POST-STRATIFICATION BASED ON THE CHOICE OF USE OF A QUANTITATIVE RANDOMIZATION DEVICE

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ABSTRACT

In this paper, we use the idea of post-stratification based on the respondents' choice of a particular randomization device in order to estimate the population mean of a sensitive quantitative variable. The proposed idea gives freedom to the respondents and is expected to result in greater cooperation from them as well as to provide some increase in the relative efficiency of the newly proposed estimator.

Keywords: Respondent's cooperation, post-stratification, quantitative sensitive variables, protection and efficiency.

1. INTRODUCTION

Warner (1965) proposed a structured interviewing/surveying technique, called Randomized Response Technique (RRT), to protect survey responder's privacy and to thus reduce a major source of bias (evasive answers or refusing to respond) in estimating the prevalence of sensitive characteristics in surveys of human populations.

An extensive amount of literature on the developments in the field of RRT can be found in Chaudhuri (2011) and Tracy and Mangat (1996). Singh and Kim (2011) pointed out that Mangat and Singh (1990) introduced a two-stage randomized response model. Leysiefer and Warner (1976), and Lanke (1975; 1976) studied different randomized response procedures at equal level of protection of the respondents, and later Nayak (1994), Bhargava (1996), Zou (1997), Bhargava and Singh (2001; 2002) and Moors (1997) found that Mangat and Singh (1990) and Warner (1965) models remain equally efficient at equal protection. While doing a face to face interview survey, experience shows that respondent's cooperation is as important as privacy/confidentiality. For one respondent, a question is too sensitive while for other respondent it is not sensitive. A respondent could prefer to the use of one randomization device over the other, thus freedom to choose a randomization device out of a couple of options of such devices, or the option of responding without the use of a randomization device, will certainly increase a respondent's cooperation.

In the next section, we suggest a new method to use randomization devices in real practice which may increase respondent cooperation in addition to some increase in the relative efficiency of the resultant estimator.

2. POST-STATIFICATION BASED ON THE CHOICE OF A QUANTITATIVE RANDOMIZATION DEVICE

Experience shows that in every face-to-face interview survey, there are respondents who are unwilling to treat the randomization device at hand but might be willing to trust a different device. Some respondents prefer to use the Additive model due to Himmelfarb and Edgell (1980), some prefer to use multiplicative model due Eichhorn and Hayre (1983), and some would be willing to respond directly without using any randomization device. Suppose we selected a simple random with replacement (SRSWR) sample of n respondents. To every selected respondent in the sample, we ask for his/her choice of method of answering out of the above four options. Assume out of n respondents: n_1 respondents prefer the Additive model, n_2 respondents prefer the Multiplicative model, and n_3 respondents prefer to respond directly without using any randomization device. Suppose $Z_{1i} = X_i + S_1,\ i = 1,2,3...,n_1$ be the scrambled responses obtained from n_1 respondents using the Additive model, $Z_{2i} = X_i S_2,\ i = 1,2,3...,n_2$ be the scrambled responses obtained from n_2 respondents using the Multiplicative model, and $Z_{3i} = X_i$, $i = 1,2,3...,n_3$ be the direct responses obtained from n_3 respondents by using no randomization device. Here S_1 and S_2 are two independent scrambling variables and their distributions are assumed to be known. In other words, $E_R(S_1) = \theta_1$, $E_R(S_2) = \theta_2$, $V_R(S_1) = \gamma_1^2$ and $V_R(S_2) = \gamma_2^2$ are assumed to be known. Here E_R and V_R denote respectively expectation and variance with respect to the randomization

device. Note that here we know preference of a respondent, unlike the optional randomization devices studied as listed in Chaudhuri (2011) and among others. If more cooperation is required then a researcher can use more options of choosing a randomization device as listed in Perri (2008), and Diana and Perri (2009, 2010).

Let $\hat{w}_1 = n_1/n$, $\hat{w}_2 = n_2/n$, and $\hat{w}_3 = n_3/n$ be the unbiased estimators of the true population proportions $W_1 = N_1/N$, $W_2 = N_2/N$, and $W_3 = N_3/N$ who prefer to use the Additive model, Multiplicative model and respond directly, respectively. Unfortunately the values of W_h, $h = 1,2,3$ remain unknown in a survey, thus their estimators $\hat{w}_h$ are used in constructing the estimators. The suffix h refers to the h[th] post-stratum. Also let $\mu_{x_1} = \frac{1}{N_1}\sum_{i=1}^{N_1} X_{1i}$ and $\sigma_{x_1}^2 = \frac{1}{N_1}\sum_{i=1}^{N_1}(X_{1i} - \mu_{x_1})^2$ be the population mean and variance of the sensitive variable of interest X in the first post-stratum; $\mu_{x_2} = \frac{1}{N_2}\sum_{i=1}^{N_2} X_{2i}$ and $\sigma_{x_2}^2 = \frac{1}{N_2}\sum_{i=1}^{N_2}(X_{2i} - \mu_{x_2})^2$ be the population mean and variance respectively of the sensitive variable X in the second post-stratum; and $\mu_{x_3} = \frac{1}{N_3}\sum_{i=1}^{N_3} X_{3i}$ and $\sigma_{x_3}^2 = \frac{1}{N_3}\sum_{i=1}^{N_3}(X_{3i} - \mu_{x_3})^2$ be the population mean and variance of the sensitive variable X in the third post-stratum.

Now we have the following theorems:

Theorem 2.1. An unbiased estimator of the population mean μ_x is given by:

$$\hat{\mu}_{\text{xp}} = \sum_{\text{h}=1}^{3} \hat{w}_h \hat{\mu}_{xh} \tag{2.1}$$

where $\hat{\mu}_{x1} = \frac{1}{n_1}\sum_{i=1}^{n_1} Z_{1i} - \theta_1$, $\hat{\mu}_{x2} = \frac{1}{n_2\theta_2}\sum_{i=1}^{n_2} Z_{2i}$ and $\hat{\mu}_{x3} = \frac{1}{n_3}\sum_{i=1}^{n_3} Z_{3i}$ such that $n_h > 0$ are defined.

Proof. Let E_2 denote the expected value for a given values of n_h, $h = 1,2,3$ over a given randomization device and E_1 is the expected value over all possible samples of size n, then we have:

$$E\left(\hat{\mu}_{\text{xp}}\right) = E_1E_2\left(\hat{\mu}_{\text{xp}} \mid n_h > 0\right) = E_1E_2\left[\sum_{\text{h}=1}^{3} \hat{w}_h \hat{\mu}_{xh} \mid n_h > 0\right]$$

$$= E_1\left[\sum_{\text{h}=1}^{3} \hat{w}_h E_2(\hat{\mu}_{xh} \mid n_h > 0)\right] = E_1\left[\sum_{\text{h}=1}^{3} \hat{w}_h \mu_{xh}\right]$$

$$= \left[\sum_{\text{h}=1}^{3} \mu_{xh} E_1(\hat{w}_h)\right] = \left[\sum_{\text{h}=1}^{3} \mu_{xh} W_h\right] = \mu_x$$

which proves the theorem.

Theorem 2.2. The variance of the unbiased estimator $\hat{\mu}_{\text{xp}}$ is given by:

$$V(\hat{\mu}_{xp}) = \frac{1}{n}\left(\sum_{h=1}^{3} \sigma_h^2 W_h\right) + \frac{1}{n}\sum_{h=1}^{3} W_h(\mu_{xh} - \mu_x)^2 \tag{2.2}$$

where

$$\sigma_1^2 = \sigma_{x_1}^2 + \gamma_1^2,\ \sigma_2^2 = \sigma_{x_2}^2\left(1 + \gamma_2^2 / \theta_2^2\right) + \mu_{x_2}^2 \text{ and } \sigma_3^2 = \sigma_{x_3}^2 .$$

Proof. Here we assume that the sample size is so large $P(n_h = 0) = 0$ (Stephen, 1945). Let V_2 denote the expected value for a given values of n_h, $h = 1,2,3$ over a given randomization device and V_1 is the expected values over all possible samples of size n, then we have

$$V(\hat{\mu}_{\text{xp}}) = E_1 V_2(\hat{\mu}_{xp} \mid n_h > 0) + V_1 E_2(\hat{\mu}_{xp} \mid n_h > 0)$$

$$= E_1 V_2\left(\sum_{h=1}^{3} \hat{w}_h \hat{\mu}_{xh} \mid n_h > 0\right) + V_1 E_2\left(\sum_{h=1}^{3} \hat{w}_h \hat{\mu}_{xh} \mid n_h > 0\right)$$

$$= E_1\left(\sum_{h=1}^{3} \hat{w}_h^2 V_2(\hat{\mu}_{xh} \mid n_h > 0)\right) + V_1\left(\sum_{h=1}^{3} \hat{w}_h E_2(\hat{\mu}_{xh} \mid n_h > 0)\right)$$

$$= E_1\left(\sum_{h=1}^{3} \hat{w}_h^2 \frac{\sigma_h^2}{n_h}\right) + V_1\left(\sum_{h=1}^{3} \hat{w}_h \mu_{xh}\right)$$

$$= \frac{1}{n} E_1\left(\sum_{h=1}^{3} \hat{w}_h \sigma_h^2\right) + \left(\sum_{h=1}^{3} \mu_{xh}^2 V_1(\hat{w}_h) + \sum_{h \neq h'=1}^{3}\sum^{3} \mu_{xh}\mu_{xh'} Cov_1(\hat{w}_h, \hat{w}_{h'})\right)$$

$$= \frac{1}{n}\left(\sum_{h=1}^{3} \sigma_h^2 E_1(\hat{w}_h)\right) + \frac{1}{n}\left[\sum_{h=1}^{3} \mu_{xh}^2 W_h(1 - W_h) - \sum_{h \neq h=1}^{3}\sum^{3} W_h W_{h'} \mu_{xh}\mu_{xh'}\right]$$

$$= \frac{1}{n}\left(\sum_{h=1}^{3} \sigma_h^2 E_1(\hat{w}_h)\right) + \frac{1}{n}\left[\sum_{h=1}^{3} \mu_{xh}^2 W_h - \left(\sum_{h=1}^{3} \mu_{xh} W_h\right)^2\right]$$

$$= \frac{1}{n}\left(\sum_{h=1}^{3} \sigma_h^2 E_1(\hat{w}_h)\right) + \frac{1}{n}\left[\sum_{h=1}^{3} \mu_{xh}^2 W_h - \mu_x^2\right], \text{where } \pi_x = \sum_{h=1}^{3} \pi_{xh} W_h$$

$$= \frac{1}{n}\left(\sum_{h=1}^{3} \sigma_h^2 W_h\right) + \frac{1}{n}\sum_{h=1}^{3} W_h(\mu_{xh} - \mu_x)^2$$

which proves the theorem.

Now we have the following corollary:

Corollary 2.1. An estimator of $V(\hat{\mu}_{\text{xp}})$ is suggested as:

$$\hat{V}\left(\hat{\mu}_{\text{xp}}\right)=\frac{1}{n}\sum_{h=1}^{3}\hat{w}_h\left[\hat{\sigma}_h^2+\left(\hat{\mu}_{xh}-\hat{\mu}_x\right)^2\right] \tag{2.3}$$

where $\hat{\sigma}_h^2$ is an unbiased estimator of σ_h^2 based on information in the h[th] post-stratum.

Corollary 2.2. If $W_1=1,\ W_2=W_3=0$ then the proposed estimator reduces to the additive model. The variance of the estimator based on the additive model is given by:

$$V\left(\hat{\mu}_{\text{add}}\right)=\frac{1}{n}\left(\sigma_x^2+\gamma_1^2\right) \tag{2.4}$$

where

$$\sigma_x^2=\sum_{h=1}^{3}W_h\sigma_{x_h}^2+\sum_{h=1}^{3}W_h\left(\mu_{x_h}-\mu_x\right)^2 \tag{2.5}$$

and

$$\mu_x=\sum_{h=1}^{3}W_h\mu_{x_h} \tag{2.6}$$

Corollary 2.3. If $W_1=0,\ W_2=1,\ W_3=0$ then the proposed estimator reduces to the multiplicative model. The variance of the estimator based on the multiplicative model is given by:

$$V\left(\hat{\mu}_{\text{mult}}\right)=\frac{1}{n}\left[\sigma_x^2\left(1+C_{\gamma 2}^2\right)+\mu_x^2\right] \tag{2.7}$$

In the next section, we consider the comparison of the proposed estimator with the other competitors considered above.

3. RELATIVE EFFICIENCY

We define the percent relative efficiency of the proposed estimator with respect to the additive model as:

$$\text{RE}(1)=\frac{V(\hat{\mu}_{add})}{V(\hat{\mu}_{xp})}\times 100 \tag{3.1}$$

and the percent relative efficiency of the proposed estimator with respect to the multiplicative model as:

$$\mathrm{RE}(2) = \frac{V(\hat{\mu}_{mult})}{V(\hat{\mu}_{xp})} \times 100 \qquad (3.2)$$

We simulated the percent relative efficiency values for different choice of parameters of the study variable and the randomization device. We have listed many results in the Appendix and we also see there is no end of such results based on a choice of parameters. We discuss only a few of these results as follows. Let $W_1 = 0.3$, $W_2 = 0.69$ and $W_3 = 0.01$, which means that 30% people in like additive model, 69% people like multiplicative model and only 1% people give direct response to a sensitive question in a population, then for $\gamma_1^2 = 169.3$, $C_{\gamma_2}^2 = 0.1$, $\mu_x = 22$, $\mu_{x_1} = 50$, $\mu_{x_2} = 10$, $\mu_{x_3} = 10$, $\sigma_{x_1}^2 = 2.5$, $\sigma_{x_2}^2 = 2.5$, $\sigma_{x3}^2 = 2.5$, and $\sigma_x^2 = 338.5$, the percent relative efficiency of the proposed estimator over the additive and multiplicative models is given by 110.8% and 186.8% respectively. Thus, from this we can see that if only 1% of the respondents agree to provide a direct response then the proposed method can be benefitted over the use of only either additive or the multiplicative model. In addition, due to many options for a respondent, more cooperation is expected.

REFERENCES

Bhargava, M. (1996). *An investigation into the efficiencies of certain randomized response strategies*. Unpublished Ph.D. thesis submitted to Punjab Agricultural University, Ludhiana, India.

Bhargava, M. and Singh, R. (2001). Efficiency comparison of certain randomized response schemes with U-model. *J. Indian Soc. Agric. Statist.*, 54(1), 19-28.

Bhargava, M. and Singh, R. (2002). On the efficiency comparison of certain randomized response strategies. *Metrika*, 55(3), 191-197.

Chaudhuri, A. (2011). *Randomized response and indirect questioning techniques in surveys.* A Chapman & Hall, CRC Press, Boca Raton, FL.

Diana, G. and Perri, P.F. (2009). Estimating a sensitive proportion through randomized response procedures based on auxiliary information. *Statistical Papers*, 50, 661-672.

Diana, G. and Perri, P.F. (2010). New scrambled response models for estimating the mean of a sensitive quantitative character. *Journal of Applied Statistics*, 37, 1875-1890.

Lanke, J. (1975). On the choice of the unrelated question in Simons version of randomized response. *J. Amer. Statist. Assoc.*, 70, 80--83.

Lanke, J. (1976). On the degree of protection in randomized interviews. *Int. Statist. Rev.*, 44, 197-203.

Leysieffer, F.W. and Warner, S. L. (1976). Respondent jeopardy and optimal designs in randomized response models. *J. Amer. Statist. Assoc*., 71, 649-656.

Mangat, N.S. and Singh, R. (1990). An alternative randomized response procedure. *Biometrika*, 77 (2): 439-442.

Moors, J.J.A. (1997). *A critical evaluation of Mangat's two-step procedure in randomized response*. Discussion paper at Center for Economic Research, Tilburg University, The Netherlands.

Nayak, T.K. (1994). On randomized response surveys for estimating a proportion. *Commun. Statist.-Theory Meth.*, 23(1), 3303-3321.

Perri, P.F. (2008). Modified randomized devices for Simmons' model. *Model Assisted Statistics and Applications*, 3(3), 233-239.

Singh, S. and Kim, Jong-Min (2011). Pseudo empirical log-likelihood estimator using scrambled responses. *Statistics and Probability Letters*, 81, 345-351.

Stephan, F.F. (1945). The expected value and variance of the reciprocal and other negative powers of a positive Bernoulli variate. *Ann. Math. Statist.*, 16, 50--61.

Tracy, D.S. and Mangat, N.S. (1996). Some developments in randomized response sampling during the last decade - A follow up of review by Chaudhuri and Mukerjee. *J. Applied Statist. Sci.*, 4(2/3),147-158.

Warner, S.L. (1965). Randomized response: a survey technique for eliminating evasive answer bias. *J. Amer. Stat. Assoc.*, 60, 63-69.

Zou, G. (1997). Two-stage randomized response procedures as single stage procedures. *Australian J. Statistics*, 39, 235-236.

APPENDIX

Table A.1. Relative efficiency of the proposed estimator with respect to the additive model and the multiplicative model for different choice of parameters.

W_1	W_2	W_3	γ_1^2	$C_{\gamma 2}^2$	μ_x	μ_{x_1}	μ_{x_2}	μ_{x_3}	$\sigma_{x_1}^2$	$\sigma_{x_2}^2$	$\sigma_{x_3}^2$	σ_x^2	$RE(1)$	$RE(2)$
0.3	0.69	0.01	169.3	0.1	22.0	50	10	10	2.5	2.5	2.5	338.5	110.8	186.8
0.3	0.69	0.01	169.3	0.3	22.0	50	10	10	2.5	2.5	2.5	338.5	110.7	201.4
0.3	0.69	0.01	169.3	0.5	22.0	50	10	10	2.5	2.5	2.5	338.5	110.6	216.0
0.3	0.69	0.01	169.3	0.1	22.0	50	10	10	2.5	2.5	4.0	338.5	110.8	186.8
0.3	0.69	0.01	169.3	0.3	22.0	50	10	10	2.5	2.5	4.0	338.5	110.7	201.4
0.3	0.69	0.01	169.3	0.5	22.0	50	10	10	2.5	2.5	4.0	338.5	110.6	216.0
0.3	0.69	0.01	169.8	0.1	22.0	50	10	10	2.5	4.0	2.5	339.5	110.8	186.5
0.3	0.69	0.01	169.8	0.3	22.0	50	10	10	2.5	4.0	2.5	339.5	110.7	201.0
0.3	0.69	0.01	169.8	0.5	22.0	50	10	10	2.5	4.0	2.5	339.5	110.5	215.5
0.3	0.69	0.01	169.8	0.1	22.0	50	10	10	2.5	4.0	4.0	339.6	110.8	186.5
0.3	0.69	0.01	169.8	0.3	22.0	50	10	10	2.5	4.0	4.0	339.6	110.7	201.0
0.3	0.69	0.01	169.8	0.5	22.0	50	10	10	2.5	4.0	4.0	339.6	110.5	215.5
0.3	0.69	0.01	169.5	0.1	22.0	50	10	10	4.0	2.5	2.5	339.0	110.8	186.7
0.3	0.69	0.01	169.5	0.3	22.0	50	10	10	4.0	2.5	2.5	339.0	110.7	201.3
0.3	0.69	0.01	169.5	0.5	22.0	50	10	10	4.0	2.5	2.5	339.0	110.6	215.9
0.3	0.69	0.01	169.5	0.1	22.0	50	10	10	4.0	2.5	4.0	339.0	110.8	186.7
0.3	0.69	0.01	169.5	0.3	22.0	50	10	10	4.0	2.5	4.0	339.0	110.7	201.3
0.3	0.69	0.01	169.5	0.5	22.0	50	10	10	4.0	2.5	4.0	339.0	110.6	215.9
0.3	0.69	0.01	170.0	0.1	22.0	50	10	10	4.0	4.0	2.5	340.0	110.8	186.4
0.3	0.69	0.01	170.0	0.3	22.0	50	10	10	4.0	4.0	2.5	340.0	110.7	201.0
0.3	0.69	0.01	170.0	0.5	22.0	50	10	10	4.0	4.0	2.5	340.0	110.5	215.4
0.3	0.69	0.01	170.0	0.1	22.0	50	10	10	4.0	4.0	4.0	340.0	110.8	186.4
0.3	0.69	0.01	170.0	0.3	22.0	50	10	10	4.0	4.0	4.0	340.0	110.7	200.9
0.3	0.69	0.01	170.0	0.5	22.0	50	10	10	4.0	4.0	4.0	340.0	110.5	215.4
0.3	0.69	0.01	168.5	0.1	22.1	50	10	20	2.5	2.5	2.5	337.1	110.7	188.1
0.3	0.69	0.01	168.5	0.3	22.1	50	10	20	2.5	2.5	2.5	337.1	110.6	202.7
0.3	0.69	0.01	168.5	0.5	22.1	50	10	20	2.5	2.5	2.5	337.1	110.5	217.3
0.3	0.69	0.01	168.6	0.1	22.1	50	10	20	2.5	2.5	4.0	337.1	110.7	188.1
0.3	0.69	0.01	168.6	0.3	22.1	50	10	20	2.5	2.5	4.0	337.1	110.6	202.7
0.3	0.69	0.01	168.6	0.5	22.1	50	10	20	2.5	2.5	4.0	337.1	110.5	217.3
0.3	0.69	0.01	169.1	0.1	22.1	50	10	20	2.5	4.0	2.5	338.1	110.7	187.8
0.3	0.69	0.01	169.1	0.3	22.1	50	10	20	2.5	4.0	2.5	338.1	110.6	202.3
0.3	0.69	0.01	169.1	0.5	22.1	50	10	20	2.5	4.0	2.5	338.1	110.4	216.8
0.3	0.69	0.01	169.1	0.1	22.1	50	10	20	2.5	4.0	4.0	338.1	110.7	187.8
0.3	0.69	0.01	169.1	0.3	22.1	50	10	20	2.5	4.0	4.0	338.1	110.6	202.3
0.3	0.69	0.01	169.1	0.5	22.1	50	10	20	2.5	4.0	4.0	338.1	110.5	216.8
0.3	0.69	0.01	168.8	0.1	22.1	50	10	20	4.0	2.5	2.5	337.5	110.7	188.0
0.3	0.69	0.01	168.8	0.3	22.1	50	10	20	4.0	2.5	2.5	337.5	110.6	202.6

0.3	0.69	0.01	168.8	0.5	22.1	50	10	20	4.0	2.5	2.5	337.5	110.5	217.2
0.3	0.69	0.01	168.8	0.1	22.1	50	10	20	4.0	2.5	4.0	337.6	110.7	188.0
0.3	0.69	0.01	168.8	0.3	22.1	50	10	20	4.0	2.5	4.0	337.6	110.6	202.6
0.3	0.69	0.01	168.8	0.5	22.1	50	10	20	4.0	2.5	4.0	337.6	110.5	217.2
0.3	0.69	0.01	169.3	0.1	22.1	50	10	20	4.0	4.0	2.5	338.6	110.7	187.7
0.3	0.69	0.01	169.3	0.3	22.1	50	10	20	4.0	4.0	2.5	338.6	110.6	202.2
0.3	0.69	0.01	169.3	0.5	22.1	50	10	20	4.0	4.0	2.5	338.6	110.5	216.7
0.3	0.69	0.01	169.3	0.1	22.1	50	10	20	4.0	4.0	4.0	338.6	110.7	187.7
0.3	0.69	0.01	169.3	0.3	22.1	50	10	20	4.0	4.0	4.0	338.6	110.6	202.2
0.3	0.69	0.01	169.3	0.5	22.1	50	10	20	4.0	4.0	4.0	338.6	110.5	216.7
0.3	0.69	0.01	168.8	0.1	22.2	50	10	30	2.5	2.5	2.5	337.7	110.7	188.9
0.3	0.69	0.01	168.8	0.3	22.2	50	10	30	2.5	2.5	2.5	337.7	110.6	203.5
0.3	0.69	0.01	168.8	0.5	22.2	50	10	30	2.5	2.5	2.5	337.7	110.6	218.1
0.3	0.69	0.01	168.8	0.1	22.2	50	10	30	2.5	2.5	4.0	337.7	110.7	188.9
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0.3	0.69	0.01	168.8	0.5	22.2	50	10	30	2.5	2.5	4.0	337.7	110.6	218.1
0.3	0.69	0.01	169.4	0.1	22.2	50	10	30	2.5	4.0	2.5	338.7	110.7	188.6
0.3	0.69	0.01	169.4	0.3	22.2	50	10	30	2.5	4.0	2.5	338.7	110.6	203.2
0.3	0.69	0.01	169.4	0.5	22.2	50	10	30	2.5	4.0	2.5	338.7	110.5	217.6
0.3	0.69	0.01	169.4	0.1	22.2	50	10	30	2.5	4.0	4.0	338.7	110.7	188.6
0.3	0.69	0.01	169.4	0.3	22.2	50	10	30	2.5	4.0	4.0	338.7	110.6	203.2
0.3	0.69	0.01	169.4	0.5	22.2	50	10	30	2.5	4.0	4.0	338.7	110.5	217.6
0.3	0.69	0.01	169.1	0.1	22.2	50	10	30	4.0	2.5	2.5	338.1	110.7	188.8
0.3	0.69	0.01	169.1	0.3	22.2	50	10	30	4.0	2.5	2.5	338.1	110.7	203.4
0.3	0.69	0.01	169.1	0.5	22.2	50	10	30	4.0	2.5	2.5	338.1	110.6	218.0
0.3	0.69	0.01	169.1	0.1	22.2	50	10	30	4.0	2.5	4.0	338.1	110.7	188.8
0.3	0.69	0.01	169.1	0.3	22.2	50	10	30	4.0	2.5	4.0	338.1	110.7	203.4
0.3	0.69	0.01	169.1	0.5	22.2	50	10	30	4.0	2.5	4.0	338.1	110.6	218.0
0.3	0.69	0.01	169.6	0.1	22.2	50	10	30	4.0	4.0	2.5	339.2	110.8	188.5
0.3	0.69	0.01	169.6	0.3	22.2	50	10	30	4.0	4.0	2.5	339.2	110.6	203.1
0.3	0.69	0.01	169.6	0.5	22.2	50	10	30	4.0	4.0	2.5	339.2	110.5	217.5
0.3	0.69	0.01	169.6	0.1	22.2	50	10	30	4.0	4.0	4.0	339.2	110.8	188.5
0.3	0.69	0.01	169.6	0.3	22.2	50	10	30	4.0	4.0	4.0	339.2	110.6	203.1
0.3	0.69	0.01	169.6	0.5	22.2	50	10	30	4.0	4.0	4.0	339.2	110.5	217.5
0.3	0.69	0.01	170.1	0.1	22.3	50	10	40	2.5	2.5	2.5	340.2	110.8	189.3
0.3	0.69	0.01	170.1	0.3	22.3	50	10	40	2.5	2.5	2.5	340.2	110.8	203.9
0.3	0.69	0.01	170.1	0.5	22.3	50	10	40	2.5	2.5	2.5	340.2	110.7	218.5
0.3	0.69	0.01	170.1	0.1	22.3	50	10	40	2.5	2.5	4.0	340.2	110.8	189.3
0.3	0.69	0.01	170.1	0.3	22.3	50	10	40	2.5	2.5	4.0	340.2	110.8	203.9
0.3	0.69	0.01	170.1	0.5	22.3	50	10	40	2.5	2.5	4.0	340.2	110.7	218.5
0.3	0.69	0.01	170.6	0.1	22.3	50	10	40	2.5	4.0	2.5	341.2	110.9	189.0
0.3	0.69	0.01	170.6	0.3	22.3	50	10	40	2.5	4.0	2.5	341.2	110.7	203.6
0.3	0.69	0.01	170.6	0.5	22.3	50	10	40	2.5	4.0	2.5	341.2	110.6	218.1

0.3	0.69	0.01	170.6	0.1	22.3	50	10	40	2.5	4.0	4.0	341.3	110.9	189.0
0.3	0.69	0.01	170.6	0.3	22.3	50	10	40	2.5	4.0	4.0	341.3	110.7	203.5
0.3	0.69	0.01	170.6	0.5	22.3	50	10	40	2.5	4.0	4.0	341.3	110.6	218.1
0.3	0.69	0.01	170.3	0.1	22.3	50	10	40	4.0	2.5	2.5	340.7	110.9	189.2
0.3	0.69	0.01	170.3	0.3	22.3	50	10	40	4.0	2.5	2.5	340.7	110.8	203.8
0.3	0.69	0.01	170.3	0.5	22.3	50	10	40	4.0	2.5	2.5	340.7	110.7	218.4
0.3	0.69	0.01	170.3	0.1	22.3	50	10	40	4.0	2.5	4.0	340.7	110.9	189.2
0.3	0.69	0.01	170.3	0.3	22.3	50	10	40	4.0	2.5	4.0	340.7	110.8	203.8
0.3	0.69	0.01	170.3	0.5	22.3	50	10	40	4.0	2.5	4.0	340.7	110.7	218.4
0.3	0.69	0.01	170.9	0.1	22.3	50	10	40	4.0	4.0	2.5	341.7	110.9	188.9
0.3	0.69	0.01	170.9	0.3	22.3	50	10	40	4.0	4.0	2.5	341.7	110.8	203.4
0.3	0.69	0.01	170.9	0.5	22.3	50	10	40	4.0	4.0	2.5	341.7	110.6	218.0
0.3	0.69	0.01	170.9	0.1	22.3	50	10	40	4.0	4.0	4.0	341.7	110.9	188.9
0.3	0.69	0.01	170.9	0.3	22.3	50	10	40	4.0	4.0	4.0	341.7	110.8	203.4
0.3	0.69	0.01	170.9	0.5	22.3	50	10	40	4.0	4.0	4.0	341.7	110.6	218.0
0.3	0.69	0.01	172.4	0.1	22.4	50	10	50	2.5	2.5	2.5	344.7	111.1	189.2
0.3	0.69	0.01	172.4	0.3	22.4	50	10	50	2.5	2.5	2.5	344.7	111.0	203.9
0.3	0.69	0.01	172.4	0.5	22.4	50	10	50	2.5	2.5	2.5	344.7	110.9	218.5
0.3	0.69	0.01	172.4	0.1	22.4	50	10	50	2.5	2.5	4.0	344.8	111.1	189.2
0.3	0.69	0.01	172.4	0.3	22.4	50	10	50	2.5	2.5	4.0	344.8	111.0	203.9
0.3	0.69	0.01	172.4	0.5	22.4	50	10	50	2.5	2.5	4.0	344.8	110.9	218.5
0.3	0.69	0.01	172.9	0.1	22.4	50	10	50	2.5	4.0	2.5	345.8	111.1	188.9
0.3	0.69	0.01	172.9	0.3	22.4	50	10	50	2.5	4.0	2.5	345.8	111.0	203.5
0.3	0.69	0.01	172.9	0.5	22.4	50	10	50	2.5	4.0	2.5	345.8	110.8	218.0
0.3	0.69	0.01	172.9	0.1	22.4	50	10	50	2.5	4.0	4.0	345.8	111.1	188.9
0.3	0.69	0.01	172.9	0.3	22.4	50	10	50	2.5	4.0	4.0	345.8	111.0	203.5
0.3	0.69	0.01	172.9	0.5	22.4	50	10	50	2.5	4.0	4.0	345.8	110.8	218.0
0.3	0.69	0.01	172.6	0.1	22.4	50	10	50	4.0	2.5	2.5	345.2	111.1	189.1
0.3	0.69	0.01	172.6	0.3	22.4	50	10	50	4.0	2.5	2.5	345.2	111.0	203.8
0.3	0.69	0.01	172.6	0.5	22.4	50	10	50	4.0	2.5	2.5	345.2	110.9	218.4
0.3	0.69	0.01	172.6	0.1	22.4	50	10	50	4.0	2.5	4.0	345.2	111.1	189.1
0.3	0.69	0.01	172.6	0.3	22.4	50	10	50	4.0	2.5	4.0	345.2	111.0	203.8
0.3	0.69	0.01	172.6	0.5	22.4	50	10	50	4.0	2.5	4.0	345.2	110.9	218.4
0.3	0.69	0.01	173.1	0.1	22.4	50	10	50	4.0	4.0	2.5	346.2	111.1	188.8
0.3	0.69	0.01	173.1	0.3	22.4	50	10	50	4.0	4.0	2.5	346.2	111.0	203.4
0.3	0.69	0.01	173.1	0.5	22.4	50	10	50	4.0	4.0	2.5	346.2	110.8	217.9
0.3	0.69	0.01	173.1	0.1	22.4	50	10	50	4.0	4.0	4.0	346.2	111.1	188.8
0.3	0.69	0.01	173.1	0.3	22.4	50	10	50	4.0	4.0	4.0	346.2	111.0	203.4
0.3	0.69	0.01	173.1	0.5	22.4	50	10	50	4.0	4.0	4.0	346.2	110.8	217.9
0.3	0.68	0.02	169.3	0.1	22.0	50	10	10	2.5	2.5	2.5	338.5	111.0	187.2
0.3	0.68	0.02	169.3	0.3	22.0	50	10	10	2.5	2.5	2.5	338.5	110.9	201.9
0.3	0.68	0.02	169.3	0.5	22.0	50	10	10	2.5	2.5	2.5	338.5	110.8	216.5
0.3	0.68	0.02	169.3	0.1	22.0	50	10	10	2.5	2.5	4.0	338.5	111.0	187.2

0.3	0.68	0.02	169.3	0.3	22.0	50	10	10	2.5	2.5	4.0	338.5	110.9	201.9
0.3	0.68	0.02	169.3	0.5	22.0	50	10	10	2.5	2.5	4.0	338.5	110.8	216.5
0.3	0.68	0.02	169.8	0.1	22.0	50	10	10	2.5	4.0	2.5	339.5	111.0	186.9
0.3	0.68	0.02	169.8	0.3	22.0	50	10	10	2.5	4.0	2.5	339.5	110.9	201.5
0.3	0.68	0.02	169.8	0.5	22.0	50	10	10	2.5	4.0	2.5	339.5	110.8	216.0
0.3	0.68	0.02	169.8	0.1	22.0	50	10	10	2.5	4.0	4.0	339.6	111.0	186.9
0.3	0.68	0.02	169.8	0.3	22.0	50	10	10	2.5	4.0	4.0	339.6	110.9	201.5
0.3	0.68	0.02	169.8	0.5	22.0	50	10	10	2.5	4.0	4.0	339.6	110.8	216.0
0.3	0.68	0.02	169.5	0.1	22.0	50	10	10	4.0	2.5	2.5	339.0	111.0	187.1
0.3	0.68	0.02	169.5	0.3	22.0	50	10	10	4.0	2.5	2.5	339.0	110.9	201.8
0.3	0.68	0.02	169.5	0.5	22.0	50	10	10	4.0	2.5	2.5	339.0	110.9	216.4
0.3	0.68	0.02	169.5	0.1	22.0	50	10	10	4.0	2.5	4.0	339.0	111.0	187.1
0.3	0.68	0.02	169.5	0.3	22.0	50	10	10	4.0	2.5	4.0	339.0	110.9	201.8
0.3	0.68	0.02	169.5	0.5	22.0	50	10	10	4.0	2.5	4.0	339.0	110.9	216.4
0.3	0.68	0.02	170.0	0.1	22.0	50	10	10	4.0	4.0	2.5	340.0	111.0	186.8
0.3	0.68	0.02	170.0	0.3	22.0	50	10	10	4.0	4.0	2.5	340.0	110.9	201.4
0.3	0.68	0.02	170.0	0.5	22.0	50	10	10	4.0	4.0	2.5	340.0	110.8	215.9
0.3	0.68	0.02	170.0	0.1	22.0	50	10	10	4.0	4.0	4.0	340.0	111.1	186.8
0.3	0.68	0.02	170.0	0.3	22.0	50	10	10	4.0	4.0	4.0	340.0	110.9	201.4
0.3	0.68	0.02	170.0	0.5	22.0	50	10	10	4.0	4.0	4.0	340.0	110.8	215.9
0.3	0.68	0.02	167.8	0.1	22.2	50	10	20	2.5	2.5	2.5	335.7	110.9	189.8
0.3	0.68	0.02	167.8	0.3	22.2	50	10	20	2.5	2.5	2.5	335.7	110.8	204.4
0.3	0.68	0.02	167.8	0.5	22.2	50	10	20	2.5	2.5	2.5	335.7	110.7	219.0
0.3	0.68	0.02	167.9	0.1	22.2	50	10	20	2.5	2.5	4.0	335.7	110.9	189.8
0.3	0.68	0.02	167.9	0.3	22.2	50	10	20	2.5	2.5	4.0	335.7	110.8	204.4
0.3	0.68	0.02	167.9	0.5	22.2	50	10	20	2.5	2.5	4.0	335.7	110.7	219.0
0.3	0.68	0.02	168.3	0.1	22.2	50	10	20	2.5	4.0	2.5	336.7	110.9	189.5
0.3	0.68	0.02	168.3	0.3	22.2	50	10	20	2.5	4.0	2.5	336.7	110.8	204.1
0.3	0.68	0.02	168.3	0.5	22.2	50	10	20	2.5	4.0	2.5	336.7	110.6	218.6
0.3	0.68	0.02	168.4	0.1	22.2	50	10	20	2.5	4.0	4.0	336.7	110.9	189.5
0.3	0.68	0.02	168.4	0.3	22.2	50	10	20	2.5	4.0	4.0	336.7	110.8	204.1
0.3	0.68	0.02	168.4	0.5	22.2	50	10	20	2.5	4.0	4.0	336.7	110.6	218.6
0.3	0.68	0.02	168.1	0.1	22.2	50	10	20	4.0	2.5	2.5	336.1	110.9	189.7
0.3	0.68	0.02	168.1	0.3	22.2	50	10	20	4.0	2.5	2.5	336.1	110.8	204.3
0.3	0.68	0.02	168.1	0.5	22.2	50	10	20	4.0	2.5	2.5	336.1	110.7	218.9
0.3	0.68	0.02	168.1	0.1	22.2	50	10	20	4.0	2.5	4.0	336.1	110.9	189.7
0.3	0.68	0.02	168.1	0.3	22.2	50	10	20	4.0	2.5	4.0	336.1	110.8	204.3
0.3	0.68	0.02	168.1	0.5	22.2	50	10	20	4.0	2.5	4.0	336.1	110.7	218.9
0.3	0.68	0.02	168.6	0.1	22.2	50	10	20	4.0	4.0	2.5	337.1	110.9	189.4
0.3	0.68	0.02	168.6	0.3	22.2	50	10	20	4.0	4.0	2.5	337.1	110.8	204.0
0.3	0.68	0.02	168.6	0.5	22.2	50	10	20	4.0	4.0	2.5	337.1	110.6	218.5
0.3	0.68	0.02	168.6	0.1	22.2	50	10	20	4.0	4.0	4.0	337.2	110.9	189.4
0.3	0.68	0.02	168.6	0.3	22.2	50	10	20	4.0	4.0	4.0	337.2	110.8	204.0

0.3	0.68	0.02	168.6	0.5	22.2	50	10	20	4.0	4.0	4.0	337.2	110.6	218.5
0.3	0.68	0.02	168.4	0.1	22.4	50	10	30	2.5	2.5	2.5	336.7	110.9	191.5
0.3	0.68	0.02	168.4	0.3	22.4	50	10	30	2.5	2.5	2.5	336.7	110.8	206.1
0.3	0.68	0.02	168.4	0.5	22.4	50	10	30	2.5	2.5	2.5	336.7	110.8	220.8
0.3	0.68	0.02	168.4	0.1	22.4	50	10	30	2.5	2.5	4.0	336.8	110.9	191.5
0.3	0.68	0.02	168.4	0.3	22.4	50	10	30	2.5	2.5	4.0	336.8	110.8	206.1
0.3	0.68	0.02	168.4	0.5	22.4	50	10	30	2.5	2.5	4.0	336.8	110.8	220.8
0.3	0.68	0.02	168.9	0.1	22.4	50	10	30	2.5	4.0	2.5	337.8	110.9	191.2
0.3	0.68	0.02	168.9	0.3	22.4	50	10	30	2.5	4.0	2.5	337.8	110.8	205.8
0.3	0.68	0.02	168.9	0.5	22.4	50	10	30	2.5	4.0	2.5	337.8	110.7	220.3
0.3	0.68	0.02	168.9	0.1	22.4	50	10	30	2.5	4.0	4.0	337.8	110.9	191.2
0.3	0.68	0.02	168.9	0.3	22.4	50	10	30	2.5	4.0	4.0	337.8	110.8	205.8
0.3	0.68	0.02	168.9	0.5	22.4	50	10	30	2.5	4.0	4.0	337.8	110.7	220.3
0.3	0.68	0.02	168.6	0.1	22.4	50	10	30	4.0	2.5	2.5	337.2	110.9	191.4
0.3	0.68	0.02	168.6	0.3	22.4	50	10	30	4.0	2.5	2.5	337.2	110.9	206.0
0.3	0.68	0.02	168.6	0.5	22.4	50	10	30	4.0	2.5	2.5	337.2	110.8	220.7
0.3	0.68	0.02	168.6	0.1	22.4	50	10	30	4.0	2.5	4.0	337.2	110.9	191.4
0.3	0.68	0.02	168.6	0.3	22.4	50	10	30	4.0	2.5	4.0	337.2	110.9	206.0
0.3	0.68	0.02	168.6	0.5	22.4	50	10	30	4.0	2.5	4.0	337.2	110.8	220.7
0.3	0.68	0.02	169.1	0.1	22.4	50	10	30	4.0	4.0	2.5	338.2	111.0	191.1
0.3	0.68	0.02	169.1	0.3	22.4	50	10	30	4.0	4.0	2.5	338.2	110.8	205.7
0.3	0.68	0.02	169.1	0.5	22.4	50	10	30	4.0	4.0	2.5	338.2	110.7	220.2
0.3	0.68	0.02	169.1	0.1	22.4	50	10	30	4.0	4.0	4.0	338.2	111.0	191.1
0.3	0.68	0.02	169.1	0.3	22.4	50	10	30	4.0	4.0	4.0	338.2	110.8	205.7
0.3	0.68	0.02	169.1	0.5	22.4	50	10	30	4.0	4.0	4.0	338.2	110.7	220.2
0.3	0.68	0.02	170.9	0.1	22.6	50	10	40	2.5	2.5	2.5	341.7	111.2	192.3
0.3	0.68	0.02	170.9	0.3	22.6	50	10	40	2.5	2.5	2.5	341.7	111.1	206.9
0.3	0.68	0.02	170.9	0.5	22.6	50	10	40	2.5	2.5	2.5	341.7	111.0	221.6
0.3	0.68	0.02	170.9	0.1	22.6	50	10	40	2.5	2.5	4.0	341.8	111.2	192.3
0.3	0.68	0.02	170.9	0.3	22.6	50	10	40	2.5	2.5	4.0	341.8	111.1	206.9
0.3	0.68	0.02	170.9	0.5	22.6	50	10	40	2.5	2.5	4.0	341.8	111.0	221.6
0.3	0.68	0.02	171.4	0.1	22.6	50	10	40	2.5	4.0	2.5	342.8	111.2	192.0
0.3	0.68	0.02	171.4	0.3	22.6	50	10	40	2.5	4.0	2.5	342.8	111.1	206.6
0.3	0.68	0.02	171.4	0.5	22.6	50	10	40	2.5	4.0	2.5	342.8	110.9	221.1
0.3	0.68	0.02	171.4	0.1	22.6	50	10	40	2.5	4.0	4.0	342.8	111.2	192.0
0.3	0.68	0.02	171.4	0.3	22.6	50	10	40	2.5	4.0	4.0	342.8	111.1	206.6
0.3	0.68	0.02	171.4	0.5	22.6	50	10	40	2.5	4.0	4.0	342.8	110.9	221.1
0.3	0.68	0.02	171.1	0.1	22.6	50	10	40	4.0	2.5	2.5	342.2	111.2	192.2
0.3	0.68	0.02	171.1	0.3	22.6	50	10	40	4.0	2.5	2.5	342.2	111.1	206.8
0.3	0.68	0.02	171.1	0.5	22.6	50	10	40	4.0	2.5	2.5	342.2	111.0	221.5
0.3	0.68	0.02	171.1	0.1	22.6	50	10	40	4.0	2.5	4.0	342.2	111.2	192.2
0.3	0.68	0.02	171.1	0.3	22.6	50	10	40	4.0	2.5	4.0	342.2	111.1	206.8
0.3	0.68	0.02	171.1	0.5	22.6	50	10	40	4.0	2.5	4.0	342.2	111.0	221.5

0.3	0.68	0.02	171.6	0.1	22.6	50	10	40	4.0	4.0	2.5	343.2	111.2	191.9
0.3	0.68	0.02	171.6	0.3	22.6	50	10	40	4.0	4.0	2.5	343.2	111.1	206.5
0.3	0.68	0.02	171.6	0.5	22.6	50	10	40	4.0	4.0	2.5	343.2	110.9	221.0
0.3	0.68	0.02	171.6	0.1	22.6	50	10	40	4.0	4.0	4.0	343.2	111.2	191.9
0.3	0.68	0.02	171.6	0.3	22.6	50	10	40	4.0	4.0	4.0	343.2	111.1	206.5
0.3	0.68	0.02	171.6	0.5	22.6	50	10	40	4.0	4.0	4.0	343.2	110.9	221.0
0.3	0.68	0.02	175.3	0.1	22.8	50	10	50	2.5	2.5	2.5	350.7	111.6	192.1
0.3	0.68	0.02	175.3	0.3	22.8	50	10	50	2.5	2.5	2.5	350.7	111.5	206.8
0.3	0.68	0.02	175.3	0.5	22.8	50	10	50	2.5	2.5	2.5	350.7	111.4	221.5
0.3	0.68	0.02	175.4	0.1	22.8	50	10	50	2.5	2.5	4.0	350.7	111.6	192.1
0.3	0.68	0.02	175.4	0.3	22.8	50	10	50	2.5	2.5	4.0	350.7	111.5	206.8
0.3	0.68	0.02	175.4	0.5	22.8	50	10	50	2.5	2.5	4.0	350.7	111.4	221.5
0.3	0.68	0.02	175.8	0.1	22.8	50	10	50	2.5	4.0	2.5	351.7	111.6	191.8
0.3	0.68	0.02	175.8	0.3	22.8	50	10	50	2.5	4.0	2.5	351.7	111.5	206.5
0.3	0.68	0.02	175.8	0.5	22.8	50	10	50	2.5	4.0	2.5	351.7	111.3	221.1
0.3	0.68	0.02	175.9	0.1	22.8	50	10	50	2.5	4.0	4.0	351.7	111.6	191.8
0.3	0.68	0.02	175.9	0.3	22.8	50	10	50	2.5	4.0	4.0	351.7	111.5	206.4
0.3	0.68	0.02	175.9	0.5	22.8	50	10	50	2.5	4.0	4.0	351.7	111.3	221.1
0.3	0.68	0.02	175.6	0.1	22.8	50	10	50	4.0	2.5	2.5	351.1	111.6	192.0
0.3	0.68	0.02	175.6	0.3	22.8	50	10	50	4.0	2.5	2.5	351.1	111.5	206.7
0.3	0.68	0.02	175.6	0.5	22.8	50	10	50	4.0	2.5	2.5	351.1	111.4	221.4
0.3	0.68	0.02	175.6	0.1	22.8	50	10	50	4.0	2.5	4.0	351.1	111.6	192.0
0.3	0.68	0.02	175.6	0.3	22.8	50	10	50	4.0	2.5	4.0	351.1	111.5	206.7
0.3	0.68	0.02	175.6	0.5	22.8	50	10	50	4.0	2.5	4.0	351.1	111.4	221.4
0.3	0.68	0.02	176.1	0.1	22.8	50	10	50	4.0	4.0	2.5	352.1	111.6	191.7
0.3	0.68	0.02	176.1	0.3	22.8	50	10	50	4.0	4.0	2.5	352.1	111.5	206.4
0.3	0.68	0.02	176.1	0.5	22.8	50	10	50	4.0	4.0	2.5	352.1	111.4	221.0
0.3	0.68	0.02	176.1	0.1	22.8	50	10	50	4.0	4.0	4.0	352.2	111.6	191.7
0.3	0.68	0.02	176.1	0.3	22.8	50	10	50	4.0	4.0	4.0	352.2	111.5	206.3
0.3	0.68	0.02	176.1	0.5	22.8	50	10	50	4.0	4.0	4.0	352.2	111.4	221.0