# A Comparison of Design-Based and Calibrated Bayes Estimates Using Data from a Health Survey

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**Abstract**
Bayesian methods have been gaining popularity as an alternative to the traditional design-based methods for estimation from complex surveys. In this paper, we apply calibrated Bayes methods to estimate vaccination rates from the National Immunization Survey (NIS). NIS is a large telephone survey, which has been continuously conducted to monitor childhood vaccination coverage among U.S. children aged 19-35 months since 1994 (http://www.cdc.gov/nchs/nis.htm). Official design-based vaccination coverage rates at the national, state, and selected urban area levels estimates using data from the NIS are available at the Website http://www.cdc.gov/vaccines/stats-surv/nis/default.htm #nis. Data from the recent NIS public-use files are used to compute and compare the Bayesian estimates with the design-based estimates. We also compare subdomain estimates based on the two methods by selected demographic characteristics.

Key Words: Bayesian, complex surveys, sample weights, NIS

## 1. Background

Bayesian methods have been gaining popularity as an alternative to the traditional design-based methods. The objective of this research is to explore an application of calibrated Bayes methods (Little, 2011) to complex surveys for inferences. We apply calibrated Bayes methods to compute estimates from a large complex health survey and evaluate the precision and bias in estimates by comparing these estimates with the corresponding design-based estimates.

The Centers for Disease Control and Prevention (CDC) conducts the National Immunization Survey (NIS) to monitor vaccination coverage among pre-school children ages 19- 35 months living in the United States. The NIS is a large list-assisted random-digit-dialing telephone survey of households followed by a mail survey of children's vaccination providers. In addition to collecting information on children's vaccination histories, the survey also collects socio-economic and demographic information on sampled children and their parents; provider's contact information and consent to contact child's vaccination provider(s) are also collected in the NIS. Generally, official unbiased estimates of childhood vaccination coverage rates are computed using design-based estimation methods for complex surveys. Several statistical software packages are commercially available to compute design-based estimates from complex surveys.

One of the objectives of the NIS is to provide vaccination coverage estimates for subpopulations of interest described by socio-demographic characteristics such as

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race/ethnicity and poverty status. Identifying groups of children with low vaccination coverage by socio-economic, demographic, and geographic areas can help in identifying specific subgroups of children for intervention. In turn, this may lead to better tailored interventions to improve vaccination coverage among those subgroups (Community Preventive Services Task Force, 2013). However, some subdomain estimates cannot be obtained using design-based estimation procedures due to small sample sizes (say n<30) from large national surveys. Increasing the sample size would be beneficial, but may be impossible or difficult from practical standpoints and may be cost prohibitive. In this paper, we apply calibrated Bayes methods to estimate vaccination coverage rates from the NIS. We also apply Bayesian methodology to explore and compare subdomain estimates by selected socio-demographic characteristics using the two methods.

## 2. Data and Methods

### 2.1 Data

We used data from the public-use 2010 NIS data file ($DHHS_a$, 2013). The overall CASRO (The Council of American Survey Research Organization) response rate for the 2010 NIS was 63.76% (a product of the resolution rate * the screening rate * the interview completion rate); 23,605 age-eligible children had completed household interviews and of those, 16,798 children (71.2%) had adequate provider data. The phrase 'adequate provider data' means that sufficient vaccination history information was obtained from children's vaccination provider(s) to determine whether the child is up-to-date (UTD) with respect to the recommended vaccine schedule. The provider level data included information on 16,798 vaccinated and unvaccinated children to compute official design-based estimates of vaccination coverage rates. ($DHHS_b$, 2013)

The 2010 NIS is based on a single stage stratified sample design. The strata are defined by geographic areas called 'estimation areas' comprising of 50 states, the District of Columbia, and selected large metropolitan areas. In each stratum, telephone numbers in each exchange area are randomly selected to identify residential numbers and households. All age-eligible children from a screened household are included in the sample. The 2010 NIS provider-phase sampling weights PROVWT are used to obtain population estimates. These sampling weights are adjusted to account for non-resolution of residential /non-residential/non-working status of a telephone number, nonresponse to the screener and household interviews, number of telephone lines in the household, noncoverage of households that do not have landline telephones and non-response by vaccination providers.

### 2.2 The 4:3:1:3:3:1 Vaccine Series

We wanted to estimate **p**, the UTD status of the vaccine series known as 4:3:1:3:3:1. A child is considered UTD for the 4:3:1:3:3:1 vaccine series if s/he has received all of the recommended numbers of doses of each recommended vaccine listed below:

- 4+DTP : 4 or more doses of diphtheria, tetanus toxoids and acellular pertussis vaccines (DTP/DT/DTaP)
- 3+Polio: 3 or more doses of poliovirus vaccine
- 1+MCV: 1 or more doses of measles-containing vaccine, including MMR
- 3+Hib: 3 or more doses of *Haemophilus Influenzae* type b (Hib) vaccine.
- 3+HepB: 3 or more doses of hepatitis B vaccine

- 1+Var: 1 or more doses of varicella vaccine received at or after age 12 months

Vaccination coverage estimates were produced for 60 geographic strata with sample sizes ranging from 179 to 382 children with adequate provider data. The official design-based vaccination coverage rates for the 4:3:1:3:3:1 vaccine series ranged from 85.8% in Florida to 61.2% in Idaho across 60 geographic areas with an overall national coverage of 74.9%. Estimates were also derived for race/ethnicity and poverty status subdomains within each estimation area. The subdomain estimates were combined to produce national Bayesian estimates, and for overall race/ethnicity and poverty status.

**2.3 Calibrated Bayes**

Bayesian methods are applied as an alternative to the traditional design-based methods for simulations and drawing inferences from complex surveys. As mentioned by Box (1980), Rubin (1984) and Little (2003, 2006, 2011), calibrated Bayesian methods lead to model-based inferences that are Bayesian but the models are chosen carefully from complex surveys to yield inferences with good frequentist properties and incorporate the sample design structure (i.e., stratification and sampling weights with adjustments for noncoverage and nonresponse).

Basically, in Bayesian methods, estimates are drawn from a posterior distribution as defined below:

Posterior distribution

= Prior distribution * Likelihood function,

where the prior distribution could be non-informative (assuming some baseline probability distribution) or informative using models based estimates from a complex survey. We used an approximation to the weighted likelihood function to obtain a posterior distribution to account for the complex sample design and weighting structure. We also used effective sample sizes to account for variation in complex sample estimates.

**2.3.1 The Prior Probability Distribution**

Recall that we are interested in finding the proportion **p** of the population, which is the proportion of children UTD for the 4:3:1:3:3:1 vaccine series within each geographic area or subdomain. For example, **p** could be the proportion of Non-Hispanic Black children aged 19-35 months living in Connecticut whom are UTD on the 4:3:1:3:3:1 series or the proportion of children aged 19-35 months living in Idaho whom are UTD on the 4:3:1:3:3:1 series.

For the Bayesian analysis (Bolstad, 2007), we used three prior distributions (two non-informative and one informative with a specific distribution):

- The uniform flat prior--------- **Beta (1,1):** $\boldsymbol{\pi(\mathbf{p}) = 1}$
- The Jeffrey's prior---------- **Beta** $\boldsymbol{(\frac{1}{2},\frac{1}{2})}$**:** $\boldsymbol{\pi(\mathbf{p}) = \frac{1}{\pi}\mathbf{p}^{-\frac{1}{2}}(1-\mathbf{p})^{-1}}$
- The informative $\text{Beta}(\alpha,\beta)$ prior----**Beta (3, 1):** $\boldsymbol{\pi(\mathbf{p}) = \frac{1}{3}\mathbf{p}^{2}}$.

where $\boldsymbol{\pi(p) = p^{\alpha-1}(1-p)^{\beta-1}\frac{1}{Beta(\alpha,\beta)}}$ is the beta probability function. We computed vaccination coverage by estimation area, poverty status, race/ethnicity, and for the nation and compared the results.

### 2.3.2 The Likelihood Function

For the $i^{th}$ child in geographic area or stratum s, let

$$\mathbf{y_{si}} = \begin{cases} \mathbf{1}, & \text{if UTD on 4:3:1:3:3:1 vaccine series} \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

Also, let $\mathbf{N_s}$ be the population and $\mathbf{n_s}$ be the sample sizes of the stratum $\mathbf{s}$. The $\mathbf{y_{s1}, \dots, y_{sN_s}}$ are independent, identically distributed and each distribution given $\mathbf{p}$ is $\mathbf{Bernoulli}$ $(\mathbf{p})$. Let

$$\mathbf{y_s} = \sum_{i=1}^{N_s} \mathbf{y_{si}} = \begin{array}{c}\text{Total number of children} \\ \text{who are UTD on 4:3:1:3:3:1} \\ \text{vaccine series in stratum } \mathbf{s}\end{array}$$

The likelihood for the population data $\mathbf{y_S} = \left(\mathbf{y_{s1}, \dots, y_{sN_s}}\right)$ given $\mathbf{p_s}$ is defined as

$$\boldsymbol{\mathcal{L}(y_s \mid p_s) = \prod_{i=1}^{N_s} p_s{}^{y_{si}}(1-p_s)^{1-y_{si}} \ for\ 0 \le p_s \le 1.}$$

The weighted likelihood function for the observed sample data using the sampling weights and effective sample sizes to account for the design features of the NIS data, is defined as

$$\boldsymbol{\mathcal{L}(y_s \mid p_s) = \prod_{i=1}^{n_s} \left[p_s{}^{y_{si}}(1-p_s)^{1-y_{si}}\right]^{w_{si}} \ for\ 0 \le p_s \le 1.}$$

where

$$\mathbf{w_{si}} = \frac{\mathbf{PROVWT_{si}} \times \mathbf{n_{seff}}}{\sum_{i=1}^{n_s} \mathbf{PROVWT_{si}}}$$

Here $\mathbf{w_{si}}$ is the normalized sampling weight for the $\mathbf{i^{th}}$ child, from geographic stratum $\mathbf{s}$ and $\mathbf{PROVWT_{si}}$ is the provider-phase sampling weight for the $\mathbf{i^{th}}$ child, in geographic stratum $\mathbf{s}$ and $\mathbf{n_{seff}}$ is the effective sample size for the stratum $\mathbf{s}$. We used the effective sample size to account for the variance in the original NIS estimates.

The log of weighted likelihood is then

$$\begin{aligned}\log \mathcal{L}(\mathbf{y_s} \mid \mathbf{p_s}) &= \log\left[\prod_{i=1}^{n_s}\left[p_s^{y_{si}}(1-p_s)^{1-y_{si}}\right]^{w_{si}}\right] \\ &= \sum_{i=1}^{n_s} w_{si}\ y_{si}\ \log p_s + w_{si}\,(1-y_{si})\log(1-p_s).\end{aligned}$$

### 2.3.3 The Posterior Distribution

The posterior distribution is then proportional to the prior times the likelihood function

$$\mathbf{g}(\mathbf{p_s}|\mathbf{y_s}) \propto \boldsymbol{\pi}(\mathbf{p_s}) \times \boldsymbol{\mathcal{L}}(\mathbf{y_s}|\mathbf{p_s})$$

which is the shape of the posterior as a function of $\mathbf{p_s}$.

For example, for Beta (3,1) the shape of the posterior distribution as a function of $\mathbf{p_s}$ is

$$\mathbf{g}(\mathbf{p_s}|\mathbf{y_s}) \propto \frac{1}{3}\mathbf{p_s}^{2}\mathbf{p_s}^{w_s y_s}(\mathbf{1}-\mathbf{p_s})^{w_s(n_s-y_s)}$$
$$\propto \mathbf{p_s}^{(w_s y_s+3)-1}(\mathbf{1}-\mathbf{p_s})^{(w_s(n_s-y_s)+1)-1}$$

where $\mathbf{w_s y_s} = \sum_{i=1}^{n_s} \mathbf{w_{si}}\ \mathbf{y_{si}}$ and $\mathbf{w_s(n_S - y_S)} = \sum_{i=1}^{n_s} \mathbf{w_{si}(n_s - \sum_{i=1}^{n_s} y_{si})}$.

The exact Beta estimates were computed using $\alpha = (w_s y_s + 3)$ and $\beta = (w_s(n_s - y_s) + 1)$ (Gelman, 2004, Appendix A).

**2.3.5 Bayesian Inference for Stratified Random Sampling**

The procedure to obtain the point estimate $\hat{p}_s$, the proportion of children who are UTD on 4:3:1:3:3:1 in stratum s, using the calibrated Bayes method is as follows. For the $i^{th}$ child in geographic area, or strata s recall

$$\mathbf{y_{si}} = \begin{cases} \mathbf{1}, & \text{if child is UTD on 4:3:1:3:3:1 vaccine series} \\ \mathbf{0}, & \text{otherwise} \end{cases}$$

Individual posterior strata means $\widehat{\mathbf{p}}_\mathbf{s}$ are derived using the Bayesian method. The posterior mean for the national 4:3:1:3:3:1 estimate, $\widehat{\mathbf{P}}$, and its variance is computed using stratum level variances from a $\text{Beta}(\alpha, \beta)$ distribution (Gelman et. al., 2004, Appendix A):

$$\widehat{\mathbf{P}} = \sum_s \mathbf{W_s}\, \widehat{\mathbf{p}}_\mathbf{s}$$

where $\mathbf{W_s} = \frac{\sum_{i=1}^{n_s} \mathbf{PROVWT_{si}}}{\sum_s \sum_{i=1}^{n_s} \mathbf{PROVWT_{si}}}$, is treated as a fixed constant

and $\mathbf{var}(\widehat{\mathbf{P}}) = \sum \mathbf{W_s}^{2}\, \mathbf{var}(\mathbf{p_s}|\mathbf{y_s})$

**2.3.5 Markov Chain Monte Carlo Methods (MCMC) Simulation**

For this research we could have computed the exact estimates using a Beta posterior distribution, however, for future model-based research using sample covariates, we wanted to use the numerical integration using MCMC simulation method. Hence, to implement the calibrated Bayes method for this research we used Proc MCMC in SAS 9.3 (SAS, 2010) with Gibbs sampling to create Markov chains that modeled the desired distribution. We initialized $\boldsymbol{p}$ at 0.5. In each stratum (i.e., in each estimation area), a separate simulation was run. A distinct random seed was chosen at the beginning of each simulation and for each simulation, we ran 10,000 iterations; first 1,000 iterations were used for burn-in.

**2.3.6 Subdomain Estimates**

To estimate the proportion of children who are UTD on the 4:3:1:3:3:1 series for a certain subgroup of the population, such as Non-Hispanic Black children, we use the same method as in 2.3.1-2.3.3 with the addition that the $\boldsymbol{y_s}$ be a member of category c. Let

$$\boldsymbol{\delta_{si}}^{(c)} = \begin{cases} \mathbf{1}, & \textit{if the } i^{th} \textit{ child in stratum } s \\ & \textit{is a member of category } c \\ \mathbf{0}, & \textit{otherwise} \end{cases}$$

Then

$$\mathbf{n_s^{(c)}} = \sum_{\mathbf{i=1}}^{\mathbf{n_a}} \boldsymbol{\delta_{si}}^{(c)} = \begin{matrix} \text{the number of children in stratum } \mathbf{s} \\ \text{who are a member of category } \mathbf{c} \end{matrix}$$

and $\mathbf{y_s}$ can be expressed as

$$\mathbf{y_s} = \sum_{\mathbf{i=1}}^{\mathbf{n_s}} \boldsymbol{\delta_{si}}^{(c)} \mathbf{y_{si}} = \begin{matrix} \text{observed number of children} \\ \text{who are UTD on 4: 3: 1: 3: 3: 1} \\ \text{vaccine series in stratum } \mathbf{s}\text{, category } \mathbf{c} \end{matrix}$$

The normalized sampling weight for the $\mathbf{i^{th}}$ child, from geographic stratum **s** in category **c** is then

$$\mathbf{w_{si}^{(c)}} = \frac{\boldsymbol{\delta_{si}}^{(c)} \mathbf{PROVWT_{si}} \mathbf{n_{seff}^{(c)}}}{\sum_{\mathbf{i=1}}^{\mathbf{n_s}} \boldsymbol{\delta_{si}}^{(c)} \mathbf{PROVWT_{si}}},$$

where $\mathbf{n_{seff}^{(c)}}$ is the effective sample size for the stratum **s,** category **c**.

Let $\hat{\mathbf{p}}_\mathbf{s}^{(\mathbf{c})}$ be the proportion of children who are UTD on the 4:3:1:3:3:1 vaccine series in stratum **s**, category **c** estimated using the Bayesian methods described in previous sections. The variance $\mathbf{var}\left(\mathbf{p_s^{(c)}} \middle| \mathbf{y_s^{(c)}}\right)$ is computed using a Beta$(\alpha, \beta)$ distribution (Gelman et. al., 2004, Appendix A). The national estimate for category **c** is defined as

$$\hat{\mathbf{P}}^{(\mathbf{c})} = \sum_{\mathbf{s}} \mathbf{W_s^{(c)}}\, \hat{\mathbf{p}}_\mathbf{s}^{(\mathbf{c})}$$

where

$$\mathbf{W_s^{(c)}} = \frac{\sum_{\mathbf{i=1}}^{\mathbf{n_s}} \boldsymbol{\delta_{si}}^{(c)} \mathbf{PROVWT_{si}}}{\sum_{\mathbf{s}} \sum_{\mathbf{i=1}}^{\mathbf{n_s}} \boldsymbol{\delta_{si}}^{(c)} \mathbf{PROVWT_{si}}}$$

and $\mathbf{var}\left(\hat{\mathbf{P}}^{(\mathbf{c})}\right) = \sum \mathbf{W_s^{(c)^2}}\, \mathbf{var}\left(\mathbf{p_s^{(c)}} \middle| \mathbf{y_s^{(c)}}\right)$

## 3. Results

### 3.1 By Geographical Area (Stratum)

The published NIS design-based and calibrated Bayes estimates using the three different priors (Beta (.5,.5), Beta (1,1), Beta(3,1)), and their standard errors, are detailed in the Appendix, Table 1. The table includes estimates for the 60 geographic areas. The strata level estimates are combined to derive the national estimate for the 4:3:1:3:3:1 vaccine series as 74.7%, 74.6%, and 74.9%, using the three Beta priors, respectively (Table 1); the corresponding design-based national NIS estimate is 74.9%. At the geographic area level, all three Bayesian methods produced estimates within -0.47% to 0.27% of the corresponding NIS design-based estimates. Assuming the published NIS estimates are true values, estimates using the Beta (.5,.5) and Beta(1,1) priors resulted in lower than

NIS estimates (most differences are below zero reference line). Appendix Figure 1 shows that the bias or difference in Bayesian estimates using the Beta (3,1) prior is uniformly closer to the zero reference line than the methods using the other priors.

Using the effective sample sizes to compute variances and standard errors, the resulting Bayesian estimates are similar to those from the design-based method. Overall, Bayesian estimates with the informative Beta (3, 1) prior gave the smallest mean of the standard errors (3.08) compared to the other two methods (Appendix Table 2). However, Beta (1,1) prior had the lowest inter-quartile range of the standard errors (1.61) across the estimation areas.

Appendix Figure 2 compares the three prior distributions using Beta (.5,.5), Beta (1,1), and Beta(3,1), and the corresponding posterior distributions for the District of Columbia, Connecticut, and Idaho (areas are selected arbitrarily). The vertical line indicates the published NIS design-based estimate. The charts show that the shape of the posterior distributions for each of the selected areas (which are also from a Beta distribution) are similar for each of the three priors. The jagged area in the middle of the curves is due to the numerical integration used in the simulations and could be made smoother using a larger number of iterations (e.g., 20,000 or 50,000) or using the exact Beta distributions. It seems that the three prior distributions have little effect on the posterior distribution in these areas with large sample sizes.

**3.2 By Race/Ethnicity**

Appendix Table 4 shows the NIS design-based and the calibrated Bayes estimates [using Beta (3,1) prior] for race/ethnicity subdomains for a few selected geographical areas. NIS design-based estimates are not published (NA) if the unweighted sample size for the denominator is <30 or the (confidence interval half width)/Estimate > 0.588 or the (confidence interval half width) >10. Calibrated Bayes estimates are not produced (NA) if the effective sample size is 0.

A large proportion of state level geographical areas do not have design-based estimates available for Hispanics and Non-Hispanic Blacks. The calibrated Bayesian method was able to produce estimates for most areas with all three Beta priors. The estimates using the three priors were similar and the standard errors were very close. Similar to design-based estimates, Bayesian estimates with very small sample sizes also resulted in very large standard errors and were inconsistent across the three Beta priors. Note that these Bayesian estimates are presented only for this methodology research and should not be used as the official estimates.

The mean of the standard errors for Non-Hispanic Whites, Non-Hispanic Blacks and Hispanic was the lowest using the Beta (3,1) prior at 4.09, 9.86, and 7.29, respectively (see Table 4). For Non-Hispanic Blacks, Bayesian estimates using the Beta (.5,.5) prior yielded the largest standard error of 28.17 for the area of Alaska (effective sample size=1). Across all races/ethnicities, Bayesian estimation using the Beta (.5,.5) prior resulted in the largest standard errors compared to using the other two priors. The standard errors are large due to small sample sizes for each ethnic group within a single geographic area.

Appendix Figures 3 and 4 show the shape of the posterior densities from MCMC simulations using Beta (3,1) prior (jagged middle curve) and the smooth distribution using the exact Beta distribution for the estimates of the 4:3:1:3:3:1 vaccine series by

race/ethnicity for the state of Arkansas. The shape of the posterior for estimation of Non-Hispanic Black with a small effective sample size of 6 is left skewed. As the sample size increases for the Hispanic race/ethnicity group (effective sample size = 26), the posterior distribution becomes more bell shaped.

### 3.3 By Poverty Status

The results of the calibrated Bayes estimates by poverty status in selected geographical area are shown in Appendix Table 5. The table also includes the NIS design-based estimates and the unweighted sample sizes. For at or above poverty level, the calibrated Bayes method using all three priors produced estimates within 0.6 of the published NIS estimates. Methods using Beta (.5,.5) and Beta (1,1) were positively biased with the mean bias of 0.16 and 0.34, respectively. The method using Beta (3,1) had the lowest average bias (0.01). For the national estimate Beta (3,1) was the closest to the published NIS estimate of 75.5 (0.7) and had the smallest standard error 75.5 (0.7).

The majority of the below poverty level estimates by geographic area were not available for the 2010 NIS. The estimate was marked as 'NA' if the unweighted sample size for the denominator is <30 or the (confidence interval half width)/Estimate > 0.588 or if the (confidence interval half width) >10. The results using the Bayesian methods show that all three Beta Priors resulted in similar point estimates where estimates were label 'NA' for the 2010 NIS. Note that these Bayesian estimates are presented only for this methodology research and should not be used as the official estimates.

## 4. Limitations

We used three prior distributions (two non-informative and one informative) for this initial methodology research. This research needs further evaluation by including informative priors based on models using various covariates from multiple years of the NIS child-level data.

Furthermore, we did not have any estimate available to benchmark the state level subdomain estimates. We need to explore the availability of those estimates from individual states. Also, we used only one year of NIS data for subdomain estimates resulting in extremely small subpopulation sample sizes; combining multiple years of data could increase the stability and power of this analysis. Problems, including estimates for sub-state geographic areas, could also be handled by using model-based empirical Bayes approach (e.g. prior distribution based on association of vaccination with race/ethnicity controlling for socio-demographic factors). However, using Bayesian or empirical Bayesian methods to provide estimates for subdomains with insufficient sample sizes for frequentist estimates raises some practical questions, including: 1) can they be explained to users, e.g., state/local immunization program managers so they are accepted and trusted?; 2) how will these estimates be used, and what level of bias and precision may be acceptable?

Little (2006) discussed several strengths and weaknesses of the Bayesian method (section 4, and Table 2 of section 5) in general and of the calibrated Bayes methods. However, it is still not clear on how to incorporate the sample design structure into the analysis. For non-Bayesian researchers, additional detail guidelines, examples, and analytic software are needed to implement this methodology in complex surveys and to compare the results with design-based estimates.

## 5. Conclusion

We applied calibrated Bayes methods separately for each stratum defined by 60 geographic areas to preserve design-based stratification and used a weighted Likelihood function with normalized weights and effective sample sizes to account for the NIS sample design and weighting structures. We used two non-informative Bernoulli priors and a Beta (3, 1) prior to evaluate methods and compare estimates. Note that these Bayesian estimates are presented only for this methodology research and should not be used as the official estimates.

Overall, the Beta (1,1) and Beta(.5,.5) methods seem to slightly underestimate the vaccination coverage while the Beta(3,1) seems to perform slightly better. When large sample sizes were available, properly calibrated Bayesian estimates using the three priors were similar to the corresponding design-based estimates or to the exact Beta estimates and standard errors (even without using any additional modeling or strong prior information). We used stratum level proportional weights to combine 60 geographic area estimates to obtain national estimate.

For subdomain estimates with small sample sizes (say n< 15 or 20), both design-based and calibrated Bayes methods performed poorly with very large standard errors.

The primary advantages of using the Bayesian methods was that it provided plausible subdomain estimates using Beta posterior distribution where design-based estimates were suppressed with 'NA'. However, the reliability of the Bayesian estimates was not verified. These estimates can change with different prior distributions. We need to further explore and benchmark those subpopulation estimates to individual state level estimates, if available, from the state health departments or from other available sources.

## References


Bolstad, WM. Introduction to Bayesian Statistics. 2nd ed. New York: Wiley. 2007

Box, GEP "Sampling and Bayes inference in scientific modelling and robustness" (with discussion), *JRSSA* 143, 383-430. 1980.

Community Preventive Services Task Force (2013). Guide to Community Preventive Services: Increasing Appropriate Vaccination (http://www.thecommunityguide.org/vaccines/index.html).

Gelman, A, Carlin, GB, Stern, HS, and Rubin, DB. Bayesian Data Analysis, 2nd ed. Chapman & Hall, CDC. 2004

Little, RJA. The Bayesian Approach to Sample Survey Inference. In Analysis of Survey Data, R.L. Chambers and C.J. Skinner (eds). New York: Wiley, 49–57. 2003.

Little, RJA. (2006). Calibrated Bayes: A Bayes/Frequentist Roadmap. *The American Statistician*, 60, 3, 213-223. {64}

Little, RJA. (2011). Calibrated Bayes, for Statistics in General, and Missing Data in Particular (with Discussion and Rejoinder). *Statistical Science* 26, 2, 162-186.

Rubin, DB (1984), "Bayesianly justifiable and relevant frequency calculations for the applied statistician", *Annals of Statistics* 12, 1151-1172.

SAS Institute Inc. (2010), SAS Software for Windows, Version 9.3, Cary, NC: SAS Institute Inc.

U.S. Department of Health and Human Services ($DHHS_a$). National Center for Health Statistics. The 2010 National Immunization Survey. Hyattsville, MD: Centers for Disease Control and Prevention, 2011. http://www.cdc.gov/nchs/nis/data_files.htm. September 2013

U.S. Department of Health and Human Services ($DHHS_b$). The 2010 National Immunization Survey: A User's Guide for the 2010 Public-Use Data File. Centers for Disease Control and Prevention. ftp://ftp.cdc.gov/pub/Health_Statistics/ NCHS/ Dataset_Documentation /NIS/NISPUF11_DUG.PDF. September 2013

U.S. Vaccination Coverage Reported via NIS. Centers for Disease Control and Prevention. http://www.cdc.gov/vaccines/stats-surv/nis/default.htm#nis. September 2013

# Appendix

**Figure 1.Percent difference between NIS design-based and calibrated Bayes estimates using three priors by estimation area for vaccine series 4:3:1:3:3:1 (Bayes-NIS), 2010**

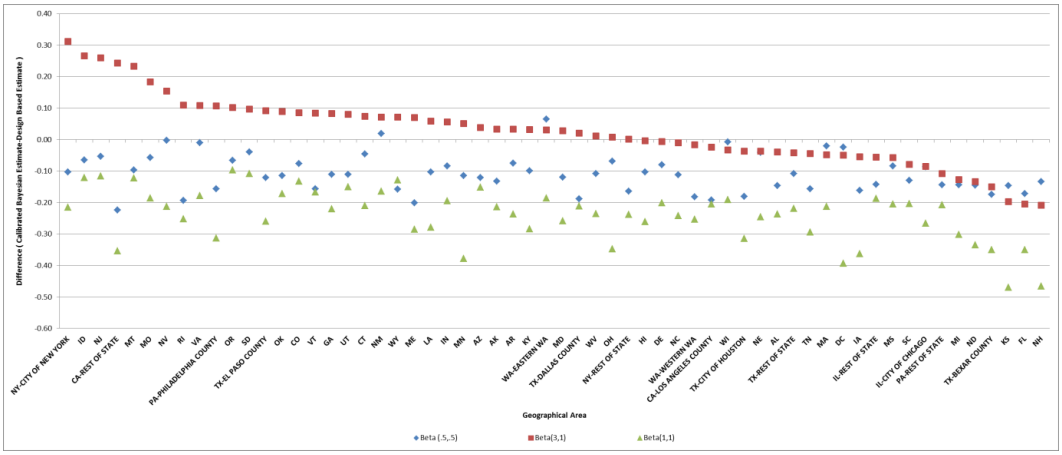
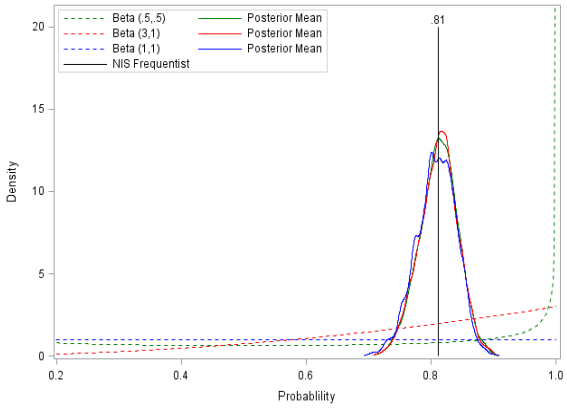
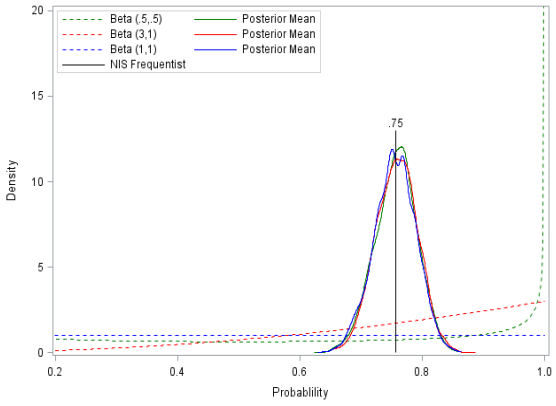



**Figure 2. Prior and posterior distributions for Bayesian estimation of vaccine series 4:3:1:3:3:1 coverage for selected geographical areas.**

District of Columbia ($n_{eff}$=151)




Connecticut ($n_{eff}$=164)




Idaho ($n_{eff}$=204)

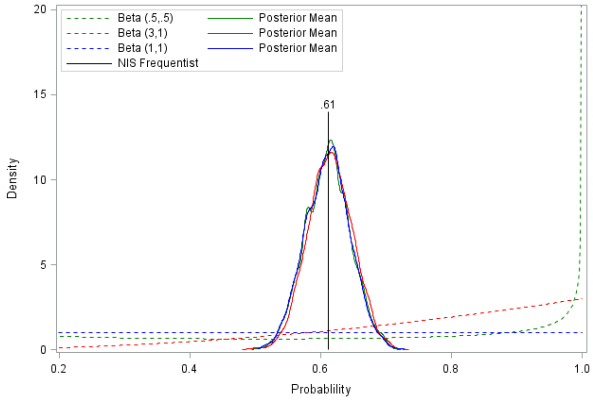


**Figure 3. Posterior densities and calibrated Bayesian estimates for vaccine Series 4:3:1:3:3:1 using Beta (3,1) Prior by race/ethnicity, Arkansas**

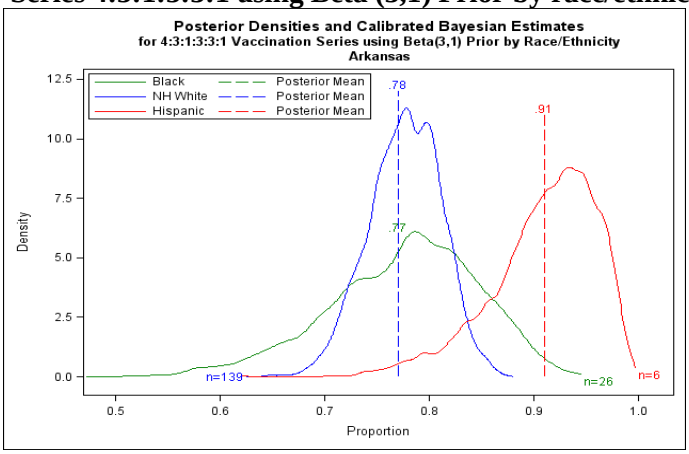



**Figure 4. Posterior densities for vaccine series 4:3:1:3:3:1 using Exact Beta$(\alpha, \beta)$ distribution by race/ethnicity, Arkansas**

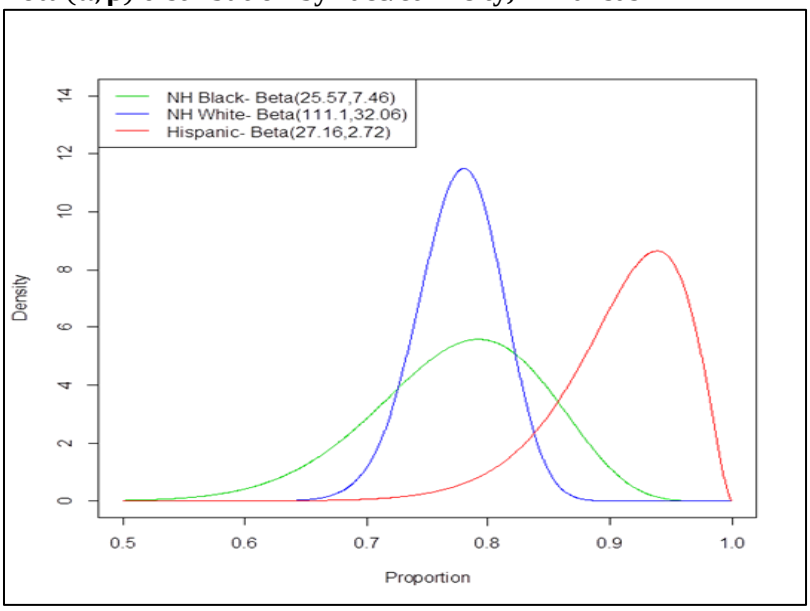


**Table 1. Comparison of estimated vaccination coverage of 4:3:1:3:3:1[†] among children aged 19-35 months by state and local areas using design-based and Bayesian methods , 2010 National Immunization Survey[§]**

| Estimation Area | NIS Sample Size, n | NIS Design-based* | Bayesian Beta (.5,.5) | Bayesian Beta (1,1) | Bayesian Beta (3,1) |
|---|---|---|---|---|---|
| **US National** | 16798 | 74.9 (0.6) | 74.4 (0.6) | 74.6 (0.6) | 74.9 (0.6) |
| **Alabama** | 336 | 77.3 (2.6) | 77.2 (2.5) | 77.1 (2.6) | 77.3 (2.6) |
| **Alaska** | 269 | 70.2 (3.1) | 70.1 (3.2) | 70.0 (3.1) | 70.2 (3.0) |
| **Arizona** | 299 | 76.3 (3.0) | 76.2 (3.0) | 76.2 (3.0) | 76.3 (2.9) |
| **Arkansas** | 341 | 79.3 (2.8) | 79.3 (2.8) | 79.1 (2.8) | 79.4 (2.7) |
| **CA-LOS ANGELES COUNTY** | 232 | 80.0 (2.9) | 79.8 (2.9) | 79.8 (2.8) | 80.0 (2.8) |
| **CA-REST OF STATE** | 179 | 68.1 (4.2) | 67.9 (4.2) | 67.7 (4.1) | 68.3 (4.2) |
| **Colorado** | 353 | 71.3 (3.1) | 71.3 (3.1) | 71.2 (3.0) | 71.4 (3.0) |
| **Connecticut** | 247 | 75.7 (3.5) | 75.7 (3.4) | 75.5 (3.4) | 75.8 (3.4) |
| **Delaware** | 347 | 72.9 (2.8) | 72.9 (2.8) | 72.7 (2.8) | 72.9 (2.9) |
| **District of Columbia** | 260 | 81.2 (3.1) | 81.2 (3.1) | 80.8 (3.2) | 81.2 (3.1) |
| **Florida** | 276 | 85.8 (2.6) | 85.7 (2.6) | 85.5 (2.6) | 85.6 (2.5) |
| **Georgia** | 306 | 73.9 (3.1) | 73.8 (3.1) | 73.7 (3.0) | 74.0 (3.1) |
| **Hawaii** | 323 | 76.0 (3.0) | 75.9 (2.9) | 75.7 (3.0) | 76.0 (2.9) |
| **Idaho** | 352 | 61.2 (3.4) | 61.1 (3.5) | 61.1 (3.4) | 61.5 (3.4) |
| **IL-CITY OF CHICAGO** | 305 | 76.5 (2.7) | 76.4 (2.7) | 76.2 (2.7) | 76.4 (2.7) |
| **IL-REST OF STATE** | 271 | 75.7 (3.1) | 75.5 (3.2) | 75.5 (3.1) | 75.6 (3.1) |
| **Indiana** | 322 | 73.9 (2.9) | 73.8 (2.9) | 73.7 (2.9) | 74.0 (2.8) |
| **Iowa** | 252 | 77.3 (3.1) | 77.1 (3.0) | 76.9 (3.0) | 77.2 (3.0) |
| **Kansas** | 270 | 77.6 (3.3) | 77.4 (3.3) | 77.1 (3.3) | 77.4 (3.3) |
| **Kentucky** | 276 | 72.5 (3.2) | 72.4 (3.1) | 72.2 (3.1) | 72.6 (3.1) |
| **Louisiana** | 226 | 73.8 (3.5) | 73.7 (3.4) | 73.6 (3.4) | 73.9 (3.4) |
| **Maine** | 220 | 70.4 (3.7) | 70.2 (3.8) | 70.1 (3.8) | 70.5 (3.7) |
| **Maryland** | 306 | 73.3 (3.5) | 73.2 (3.5) | 73.0 (3.5) | 73.3 (3.5) |
| **Massachusetts** | 297 | 79.9 (3.0) | 79.9 (3.0) | 79.7 (3.0) | 79.9 (3.0) |
| **Michigan** | 270 | 83.4 (2.5) | 83.2 (2.6) | 83.1 (2.5) | 83.2 (2.5) |
| **Minnesota** | 269 | 74.3 (3.3) | 74.2 (3.3) | 74.0 (3.3) | 74.4 (3.3) |
| **Mississippi** | 323 | 79.3 (2.5) | 79.2 (2.5) | 79.1 (2.5) | 79.2 (2.5) |
| **Missouri** | 325 | 70.3 (3.0) | 70.3 (3.0) | 70.2 (3.0) | 70.5 (2.9) |
| **Montana** | 264 | 64.3 (3.5) | 64.2 (3.4) | 64.1 (3.5) | 64.5 (3.5) |
| **Nebraska** | 224 | 78.9 (3.0) | 78.8 (3.0) | 78.6 (3.0) | 78.8 (3.0) |
| **Nevada** | 230 | 66.6 (3.5) | 66.6 (3.5) | 66.4 (3.5) | 66.7 (3.5) |
| **New Hampshire** | 243 | 84.1 (3.1) | 84.0 (3.1) | 83.6 (3.2) | 83.9 (3.1) |
| **New Jersey** | 229 | 66.4 (3.5) | 66.4 (3.4) | 66.3 (3.4) | 66.7 (3.4) |
| **New Mexico** | 232 | 70.9 (3.0) | 70.9 (2.9) | 70.7 (3.0) | 71.0 (2.9) |
| **North Carolina** | 332 | 77.0 (2.7) | 76.8 (2.7) | 76.7 (2.7) | 77.0 (2.7) |
| **North Dakota** | 318 | 76.0 (2.9) | 75.9 (2.9) | 75.7 (2.9) | 75.9 (2.9) |
| **NY-CITY OF NEW YORK** | 310 | 65.1 (4.0) | 65.0 (4.0) | 64.9 (3.9) | 65.4 (3.8) |
| **NY-REST OF STATE** | 253 | 73.0 (3.6) | 72.9 (3.6) | 72.8 (3.6) | 73.0 (3.5) |
| **Ohio** | 243 | 76.0 (3.4) | 75.9 (3.4) | 75.6 (3.4) | 76.0 (3.3) |
| **Oklahoma** | 326 | 70.3 (3.2) | 70.2 (3.2) | 70.1 (3.2) | 70.4 (3.2) |
| **Oregon** | 273 | 69.3 (3.2) | 69.2 (3.1) | 69.2 (3.2) | 69.4 (3.1) |
| **PA-PHILADELPHIA COUNTY** | 268 | 74.3 (3.1) | 74.2 (3.0) | 74.0 (3.2) | 74.4 (3.0) |
| **PA-REST OF STATE** | 294 | 79.6 (2.7) | 79.5 (2.6) | 79.4 (2.7) | 79.5 (2.6) |
| **Rhode Island** | 231 | 75.3 (3.5) | 75.1 (3.6) | 75.0 (3.6) | 75.4 (3.5) |
| **South Carolina** | 300 | 77.7 (2.7) | 77.5 (2.8) | 77.5 (2.7) | 77.6 (2.8) |
| **South Dakota** | 285 | 73.2 (3.1) | 73.2 (3.1) | 73.1 (3.1) | 73.3 (3.1) |
| **Tennessee** | 319 | 82.3 (2.6) | 82.2 (2.6) | 82.0 (2.7) | 82.3 (2.6) |
| **TX-BEXAR COUNTY** | 277 | 78.4 (3.0) | 78.2 (3.0) | 78.1 (2.9) | 78.3 (2.9) |
| **TX-CITY OF HOUSTON** | 252 | 74.5 (3.1) | 74.3 (3.1) | 74.2 (3.1) | 74.4 (3.0) |
| **TX-DALLAS COUNTY** | 201 | 72.6 (3.4) | 72.5 (3.3) | 72.4 (3.3) | 72.7 (3.3) |
| **TX-EL PASO COUNTY** | 330 | 72.6 (3.5) | 72.5 (3.5) | 72.3 (3.5) | 72.7 (3.5) |
| **TX-REST OF STATE** | 277 | 74.9 (2.9) | 74.8 (2.9) | 74.7 (2.8) | 74.9 (2.8) |
| **Utah** | 348 | 70.6 (3.3) | 70.4 (3.3) | 70.4 (3.2) | 70.6 (3.2) |
| **Vermont** | 235 | 71.0 (3.3) | 70.8 (3.4) | 70.8 (3.4) | 71.1 (3.3) |
| **Virginia** | 382 | 74.2 (3.1) | 74.2 (3.1) | 74.0 (3.0) | 74.3 (3.0) |
| **WA-EASTERN WA** | 206 | 79.1 (3.1) | 79.1 (3.1) | 78.9 (3.0) | 79.1 (3.0) |
| **WA-WESTERN WA** | 221 | 72.0 (3.5) | 71.8 (3.5) | 71.8 (3.5) | 72.0 (3.5) |
| **West Virginia** | 304 | 73.0 (2.9) | 72.9 (2.9) | 72.8 (2.9) | 73.0 (2.8) |
| **Wisconsin** | 296 | 81.7 (2.7) | 81.7 (2.7) | 81.5 (2.7) | 81.7 (2.7) |
| **Wyoming** | 243 | 67.5 (3.6) | 67.3 (3.5) | 67.3 (3.4) | 67.5 (3.5) |

† 4 or more doses of DTaP, 3 or more doses of poliovirus vaccine, 1 or more doses of any MMR vaccine, 3 or more doses of Hib vaccine of any type, 3 or more doses of HepB vaccine, and 1 or more doses of varicella vaccine.

§ Children in the Q1/2010-Q4/2010 National Immunization Survey were born from January 07 through July 09.

*Official NIS 2010 Estimates; Bayesian estimate are only for methodology research.

**Table 2. Summary statistics of standard errors for vaccine series 4:3:1:3:3:1**

| Summary Statistics | NIS Design-Based | Beta (.5,.5) | Beta (1,1) | Beta (3,1) |
|---|---|---|---|---|
| Mean | 3.14 | 3.11 | 3.12 | 3.08 |
| Min | 2.51 | 2.50 | 2.53 | 2.48 |
| Max | 4.19 | 4.15 | 4.14 | 4.17 |
| Interquartile Range | 1.68 | 1.65 | 1.61 | 1.69 |

**Table 3. Summary statistics of standard errors for vaccine series 4:3:1:3:3:1 by race/ethnicity**

| Summary Statistics | Non-Hispanic White | | | |
|---|---|---|---|---|
| | NIS Design-Based | Beta (.5,.5) | Beta(1,1) | Beta(3,1) |
| Mean | 3.81 | 4.19 | 4.19 | 4.1 |
| Min | 2.70 | 2.80 | 2.78 | 2.7 |
| Max | 5.10 | 8.18 | 8.49 | 7.7 |
| | **Non-Hispanic Black** | | | |
| | NIS Design-Based | Beta (.5,.5) | Beta(1,1) | Beta(3,1) |
| Mean | 4.46 | 11.51 | 11.12 | 9.86 |
| Min | 3.90 | 4.01 | 3.99 | 3.95 |
| Max | 5.00 | 28.17 | 24.55 | 18.09 |
| | **Hispanic** | | | |
| | NIS Design-Based | Beta (.5,.5) | Beta(1,1) | Beta(3,1) |
| Mean | 3.93 | 7.94 | 7.86 | 7.3 |
| Min | 3.42 | 2.49 | 2.99 | 2.9 |
| Max | 4.44 | 20.21 | 18.96 | 14.9 |

**Table 4. Examples of estimated vaccination coverage* of 4:3:1:3:3:1† series among children aged 19-35 months by race/ethnicity‡ and by selected states using design-based and calibrated Bayes methods, 2010 National Immunization Survey§**

| Estimation Area | Non-Hispanic White | | | Non- Hispanic Black | | | Hispanic | | |
|---|---|---|---|---|---|---|---|---|---|
| | NIS Sample Size, n | NIS** Design-based | Bayesian Beta (3,1) | NIS Sample Size, n | NIS** Design-based | Bayesian Beta (3,1) | NIS Sample Size, n | NIS** Design-based | Bayesian Beta (3,1) |
| **US National** | 10,505 | 73.6 (0.8) | 73.8(0.8) | 1,589 | 74.5 (1.6) | 71.8(1.6) | 2,980 | 77.2 (1.4) | 77.0(1.4) |
| **Alabama** | 214 | 80.2 (3.0) | 80.0 (2.9) | 77 | **NA** | 67.8 (5.7) | 22 | **NA** | 96.1 (2.9) |
| **Alaska** | 145 | 71.1 (4.1) | 71.2 (4.0) | 2 | **NA** | 72.8 (18.1) | 13 | **NA** | 51.0 (12.3) |
| **Arizona** | 150 | 72.3 (4.6) | 72.3 (4.6) | 6 | **NA** | **NA** | 111 | 80.4 (4.3) | 80.3 (4.2) |
| **Arkansas** | 253 | 77.7 (3.5) | 77.6 (3.5) | 39 | **NA** | 77.5 (7.2) | 27 | **NA** | 91.0 (5.1) |
| **Colorado** | 235 | 68.2 (4.0) | 68.5 (3.9) | 12 | **NA** | **NA** | 80 | **NA** | 72.5 (5.3) |
| **Connecticut** | 176 | 77.7 (3.9) | 77.6 (4.0) | 18 | **NA** | 69.2 (10.2) | 31 | **NA** | 72.8 (8.3) |
| **Delaware** | 194 | 72.0 (4.1) | 72.1 (3.9) | 51 | 91.4 (3.9) | 90.3 (4.0) | 60 | **NA** | 57.5 (7.0) |
| **Florida** | 140 | 84.9 (3.8) | 84.5 (3.8) | 33 | **NA** | 81.6 (7.9) | 76 | 88.2 (3.7) | 87.5 (3.7) |
| **Georgia** | 165 | 80.9 (3.9) | 80.8 (3.8) | 83 | **NA** | 73.7 (5.4) | 37 | **NA** | 63.1 (7.7) |
| **Hawaii** | 48 | **NA** | 62.3 (7.7) | 8 | **NA** | **NA** | 42 | **NA** | 75.2 (6.7) |
| **Maryland** | 178 | 74.7 (4.6) | 74.8 (4.5) | 56 | **NA** | 70.9 (6.4) | 33 | **NA** | 83.3 (7.2) |
| **Michigan** | 200 | 80.9 (3.3) | 80.7 (3.2) | 29 | **NA** | 91.7 (4.5) | 16 | **NA** | **NA** |
| **Mississippi** | 179 | 78.8 (3.4) | 78.6 (3.3) | 118 | 80.4 (4.0) | 80.2 (4.0) | 13 | **NA** | **NA** |
| **Missouri** | 246 | 70.8 (3.5) | 70.9 (3.5) | 30 | NA | 80.1 (7.0) | 20 | **NA** | 71.1 (9.5) |
| **Nebraska** | 190 | 76.1 (3.7) | 76.1 (3.7) | 7 | **NA** | **NA** | 29 | **NA** | 88.1 (6.0) |
| **Nevada** | 104 | 70.2 (5.0) | 70.6 (4.7) | 16 | **NA** | 67.0 (10.9) | 87 | **NA** | 68.4 (5.3) |
| **New Jersey** | 200 | 67.1 (4.5) | 67.4 (4.4) | 21 | **NA** | 62.9 (11.0) | 64 | **NA** | 70.6 (6.0) |
| **New Mexico** | 120 | **NA** | 61.5 (5.1) | 5 | **NA** | NA | 168 | 71.7 (4.0) | 71.7 (4.0) |
| **Virginia** | 266 | 73.6 (3.8) | 73.7 (3.8) | 42 | **NA** | 67.5 (8.1) | 31 | **NA** | 76.8 (7.5) |
| **Wisconsin** | 250 | 82.6 (2.7) | 82.6 (2.7) | 9 | **NA** | **NA** | 23 | **NA** | 88.3 (7.0) |
| **Wyoming** | 200 | 67.3 (3.9) | 67.4 (4.0) | 3 | **NA** | **NA** | 20 | **NA** | 72.9 (9.7) |

* Estimate=NA (Not Available) if the unweighted sample size for the denominator was <30 or (CI half width)/Estimate > 0.588 or (CI half width) >10. ;
Estimate= NA for Bayesian estimate if sample size <16; estimates for n>16 included for methodology research only.

† 4 or more doses of DTaP, 3 or more doses of poliovirus vaccine, 1 or more doses of any MMR vaccine, 3 or more doses of Hib vaccine of any type, 3 or more doses of HepB vaccine, and 1 or more doses of varicella vaccine.

‡ Self-reported by respondent. Children of Hispanic ethnicity may be of any race.

** Official NIS 2010 Estimates; Bayesian estimate are only for methodology research.

**Table 5. Examples of estimated vaccination coverage* of 4:3:1:3:3:1† series among children aged 19-35 months by poverty status and in selected states using design-based and calibrated Bayes methods, 2010 National Immunization Survey§**

| Estimation Area | At or Above Poverty | | | Below Poverty | | |
|---|---|---|---|---|---|---|
| | NIS Sample Size, n | NIS** Design-based | Bayesian Beta (3,1) | NIS Sample Size, n | NIS** Design-based | Bayesian Beta (3,1) |
| **US National** | 12552 | 75.5 (0.7) | 75.5 (0.7) | 3615 | 73.5 (1.2) | 73.5 (1.2) |
| **Alaska** | 214 | 68.6 (3.6) | 68.7 (3.5) | 45 | NA | 71.7 (3.4) |
| **Arizona** | 194 | 77.3 (3.8) | 77.1 (3.7) | 92 | 75.8 (4.8) | 75.7 (3.7) |
| **Arkansas** | 237 | 82.7 (2.8) | 82.6 (2.6) | 87 | NA | 72.6 (3.2) |
| **Colorado** | 297 | 71.0 (3.5) | 71.1 (3.4) | 47 | NA | 73.5 (3.3) |
| **Connecticut** | 220 | 79.1 (3.3) | 79.0 (3.2) | 21 | NA | 55.8 (3.9) |
| **Delaware** | 266 | 73.0 (3.3) | 73.0 (3.2) | 67 | NA | 78.0 (2.9) |
| **Florida** | 205 | 87.1 (2.6) | 86.8 (2.6) | 56 | NA | 81.1 (2.9) |
| **Georgia** | 209 | 69.0 (4.1) | 69.3 (3.8) | 80 | 79.3 (4.8) | 79.2 (3.3) |
| **Hawaii** | 253 | 77.0 (3.4) | 77.0 (3.3) | 56 | NA | 71.3 (3.5) |
| **Idaho** | 216 | 61.3 (3.7) | 61.5 (3.5) | 50 | NA | 61.6 (3.5) |
| **Maryland** | 259 | 78.8 (3.5) | 78.8 (3.3) | 31 | NA | 53.5 (4.1) |
| **Michigan** | 209 | 81.9 (3.1) | 81.8 (2.8) | 50 | 86.1 (4.9) | 85.9 (2.6) |
| **Minnesota** | 235 | 76.2 (3.4) | 76.1 (3.4) | 28 | NA | 66.0 (3.8) |
| **Mississippi** | 181 | 79.0 (3.3) | 78.9 (3.1) | 129 | 79.8 (3.9) | 79.6 (3.1) |
| **Missouri** | 249 | 74.8 (3.3) | 74.7 (3.1) | 64 | NA | 63.7 (3.5) |
| **Nebraska** | 201 | 77.2 (3.5) | 77.1 (3.5) | 36 | NA | 79.9 (3.3) |
| **Nevada** | 159 | 68.2 (4.2) | 68.5 (3.8) | 57 | NA | 65.6 (3.9) |
| **New Jersey** | 264 | 67.6 (3.8) | 67.8 (3.5) | 50 | NA | 66.2 (3.6) |
| **New Mexico** | 207 | 67.1 (3.9) | 67.2 (3.8) | 97 | 77.1 (4.7) | 77.1 (3.4) |
| **Vermont** | 202 | 73.7 (3.2) | 73.9 (3.1) | 28 | NA | 58.7 (3.5) |
| **West Virginia** | 214 | 78.5 (3.1) | 80.6 (3.5) | 81 | NA | 62.3 (3.5) |
| **Wyoming** | 212 | 65.4 (3.9) | 78.5 (3.0) | 25 | NA | 76.4 (3.4) |

* Estimate=NA (Not Available) if the unweighted sample size for the denominator was <30 or (CI half width)/Estimate > 0.588 or (CI half width) >10.

† Children in the Q1/2010-Q4/2010 National Immunization Survey were born from January 2007 through July 2009.

^ Poverty status was based 2009 U.S. Census poverty thresholds (available at http://www.census.gov/hhes/www/poverty.html).

**Official NIS 2010 Estimates; Bayesian estimate are only for methodology research.