples (1,000 of them) from the universe using the ASPEP sampling design and then generated 200 samples using the EBLUP model, called parametric bootstrap samples. In total, we have 1,000 replicates, each has 200 bootstrap samples, and each bootstrap sample contains 29 small areas. The average of the RRMSEs from 1,000 replicates was compared with simulated true RRMSE for each small area.

U.S. Census Bureau conducts Censuses of about 90,000 state and local government units every five years in order to collect data on the number of full-time and part-time state and local government employees and payroll. Between two consecutive Censuses (years ending with 2 and 7, e.g., 2002, 2007, and 2012), U.S. Census Bureau also conducts the Annual Survey of Public Employment and Payroll, a nationwide sample survey covering all state and local governments in the United States, which include five types of governments: counties, cities, townships, special districts, and school districts. The first three types of government are referred to as general-purpose governments, because they generally provide multiple government activities. Activities in ASPEP are designated by function codes (see Appendix). School districts cover only education functions. Special districts usually provide only one function, but can provide multiple functions, like Natural Resources, or Sewerage. ASPEP is the only source of public employment data by program function and full-time and part-time break. Data on employment include the number of full-time and part-time employees and gross pay as well as hours paid for part-time employees. All data are reported for the government's pay period covering March 12. Data collection begins in March and continues for about seven months. For more information on the survey, we refer to http://www.Census.gov/govs/apes.

In 2014, a new sample for ASPEP was selected based on the 2012 Census of Governments: Employment (CoG:E). The sample design was changed slightly as compared to the 2009 design. Initial certainty criteria were not used, and more samples were allocated to school district strata of small states. Instead of regular  $\pi$ PS (proportion to size without replacement), systematic  $\pi$ PS sampling was performed in all strata after sorting by population (for general-purpose governments), enrollment (for school districts), and function (for special districts). In the second stage of the design, we again used modified cut-off sampling to select a subsample of small-size units. The 2014 ASPEP sample contains about 10,000 units from the 2012 Cog: E frame of about 90,000 units.

The ASPEP survey is designed to produce reliable estimates of the number of full-time and part-time employees and payroll at the national level and for large domains (e.g., government functions such as elementary and secondary education, higher education, police protection, fire protection, financial administration, judicial and legal, etc., at the national level, and states aggregates of all function codes). However, it is also required to estimate the parameters for individual function codes within each state. This requirement leads us to explore small area estimation methodology that borrows strength from previous Census data instead of collecting expensive additional data for small cells. We refer to Rao (2003) and Jiang and Lahiri (2006) for a comprehensive account of small area estimation theory and applications.

#### 2. Estimation Methods

#### 2.1 EBLUP Estimators (are-level and unit-level models)

In this paper, the variable of interest is the number of full-time employees. Our data is skewed; therefore, we transformed the variable in a log scale (see Figure 2). We proposed two models: area-level model and unit-level on the auxiliary variable (see model (2) and model (5) below).

#### Area-level Model

Let  $y_{ij}$  denote the number of full-time employees for the j<sup>th</sup> governmental unit within the i<sup>th</sup> small area ( $i = 1, \dots, m$ ;  $j = 1, \dots, N_i$ ). The small area in this paper we refer to the cell (state, function). In this paper, we are interested in estimating the total number of fulltime employees for the i<sup>th</sup> small area given by  $Y_i = \sum_{i=1}^{N_i} y_{ij}$  ( $i = 1, \dots, m$ ). An estimator of  $Y_i$  is given by:

$$\hat{Y}_{i}^{EB} = N_{i} \left[ f_{i} \overline{y}_{i} + (1 - f_{i}) \hat{\overline{Y}}_{ir} \right]$$
(1)

where  $\overline{y}_i = n_i^{-1} \sum_{j=1}^{n_i} y_{ij}$  is the sample mean;  $f_i = n_i / N_i$ ,  $N_i$  and  $n_i$  are the sampling fraction, number of government units in the population and sample for area *i*, respectively;  $\hat{Y}_{ir}$  is a model-dependent predictor of the mean of the non-sampled part of area *i* (*i* = 1,...,*m*).

In this paper, we obtain  $\hat{\vec{Y}}_{ir}$  using the following nested error regression model on the logarithm of the number of full-time employees at the government unit level:

$$\log(y_{ij}) = \beta_0 + \beta_1 \log(\overline{X}_i) + v_i + \varepsilon_{ij}, \qquad (2)$$

$$v_i \sim N(0, \tau^2) \text{ and } \varepsilon_{ij} \sim N(0, \sigma^2),$$
 (3)

where  $\overline{X}_i$  is the average number of full-time employees for the *i*<sup>th</sup> small area obtained from the previous Census;  $\beta_0$  and  $\beta_1$  are unknown intercept and slope, respectively;  $v_i$ are small area specific random effects. The distribution of the random effects describes deviations of the area means from values  $\beta_0 + \beta_1 \log(\overline{X}_i)$ ;  $\varepsilon_{ij}$  are errors in individual observations ( $j = 1, ..., N_i$ ; i = 1, ..., m). The random variables  $v_i$  and  $\varepsilon_{ij}$  are assumed to be mutually independent. We assume that sampling is non-informative for the distribution of measurements  $y_{ij}$  ( $j = 1, ..., N_i$ ; i = 1, ..., m). A similar model without logarithmic transformation can be found in Battese et al. (1988). The logarithmic transformation is taken to reduce the extent of heteroscedasticity in the employment data. Similar model using unit level auxiliary information was considered by Bellow and Lahiri (2012) in the context of estimating total hectare under corn for U.S. counties. We use the following model-based predictor of  $\overline{Y}_{ir}$ :

$$\hat{\overline{Y}}_{ir} \approx \exp\left[\hat{\beta}_0 + \hat{\beta}_1 \log(\overline{X}_i) + \hat{v}_i + \frac{1}{2}(\hat{\sigma}^2 + \hat{\delta}_i^2)\right]$$
(4)

where  $\hat{\beta}_0$ ,  $\hat{\beta}_1$ ,  $\hat{v}_i$ ,  $\hat{\sigma}^2$ , and  $\hat{\delta}_i^2$  (standard error of  $\hat{v}_i$ ) are obtained by fitting (2) using PROC MIXED of SAS. We obtain our estimate of total number of full-time employees in area *i* using equations (1) and (4).

#### **Unit-level Model**

Besides area-level (model 2), we also performed the unit-level ( $X_{ii}$ ) model as below.

$$\log(y_{ij}) = \beta_0 + \beta_1 \log(X_{ij}) + v_i + \varepsilon_{ij}, \qquad (5)$$

$$v_i \stackrel{iid}{\sim} N(0, \tau^2) \text{ and } \varepsilon_{ij} \stackrel{iid}{\sim} N(0, \sigma^2),$$
 (6)

After estimating the models parameters, the estimate will be obtained by two different ways: simple back transformed, and log-normal back transformed given as follows:

# Simple $\hat{Y}_{i}^{EB} = \sum_{i \in S} y_{i} + \sum_{i \notin S} \exp(\hat{\beta}_{0} + \hat{\beta}_{1} \log(X_{ij}) + \hat{v}_{i}) \text{ (simple)}$ Log-Normal Back Transformation $\hat{Y}_{i}^{EB} = N_{i} (f_{i} \bar{y}_{i} + (1 - f_{i}) \hat{Y}_{ir}, \text{ where}$ $\hat{\bar{Y}}_{ir} = \hat{\alpha}_{ir} \exp(\hat{v}_{i} + \frac{1}{2} (\hat{\sigma}^{2} + \hat{\delta}_{i}^{2})), \text{and } \hat{\alpha}_{ir} = (N_{i} - n_{i})^{-1} \sum_{j \notin S_{i}} \exp(\hat{\beta}_{0} + \hat{\beta}_{1} \log(X_{ij}))$

# 2.2 Parametric Bootstrap

As mentioned above, the universe is the intersection of the 2007 CoG: E and 2012 CoG: E. One thousand samples (sample replicates) were drawn from the universe using the 2014 ASPEP sample design. From each replicate, 200 samples (parametric bootstrap samples) were generated from the EBLUP model. There are 29 small areas (29 function codes-see Appendix) in each bootstrap sample. As a result, we have the average of relative root mean square errors being compared with simulated true root mean square error. The steps of the analysis are described as below. Hereafter, the notations for the indices are:

k is for sample replicates, i for small area i<sup>th</sup>, b for bootstrap b<sup>th</sup>, and j is unit j<sup>th</sup>

# Step 1

Create 1,000 sample replicates,  $S_1, S_2, ..., S_{1000}$  using production sample design

#### Step2

With each  $S_k$  run a mixed model to generate the estimates of the parameters denoted as  $\hat{\theta}_{k,i} = (\hat{\tau}_k^2, \hat{\sigma}_k^2, \hat{\beta}_{k,0}, \hat{\beta}_{k,1}, \hat{v}_{k,i})$ .  $\hat{\tau}_k^2, \hat{\sigma}_k^2, \hat{\beta}_{k,0}, \hat{\beta}_{k,1}$  are sample dependent,  $\hat{v}_{k,i}$  is sample and function code dependent.

#### Step 3

For each  $S_k$  using  $\hat{\theta}_{k,i} = (\hat{\tau}_k^2, \hat{\sigma}_k^2, \hat{\beta}_{k,0}, \hat{\beta}_{k,1}, \hat{v}_{ki})$  generate B (B= 200) parametric bootstrap samples  $S_{k,1}, S_{k,2}, \dots, S_{k,200}$ 

#### Step 4

Perform mixed model on  $S_{k,b}$  where all the units in the sample now are  $y_{kij}^{(b)}$  to obtain  $\hat{\theta}_{kbi} = \left(\hat{\tau}_{k}^{2(b)}, \hat{\sigma}_{k}^{2(b)}, \hat{\beta}_{k,0}^{(b)}, \hat{\beta}_{k,1}^{(b)}, \hat{v}_{ki}^{(b)}\right)$  for each bootstrap  $b^{th}$  sample from the sample replicate k.

#### Step 5

Then estimate the EB for area  $i^{th}$  for each bootstrap  $b^{th}$  from the sample replicate  $k^{th}$ .  $\hat{Y}_{k,i}^{EB,(b)} = N_i (f_i \bar{y}_{ki}^{(b)} + (1 - f_i)) \hat{Y}_{k,ir}^{(b)}$ ) where  $\hat{Y}_{k,ir}^{(b)} = \exp(\hat{\beta}_{k,0}^{(b)} + \hat{\beta}_{k,1}^{(b)} \log(\bar{X}_i) + \hat{v}_{ki}^{(b)} + \frac{1}{2}(\hat{\sigma}_{k}^{2} + \hat{\delta}_{k,i}^{2,(b)}))$ , and  $\hat{\delta}_{k,i}^{2,(b)} = Var(v_{k,i}^{(b)} | data)$ ,  $f_i$  is the sampling rate in each small area.

#### Step 6

Compute the parametric bootstrap mse estimate for the area  $i^{th}$  for each  $S_k$  as follows:  $\widehat{mse}_{k,i} = \frac{1}{B} \sum_{b=1}^{B} (\widehat{Y}_{k,i}^{EB,(b)} - Y_{k,i}^{(b)})^2$ There are 500 bootstrap samples in each  $S_k$  produce 29 (item codes)  $\widehat{mse}_{k,i}$ . In total, we have 29\*1000  $\widehat{mse}_{k,i}$ 

Where 
$$Y_{k,i}^{(b)} = \exp(\hat{\beta}_{k,0} + \hat{\beta}_{k,1}\log(\bar{X}_i) + v_{ki}^{(b)})$$
,  
 $\hat{\beta}_{k,0}, \hat{\beta}_{k,1}$  are obtained from step 2, and  $v_{ki}^{(b)}$  is obtained in step 3.

### Step 7

Approximate the true mse for each function code *i*.  $truemse_i = \frac{1}{1000} \sum_{k=1}^{1000} (\hat{Y}_{k,i}^{EB} - Y_i)^2$ where  $\hat{Y}_{k,i}^{EB}$  is obtained the same manner as step 5 but now applied on sample replicate,  $Y_i$ is the true total in the universe for small area *i*<sup>th</sup>. We have 29 *truemse<sub>i</sub>*.

## Step 8

Evaluation: Performance of parametric bootstrap mse to approximate the true mse, where

Percent Relative Bias:  $RB_i = 100 * \frac{\frac{1}{1000} \sum_{k=1}^{1000} \widehat{mse}_{k,i} - truemse_i}{truemse_i}$ Percent Relative Root MSE  $RRMSE_i = 100 * \frac{\sqrt{\frac{1}{1000} \sum_{k=1}^{1000} (\widehat{mse}_{ki} - truemse_i)^2}}{truemse_i}$ 

# **3. Results & Evaluations**

Figure 1 and Figure 2 show the data for California before and after log transform respectively, and Figure 3 shows the normality of the residuals after the transformation. As you can see the normality of the residuals confirms the validity of the model assumption. Figure 4 shows that the unit-covariate model outperforms the area-covariate model in terms of the relative *mse*.

Table 1 and 2 are the main work of this paper that shows the performance of the parametric bootstrap when using area-covariate, and unit-covariate models.

Figure 1: Skewed Data (California Data 2007)



Figure 2: The Data after Log Transformed



Figure 3 shows the distribution of the residuals after log transformation. As we can see the normality assumption in the model is satisfied very well.





Figure 4 is comparing the two *mse* estimates between the Area-covariate and Unit-covariate models.





True MSE: Area vs Unit Covariate

Table 1:	Statistics for RRMSE of 29 Small Areas compared to Design-based True
	RRMSE if the True Total of the Area Known (Area-covariate model)

								Design-based
Function's Description	Function	Min	mean	Median	Max	25%	75%	RRMSE
Air Transportation	001	0.96%	1.40%	1.33%	3.31%	1.23%	1.45%	0.57%
Correction	005	1.00%	1.82%	1.66%	4.76%	1.51%	1.88%	0.46%
Elementary and Secondary - Instruction	012	4.47%	11.3%	9.25%	42.4%	7.92%	11.3%	4.45%
Higher Education-Other	016	2.08%	3.02%	2.92%	6.13%	2.72%	3.19%	2.35%
Higher Education - Instructional	018	1.94%	2.90%	2.83%	5.17%	2.65%	3.09%	0.54%
Financial Administration	023	4.20%	8.72%	8.30%	17.7%	7.07%	9.81%	5.13%
Firefighters	024	1.32%	2.87%	2.51%	7.37%	2.21%	2.98%	1.27%
Judical & Legal	025	2.23%	4.76%	4.64%	9.07%	3.96%	5.29%	2.21%
Other Government Administration	029	5.03%	11.5%	10.4%	20.0%	9.25%	12.7%	6.51%
Health	032	4.88%	6.47%	6.50%	10.1%	6.23%	6.77%	2.30%
Hospitals	040	0.24%	0.66%	0.54%	2.51%	0.46%	0.63%	0.68%
Highways	044	7.06%	15.8%	13.8%	35.1%	12.2%	16.8%	<mark>8.56%</mark>
Housing & Community Development	050	0.32%	0.74%	0.63%	2.04%	0.53%	0.80%	0.25%
Libraries	052	5.06%	10.4%	9.58%	28.6%	8.41%	11.3%	6.07%
Natural Resources	059	4.66%	8.30%	7.73%	23.1%	6.82%	9.01%	8.11%
Parks & Recreation	061	3.42%	6.72%	6.36%	14.6%	5.66%	7.74%	2.70%
Police Protection - Officers	062	0.91%	2.08%	1.88%	6.35%	1.65%	2.23%	1.37%
Public Welfare	079	1.52%	3.23%	3.08%	7.91%	2.76%	3.47%	1.05%
Sewerage	080	2.31%	6.69%	6.56%	13.8%	5.61%	7.76%	2.09%
Solid Waste Management	081	1.99%	3.76%	3.60%	7.50%	3.19%	4.29%	1.29%
Water Transport & Terminals	087	1.24%	5.51%	3.08%	11.7%	2.58%	11.7%	5.35%
All Other & Unallocable	089	3.77%	5.35%	5.24%	7.39%	4.82%	5.89%	1.85%
Water Supply	091	4.65%	9.60%	9.09%	18.8%	8.15%	10.8%	4.51%
Transit	094	0.03%	0.08%	0.07%	0.17%	0.06%	0.08%	0.03%
Elementary and Secondary-Other	112	3.81%	11.3%	8.87%	47.3%	7.55%	11.1%	8.09%
Fire-Other	124	1.13%	5.62%	5.31%	16.3%	4.37%	6.50%	3.02%
Police-Other	162	1.38%	3.41%	3.13%	7.68%	2.73%	3.86%	1.00%

Function's Description	Function	Min	mean	Median	Max	25%	75%	Design-based True RRMSE
Air Transportation	001	0.61%	1.63%	1.62%	2.35%	1.54%	1.69%	0.89%
Correction	005	1.01%	1.73%	1.70%	2.75%	1.49%	1.95%	0.43%
Elementary and Secondary - Instruction	012	4.74%	11.8%	11.6%	19.5%	9.64%	14.0%	3.54%
Higher Education-Other	016	0.56%	2.76%	2.70%	4.08%	2.48%	3.01%	1.59%
Higher Education - Instructional	018	2.18%	3.11%	3.05%	4.40%	2.82%	3.35%	0.85%
Financial Administration	023	1.03%	4.28%	4.18%	8.61%	3.17%	5.35%	1.72%
Firefighters	024	0.46%	1.35%	1.33%	3.56%	1.05%	1.61%	0.31%
Judical & Legal	025	0.63%	2.35%	2.32%	4.80%	1.76%	2.90%	0.64%
Other Government Administration	029	1.48%	5.03%	4.97%	10.1%	3.94%	6.11%	2.16%
Health	032	3.84%	4.74%	4.74%	6.32%	4.51%	4.96%	0.79%
Hospitals	040	0.18%	0.51%	0.50%	0.82%	0.43%	0.60%	0.18%
Highways	044	1.34%	5.92%	5.84%	12.1%	4.47%	7.35%	2.16%
Housing & Community Development	050	0.15%	0.44%	0.39%	1.16%	0.31%	0.48%	0.14%
Libraries	052	1.46%	4.86%	4.80%	9.95%	3.73%	5.82%	1.32%
Natural Resources	059	4.54%	6.93%	6.81%	12.4%	6.30%	7.42%	6.56%
Parks & Recreation	061	0.94%	3.43%	3.39%	7.19%	2.74%	4.08%	1.16%
Police Protection - Officers	062	0.44%	1.20%	1.19%	2.43%	0.93%	1.46%	0.34%
Public Welfare	079	2.37%	4.00%	3.98%	5.83%	3.49%	4.45%	0.87%
Sewerage	080	1.14%	3.55%	3.46%	9.64%	2.75%	4.35%	1.33%
Solid Waste Management	081	0.71%	1.99%	1.97%	4.48%	1.58%	2.38%	0.53%
Water Transport & Terminals	087	0.41%	2.28%	2.32%	5.96%	2.07%	2.59%	0.53%
All Other & Unallocable	089	2.12%	3.27%	3.24%	5.55%	2.98%	3.54%	0.73%
Water Supply	091	1.15%	4.43%	4.33%	11.0%	3.36%	5.43%	1.11%
Electric Power	092	3.74%	11.9%	11.4%	23.3%	9.94%	13.5%	3.62%
Gas Supply	093	8.73%	17.7%	15.8%	74.0%	14.4%	17.4%	15.1%
Transit	094	0.02%	0.05%	0.05%	0.10%	0.04%	0.06%	0.02%
Elementary and Secondary-Other	112	1.89%	7.44%	7.30%	15.8%	5.39%	9.41%	3.12%
Fire-Other	124	0.79%	2.35%	2.17%	5.78%	1.59%	2.92%	1.20%
Police-Other	162	0.68%	1.89%	1.81%	4.52%	1.39%	2.32%	0.82%

 Table 2: Statistics for RRMSE of 29 Small Areas compared to Design-based True RRMSE if the True Total of the Area Known (Unit-covariate model)

# Conclusion

As we can see, the unit covariate model produces better estimates of employment totals than the corresponding area-covariate model. Also, unit-covariate model produces parametric bootstrap MSE estimates closer to the true design-based MSE than the corresponding area-covariate model. The parametric bootstrap MSE overestimates the true design-based MSE. The increases of the number of sample replicates and/or the number of parametric bootstrap samples may help to increase the estimates of the RRMSEs from parametric bootstrap method. In practice, many times we do not always have unit-level auxiliary variable information, area-covariate model is good to use. In the future, we will examine performance of double bootstrap or some suitable adjustments to the single bootstrap formula.

#### Acknowledgement

I have great pleasure in expressing my profound gratitude to my research guide from Dr. Lahiri, University of Maryland JPSM, College Park, U.S.A for his encouragement, guidance and valuable suggestions throughout the research in Small Area Estimation.

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# Appendix

Function	Descriptions
000	Totals for Government
001	Air Transportation
002	Space Research & Technology (Federal)
005	Correction
006	Nat Defense & International Relations (Federal)
012	Elementary and Secondary - Instruction
014	Postal Service (Fed)
016	Higher Education - Other
018	Higher Education - Instructional
021	Other Education (State)
022	Social Insurance Administration (State)
023	Financial Administration
024	Firefighters
025	Judicial & Legal
029	Other Government Administration
032	Health
040	Hospitals
044	Highways
050	Housing & Community Development
052	Libraries
059	Natural Resources
061	Parks & Recreation
062	Police Protection - Officers
079	Public Welfare
080	Sewerage
081	Solid Waste Management
087	Water Transport & Terminals
089	All Other & Unallocable
090	Liquor Stores (State)
091	Water Supply
092	Electric Power
093	Gas Supply
094	Transit
112	Elementary and Secondary - Other
124	Fire - Other
162	Police-Other