

Incorporating a Finite Population Correction Factor into American Community Survey Variance Estimates

Michael Starsinic¹

¹U.S. Census Bureau, Washington, DC, 20233

Abstract

The American Community Survey (ACS) produced its first nationwide 5-year estimates in 2010, using sample data from 2005 through 2009. With five years' worth of sample, the combined sample size in some areas would be large enough that a finite population correction (FPC) factor might have a noticeable impact on variances. This paper discusses the methodology used to incorporate an FPC factor into the 5-year ACS variance estimates, and how the method was adapted to account for the subsampling of nonrespondents. Results comparing the impact on the variance of using the FPC across a broad spectrum of estimates and geographic areas are also presented. Preliminary work indicated improvements in the standard error estimates of between two and four percent could be achieved.

Key Words: American Community Survey, variance estimation, finite population correction

1. Introduction[†]

The American Community Survey (ACS) is a continuous monthly survey that collects the data historically collected by the decennial census long form sample. Full implementation of the ACS began in January 2005, with the sample expanding to a size of approximately three million housing unit addresses, with sample selected from all counties and county equivalents in the 50 states, the District of Columbia, and Puerto Rico.

A single year's worth of sample in the ACS is not adequate to publish estimates for all geographic areas for which long form estimates were published in Census 2000. Instead, single-year estimates are published only for geographic areas with a population of at least 65,000. For smaller areas, several years of ACS sample are pooled together to create period estimates. The first estimates based on three years of pooled ACS data were published in 2008 for all areas with a population of at least 20,000 using data from 2005 through 2007. All geographic areas, including Census tracts and block groups, can be published using five years' worth of pooled ACS data. The five-year data were first published in 2010 using sample from the years 2005-2009. (U.S. Census Bureau, 2010b)

The ACS had not previously used a finite population correction (FPC) factor in its variance estimation methodology. One-year ACS samples are not large enough for an

[†] This report is released to inform interested parties of ongoing research and to encourage discussion of work in progress. The views expressed are those of the author and not necessarily those of the U.S. Census Bureau.

FPC to have much impact on variances of published estimates. However, with 5-year ACS estimates, up to 50 percent of housing units in certain blocks may have been in sample over the 5-year period. The Census 2000 long form, which used essentially the same variance estimation methodology as the ACS does currently, did include such a factor. The motivation behind this research into applying an FPC factor was the impending release of the first ACS 5-year sample data (using sample from 2005 through 2009), and the belief that using an FPC factor could enable more accurate estimates of the variance, particularly for small areas. Using an FPC should decrease most variance estimates. The true sampling error is, of course, unchanged, but the FPC should allow a more accurate estimate of the variance.

This paper will describe the FPC methodology used by the ACS, and will discuss the improvements in the estimation of standard errors seen for the 5-year 2005-2009 ACS data products due to the use of the FPC.

2. Statistical Background of the FPC

For a simple random sample without replacement, the variance of a sample estimate is

$$v(\hat{Y}) = \left(1 - \frac{n}{N}\right) \times \frac{N^2 s^2}{n}$$

where n is the sample size, N is the population size, n/N is the sampling fraction, and the $(1 - n/N)$ multiplicative factor is the FPC. (Cochran, 1977) As the sample size approaches the population size, the factor and the variance decrease. This makes intuitive sense – if a large proportion of a population is included in sample, the variability of the resulting estimates should be reduced. If all of the population is in sample, the FPC and the variance are zero, as there is no sampling variability.

If the sampling fraction is small, then the FPC makes very little difference to the variance calculation. For many large surveys, where N may be the population of the United States, n is much, much smaller, and the FPC is ignored when calculating the variance. Cochran (1977) suggests that the FPC can be ignored for sampling fractions up to 10 percent.

In 2009, the number of initially selected addresses for the ACS sample was 2,897,256, while the July 1, 2009 official estimate of housing units was 129,949,960, for a sampling rate of about 2.23 percent. (U.S. Census Bureau, 2010a) Using this rate in an FPC would decrease the standard error (the square root of the variance) by about 1.1 percent, which is not much of an impact.

FPC theory, however, is dependent on complete response. If a 90 percent sample of the population is selected, but only five percent responds, an FPC of $1 - 90/100 = 0.1$ isn't appropriate. Instead, let the effective sample size, n_{eff} , be defined as the number of actual respondents, or $n * r$, where r is the response rate. Then, n_{eff} would be used in the FPC instead of n .

$$v_r(\hat{Y}) = \left(1 - \frac{nr}{N}\right) \times \frac{N^2 s^2}{nr}$$

The nonresponse means that this version of the variance estimator with the adjusted FPC is no longer an unbiased estimate of the variance, however.

3. Census 2000 Long Form FPC Application

The Census 2000 long form selected a sample of about 1-in-6 housing units. However, the sampling rate varied by Census block, with target rates of 1-in-8, 1-in-6, 1-in-4, and 1-in-2. (U.S. Census Bureau, 2003) With sampling rates this large, an FPC could have a noticeable impact on the magnitude of the variance estimates.

The long form variances were calculated using the successive differences replication methodology (Fay & Train, 1995), which was developed to better handle systematically selected samples such as the long form, the ACS, and the Current Population Survey. A pre-specified number of replicate weights (52 for the long form, 80 for the ACS) are created by first applying sets of replicate factors to each observation's initial base weight, and then reprocessing the weighting methodology independently on each set of replicate initial weights. The variance of an estimate is calculated from the sum of the squared differences between the production estimate (which was calculated without replicate factors) and the replicate estimates, created from the sets of replicate weights (denoted by "k", below).

$$Var(x_0) = \frac{4}{k} \sum_{i=1}^k (x_i - x_0)^2$$

The replicate factors, $f_{i,j}$ (i = replicate #, j = sample unit #), in a typical successive differences variance estimator are defined as follows:

$$f_{i,j} = 1 + 2^{-1.5} a_{i1,j} - 2^{-1.5} a_{i2,j} = \begin{cases} 1 \\ 1 + 1/\sqrt{2} \\ 1 - 1/\sqrt{2} \end{cases}$$

where $a_{i1,j} = \pm 1$ and $a_{i2,j} = \pm 1$ are the appropriate cells from a Hadamard matrix.

The long form applied an FPC factor directly to the replicate factors:

$$f_{i,j}^* = 1 + \left(2^{-1.5} a_{i1,j} - 2^{-1.5} a_{i2,j} \right) \times \sqrt{FPC}, \text{ where } FPC = 1 - n_{eff} / N$$

where n_{eff} is defined as the observed number of long form sample respondents, and N is defined as the uncorrected census count (Gbur & Fairchild, 2002). The FPC is typically applied as a multiplicative factor "outside" the variance formula. However, under certain simplifying assumptions, the variance using the replicate factors after applying the FPC factor is equal to the original variance multiplied by the FPC factor.

We can explore the algebra associated with this FPC adjustment using the simplified assumption that there are no further weighting adjustments after the initial weights are assigned. First, define

$$g_{i,j} = 2^{-1.5} a_{i1,j} - 2^{-1.5} a_{i2,j}$$

So, $f_{ij} = 1 + g_{ij}$.

Let w_{0j} be the production weight for the j^{th} sample unit. Then the replicate weight w_{ij} is

$$w_{ij} = w_{0j} * f_{ij} = w_{0j} * (1 + g_{ij})$$

Now let's define the estimate and replicate estimates as weighted sums over the n units in sample.

$$x_0 = \sum_{j=1}^n w_{0,j} \quad \text{and} \quad x_i = \sum_{j=1}^n w_{i,j}$$

Using the ACS with its 80 replicates for this example, the variance of x_0 is then

$$\begin{aligned} \text{Var}(x_0) &= \frac{4}{80} \sum_{i=1}^{80} (x_i - x_0)^2 = \frac{4}{80} \sum_{i=1}^{80} \left(\sum_{j=1}^n w_{i,j} - \sum_{j=1}^n w_{0,j} \right)^2 = \frac{4}{80} \sum_{i=1}^{80} \left(\sum_{j=1}^n (w_{i,j} - w_{0,j}) \right)^2 \\ &= \frac{4}{80} \sum_{i=1}^{80} \left(\sum_{j=1}^n (w_{0,j} (1 + g_{i,j}) - w_{0,j}) \right)^2 \\ &= \frac{4}{80} \sum_{i=1}^{80} \left(\sum_{j=1}^n w_{0,j} g_{i,j} \right)^2 \end{aligned}$$

The FPC-adjusted replicate factor is

$$f_{i,j}^* = 1 + (2^{-1.5} a_{i1,j} - 2^{-1.5} a_{i2,j}) \times \sqrt{FPC} = 1 + g_{i,j} \times \sqrt{FPC}$$

The variance using the FPC-adjusted replicate factors is

$$\begin{aligned} \text{Var}^*(x_0) &= \frac{4}{80} \sum_{i=1}^{80} (x_i^* - x_0)^2 = \frac{4}{80} \sum_{i=1}^{80} \left(\sum_{j=1}^n w_{i,j}^* - \sum_{j=1}^n w_{0,j} \right)^2 = \frac{4}{80} \sum_{i=1}^{80} \left(\sum_{j=1}^n (w_{i,j}^* - w_{0,j}) \right)^2 \\ &= \frac{4}{80} \sum_{i=1}^{80} \left(\sum_{j=1}^n (w_{0,j} (1 + g_{i,j} \times \sqrt{FPC}) - w_{0,j}) \right)^2 \\ &= \frac{4}{80} \sum_{i=1}^{80} \left(\sum_{j=1}^n w_{0,j} g_{i,j} \times \sqrt{FPC} \right)^2 \\ &= FPC \times \frac{4}{80} \sum_{i=1}^{80} \left(\sum_{j=1}^n w_{0,j} g_{i,j} \right)^2 \\ &= FPC \times \text{Var}(x_0) \\ SE^*(x_0) &= \sqrt{FPC} \times SE(x_0) \end{aligned}$$

So, in this simplified example, applying the FPC to the replicate factors yields exactly the original standard error (SE) multiplied by the square root of the FPC. It is expected that the reduction in the variance estimate will carry through the weighting, and will be seen when the final weights are used.

Why was the FPC adjustment made to the replicate factor and not on the “outside” of the variance calculation? For simplicity, this example only included one FPC adjustment. The long form applied individual FPCs for each long form “weighting area” (frequently equivalent to a census tract). For an estimate that crossed multiple weighting areas, the FPCs would have to be adjusted depending on what weighting areas the observations comprising the estimate were included in. By applying the FPC to the replicate factor, that step has already been taken care of, and the FPC adjustment does not need to be recalculated for each estimate.

4. Application of the FPC to the ACS

Like the long form, the ACS target sampling rates differ at the census block level. Although the overall sampling rate for 2009 was just 2.23 percent, target block rates ranged from 1.49 percent to 10 percent. The block-level sampling rates were based in part on the expected number of occupied housing units in the governmental units (e.g. counties, places, minor civil divisions (MCDs), American Indian areas, etc.) that contains the given block. The 10 percent sampling rate is assigned to blocks that are contained within a governmental unit which has 200 or fewer estimated occupied housing units. For these blocks, the ACS 5-year estimates, based on 5-years worth of sample, will have combined sampling rates of up to about 50 percent. Table 1 shows selected 2005-2009 combined sampling rates.

Table 1: ACS 2005-2009 Sampling Rates for Selected Areas

Sampling Rate Category	2005-2009 Combined Sampling Rate
Blocks in smallest governmental units (< 200 occupied housing units)	50.0%
Blocks in smaller governmental units (200-799 occupied housing units)	33.5%
Blocks in small governmental units (800-1200 occupied housing units)	16.9%

Source: 2005-2009 American Community Survey, <http://www.census.gov/www/acs>

The sampling rates for those three classes of blocks seemed large enough that trying to apply an FPC could be worthwhile. All other blocks have a combined 5-year sampling rate of about 11 percent or less. (U.S. Census Bureau, 2010b)

Research (and experience) has shown that ACS variance estimates for small areas can be very high. (Starsinic & Tersine, 2007) The highest sampling rates are in blocks in small governmental units. So, the application of the FPC should result in the largest reductions in the estimate of the SEs in those small governmental units.

The ACS differs from the long form in having an additional phase of sampling. Housing units in the initial sample which do not respond by mail or by a computer-assisted telephone interview (CATI) are subsampled for a computer-assisted personal interview (CAPI) at rates roughly between 1-in-3 and 2-in-3. (U.S. Census Bureau, 2010a)

Because of the ACS CAPI sub-sampling, a single FPC factor was not deemed suitable. To develop an ACS application we considered basic features of the ACS sampling and weighting methodology, mainly the sampling of mail/CATI non-respondents. To develop an alternate method, we considered the mail/CATI response under a fixed response

model. The fixed response model assumes two distinct populations, one containing units that, if in sample, would always respond, and the other containing units that would never respond. (Bethlehem, 2009) Those in the ACS sample and responding by mail or CATI would be in the first “stratum”, while the CAPI respondents (who were non-respondents by mail and CATI), would be in the second “stratum”. Separate FPC factors would be computed for each stratum.

$$F_1 = \frac{n_{mail} + n_{CATI}}{N \times R}, \text{ and}$$

$$F_2 = \frac{n_{CAPI}}{N \times (1 - R)}, \text{ where } R = \frac{w_{mail} + w_{CATI}}{w_{samp}}$$

Here, N is the unweighted sample universe count, and n_{mail} , n_{CATI} , and n_{CAPI} are the unweighted counts of respondents by mode. R is the weighted proportion (using the unbiased sampling weights) of those who respond by mail or CATI, so $N * R$ is an estimate of the population of the respondent stratum, and $N * (1-R)$ is an estimate of the population of the nonrespondent stratum.

For mail and CATI respondents,

$$FPC_{mail/CATI} = 1 - F_1$$

and for CAPI respondents,

$$FPC_{CAPI} = 1 - F_2$$

Weighting areas for the ACS are defined at the county level, unlike the long form. Since the ACS initial sampling rate variation occurs at smaller areas than county, it was decided to calculate the FPC factors at the census tract level. Tracts were thought to be small enough to capture some of the variability, and are generally equivalent to the long form's weighting areas.

Also, the FPC was only applied to housing units and the housing unit population. No FPC was applied to persons in group quarters

If the decision had been made to use an overall FPC “in front” of the variance estimation formula, $FPC_{mail/CATI}$ would have been 0.895, and FPC_{CAPI} would have been 0.962. A combined, naive FPC value that ignored the subsampling for non-response would have been 0.926. This would have reduced SEs about 3.8 percent. However, this would have ignored the local variability in the sampling rate that we were trying to capture using the replicate factor adjustment method described above.

With the methodology decided, a simulation was conducted using 2006-2008 3-year data. A set of replicate weights were created using the FPC factor. SEs for a number of estimates were computed for selected geographic types and areas, and directly compared against the production SEs which did not have the FPC factors. The with-FPC SEs were, on average, 1.8 to 2.5 percent less than the without-FPC SEs, which was about the change that was expected. The expectation was that the decrease in the 5-year SEs would be even larger. Also, very few 3-year estimates saw an increase in their SEs. With the

successful results from the simulation, the decision was made to apply the FPCs to the 3-year 2007-2009 and 5-year 2005-2009 data tabulations. Only analysis of the 5-year 2005-2009 SEs is included in this paper.

5. Evaluation of 5-Year FPC Impact

As with the simulation, the analysis of the production data used two sets of SEs: the actual 5-year 2005-2009 production data created from replicate weights with the FPC adjustments, and one set for this research using replicate weights without the FPC adjustments. (Since the FPC only affects the replicate weights, the estimates from the with-FPC and without-FPC datasets are identical.) The evaluation would be conducted on the ACS data profile. The data profile contains estimates of over 400 unique characteristics, and covers a broad set of subjects (such as age, race, education, and income) and estimate types (including counts, percents, means, and ratios). Published data profiles (using the FPC) were available for each of the geography types we wanted to investigate – nation, state, county, MCD (including Census County Divisions), place, tract, and American Indian/Alaska Native (AIAN) area, which also include Hawaiian home lands. The profile's SEs were recreated for these geographic areas using the without-FPC set of replicate estimates, to provide a direct comparison with the published SEs.

Certain estimate values were omitted from the evaluation because their SEs are handled as special cases that do not use the replicate weight values. These include estimates of zero (for counts and percents), estimates of 100 percent, and estimates which are controlled during the weighting process to equal a value from the independent population controls used by the ACS.

Table 2 shows the distribution of the ratio of the SE using the FPC to the SE without the FPC. Since the FPC is less than one, this ratio should be less than one as well, if the adjustment worked as expected.

Table 2: Distribution of SE Ratio by Summary Level

Summary Level	1st Percentile	25th Percentile	Median	75th Percentile	99th Percentile
Nation	0.937	0.962	0.971	0.980	1.002
State	0.930	0.959	0.967	0.973	0.993
County	0.818	0.936	0.957	0.969	0.999
MCD	0.746	0.875	0.927	0.960	1.000
Tract	0.849	0.948	0.963	0.974	0.999
Place	0.798	0.925	0.955	0.971	1.008
AIAN Area	0.788	0.882	0.924	0.956	1.020

Source: 2005-2009 American Community Survey (special tabulation), <http://www.census.gov/www/acs>

As expected, the nation and state SEs are the least affected by the adjustment, with median ratios around 0.97, or, equivalently, a 3 percent decrease. The largest decreases are seen at the MCD and AIAN Area levels. Small governmental units are eligible for the highest initial sampling rates, so it makes sense that these areas also see the largest change. A relatively small number of estimates do show an increase in the SE – for several of the geographic area types, the 99th percentile of the SE ratio is at or over 1.0,

but these are not large changes. However, we could find no systematic pattern in the estimates with increased SEs. We believe this is just the variance of the variance.

Does the FPC adjustment have a differential impact on different types of estimates? For example, maybe SEs of percent estimates were less affected than SEs of counts. Table 3 shows the median of the ratio of SEs by the type of estimate for tracts and places.

Table 3: Median of SE Ratio by Estimate Type for Tracts and Places

Estimate Type	Tract	Place
	Median	Median
Household Count	0.963	0.954
Housing Unit Count	0.963	0.955
Population Count	0.963	0.955
Percent	0.964	0.956
Ratio or Mean	0.960	0.951

Source: 2005-2009 American Community Survey (special tabulation), <http://www.census.gov/www/acs>

There appears to be no appreciable difference between counts, percents, and ratios for either tracts or places. Other geographic types show a similar distribution.

Does the FPC adjustment have a differential impact on different characteristics at lower levels of geography? The many estimates in the data profile can be easily broken down into broad topics, which may include only one estimate or many. The SE ratio distribution for 17 selected topic groupings at the tract level are presented in Table 4.

Table 4: Distribution of SE Ratio for Selected Topic Groupings for Tracts

Topic	1st	25th	Median	75th	99th
	Percentile	Percentile	Percentile	Percentile	Percentile
AIAN Groups	0.805	0.936	0.953	0.969	1.000
Grandparents	0.820	0.941	0.955	0.968	0.991
Ancestry	0.827	0.943	0.956	0.968	1.000
Asian Groups	0.841	0.944	0.956	0.968	1.000
Housing Value	0.837	0.942	0.957	0.969	0.990
Household Income	0.832	0.945	0.960	0.972	0.997
Language Spoken at Home	0.835	0.947	0.962	0.974	1.000
Educational Attainment	0.854	0.950	0.963	0.973	0.995
Poverty (Persons)	0.849	0.949	0.964	0.975	0.992
Household Type	0.867	0.952	0.966	0.976	0.996
Age & Sex	0.873	0.955	0.967	0.977	1.000
Residence One Year Ago	0.856	0.952	0.968	0.978	1.000
Tenure	0.882	0.958	0.970	0.979	1.003
Total Population	0.885	0.963	0.973	0.981	1.019
Total Housing Units	0.882	0.958	0.976	0.991	1.062
Total Households	0.895	0.962	0.976	0.988	1.029
Vacancy Rate	0.882	0.971	0.980	0.986	0.996

Source: 2005-2009 American Community Survey (special tabulation), <http://www.census.gov/www/acs>

The median SE ratios range from 0.953 for American Indian tribal groups to 0.980 for vacancy rates. The distribution of the median values among the topics is fairly smooth, with no large outliers with very high or low median ratios. The FPC adjustment has less

of an impact on the population, housing unit, and household totals, but SEs for those characteristics are generally small to begin with.

Does the FPC have a differential impact on the standard error for smaller or larger geographic areas? Table 5 shows the SE ratio distribution for the tract and place summary levels, with each broken into 10 population size groupings. The population size ranges were chosen so that approximately 10 percent of that type of geographic area is contained within in. For example, about 10 percent of all tracts have a population between 1,901 and 2,600.

Table 5: Distribution of SE Ratio by Population Size for Tracts and Places

Size Range	Geo	1st Percentile	25th Percentile	Median	75th Percentile	99th Percentile
1-1,900	Tract	0.800	0.934	0.953	0.969	1.000
1,901-2,600	Tract	0.823	0.938	0.955	0.969	0.999
2,601-3,100	Tract	0.837	0.941	0.957	0.970	0.998
3,101-3,600	Tract	0.847	0.944	0.958	0.971	0.998
3,601-4,100	Tract	0.857	0.946	0.960	0.972	0.997
4,201-4,700	Tract	0.866	0.948	0.962	0.973	0.998
4,701-5,400	Tract	0.879	0.952	0.965	0.975	0.998
5,401-6,300	Tract	0.896	0.956	0.968	0.977	0.998
6,301-7,700	Tract	0.905	0.959	0.970	0.979	0.998
>7,700	Tract	0.925	0.962	0.972	0.980	0.998
1-150	Place	0.746	0.875	0.918	0.953	1.043
151-310	Place	0.759	0.885	0.925	0.957	1.035
311-540	Place	0.771	0.890	0.928	0.957	1.025
541-870	Place	0.786	0.899	0.933	0.959	1.010
871-1,390	Place	0.800	0.904	0.937	0.961	1.003
1,391-2,260	Place	0.818	0.916	0.944	0.964	1.000
2,271-3,660	Place	0.865	0.939	0.956	0.970	1.000
3,661-6,950	Place	0.910	0.953	0.965	0.975	1.000
6,950-16,230	Place	0.922	0.956	0.967	0.976	0.999
>16,230	Place	0.932	0.961	0.969	0.977	0.996

Source: 2005-2009 American Community Survey (special tabulation), <http://www.census.gov/www/acs>

For both geography types the smaller areas in population generally have a smaller SE ratio, indicating a larger decrease in the value of the SE after applying the FPC. This is more pronounced for places, as places with small populations are likely to be sampled at higher rates, and have correspondingly higher sampling fractions and smaller FPC factors.

The published ACS data profiles and other products include the 90 percent margin of error (MOE) for each estimate, that is, the SE multiplied by 1.645. Percent estimates and their margins of error are presented with one decimal place of accuracy. If a user had without-FPC and with-FPC margins of error side-by-side, would the small decreases in the SEs we have seen make a noticeable difference in the values in the published data profile?

Table 6 answers this question for several summary levels. It shows that, for many areas, the as-published MOE would be lower with the FPC than without the FPC.

Table 6: Observable Change in MOE for Percent Estimates by Summary Level

Summary Level	MOE Higher	No Change	MOE Lower
Nation	0.0%	100.0%	0.0%
State	0.0%	91.7%	8.3%
County	0.0%	41.1%	58.9%
MCD	0.3%	11.6%	88.1%
Tract	0.1%	13.3%	86.6%
Place	1.0%	14.7%	84.4%
AIAN Area	2.1%	10.4%	87.5%

Source: 2005-2009 American Community Survey (special tabulation), <http://www.census.gov/www/acs>

More than 80 percent of MCDs, tracts, places, and AIAN areas would display a smaller MOE. At the nation and state, levels, with lower SEs due to more sample, fewer percent estimates would show a change in the MOE. If the SE is small enough such that the without FPC MOE is 0.1 percent, then no amount of decrease due to the FPC would make a difference in the published MOE.

6. Conclusions and Further Research

After initial research, the FPC factor methodology was applied to the 3-year 2007-2009 and 5-year 2005-2009 ACS data published in 2010. This evaluation of the 5-year 2005-2009 data indicates that the change in estimates of the SEs, as measured by the ratio of the with-FPC to without-FPC SEs, shows a consistent pattern of improvement (i.e. lower SEs with the FPC). The changes are within a reasonable range of values, and few estimates have SEs that increase at all, much less substantially. The largest reductions are seen at geographic types (MCDs, places, and AIAN areas) as would be expected based on the ACS sampling methodology.

We plan to continue using this methodology for the 3-year and 5-year ACS data published in 2011, and to continue to monitor its impact. There are currently no plans to apply this to the 1-year ACS data, but further research could indicate its utility.

Acknowledgements

The author would like to acknowledge the work done by Alfredo Navarro and Mark Asiala on this research.

References

- Bethlehem, J. (2009). *Applied Survey Methods: A Statistical Perspective*. Hoboken, NJ: John Wiley & Sons.
- Cochran, W. (1977). *Sampling Techniques, 3rd Edition*. New York, NY: John Wiley & Sons.
- Fay, R., and G. Train. (1995). "Aspects of Survey and Model-Based Postcensal Estimation of Income and Poverty Characteristics for States and Counties". Proceedings of the Section on Government Statistics. Alexandria, VA: American Statistical Association, pp. 154–159.

- Gbur, P., and L. Fairchild. (2002). “Overview of the U.S. Census 2000 Long Form Direct Variance Estimation”. Proceedings of the Section on Survey Research Methods. Alexandria, VA: American Statistical Association, pp. 1139–1144.
- Starsinic, M., and A. Tersine. (2007). “Analysis of Variance Estimates from American Community Survey Multiyear Estimates”. Proceedings of the Section on Survey Research Methods. Alexandria, VA: American Statistical Association, pp. 3011-3017.
- U.S. Census Bureau (2002), “Summary File 3 Technical Documentation”, <http://www.census.gov/prod/cen2000/doc/sf3.pdf>.
- U.S. Census Bureau. (2010a). “American Community Survey Accuracy of the Data (2009)”. http://www.census.gov/acs/www/Downloads/data_documentation/Accuracy/ACS_Accuracy_of_Data_2009.pdf
- U.S. Census Bureau. (2010b). “American Community Survey Multiyear Accuracy of the Data (3-year 2007-2009 and 5-year 2005-2009)”. http://www.census.gov/acs/www/Downloads/data_documentation/Accuracy/MultiyearACSAccuracyofData2009.pdf