Reweighting in the Presence of Nonresponse in Stratified Designs

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Abstract
Reweighting a sample using weighting class adjustments is a common approach to deal with nonresponse. This approach uses a response model defined as a set of assumptions about the true but unknown response distribution that corresponds to the weighting class. A reweighted estimator is unbiased if the model coincides with the response distribution. However, in most cases, the response model will differ from the true response distribution. In this paper we examine the effect of using weights for reweighting when the model fails in stratified designs. The majority of results on model failure in nonresponse in the literature assume a simple random sampling where the weights are constant.

Key Words: Reweighting, nonresponse, stratification, weighting classes

1. Reweighting

A common strategy to reduce potential biases in estimates from surveys with nonresponse is to apply an adjustment factor based on a response model to the sampling weights. This is sometimes called reweighting. In order to define reweighting, we assume a finite population $Y = (y_1, \ldots y_N)$ and the total of Y is $Y = \sum_N y_i$. Under a sample design, $p(s)$ with a defined probability of selection π_i for all units in Y, an unbiased estimator of the total Y is $\hat{Y} = \sum_s y_i/\pi_i$.

With nonresponse, we only observe a subset r (i.e., $r \subset s$). The reweighted estimator of the total would be $\hat{Y}^* = \sum_r y_i/\pi_i\theta_i$, where θ_i is the probability that the element i responds given that the sample s was selected if θ_i were known. Using the theory of double sampling, the estimator $\hat{Y}^*$ is unbiased (Särndal, et al., 1992) if we know θ_i for all i.

The quantity $f_i = 1/\theta_i$ is an adjustment factor made to the sampling weight $w_i = 1/\pi_i$ that eliminates any bias due to nonresponse. In practice, the response probabilities, the θ_i's, are not known and must be estimated ($\hat{\theta}_i$). The reweighted estimator of the total substitutes $\hat{\theta}_i$ for θ_i

$$\hat{Y}^R = \sum_r \frac{y_i}{\pi_i\hat{\theta}_i} = \sum_r \hat{f}_i w_i y_i . \qquad (1)$$

The reweighted estimate $\hat{Y}^R$ may not be unbiased, and the bias depends in part on how well the probabilities of response θ_i are estimated by $\hat{\theta}_i$.

We restrict our study to the class of reweighting estimators that estimate response propensities using classes or nonresponse adjustment cells denoted by c, for c = 1,…,C. The classes are often constructed so that the units within the same class have the same response propensity. Auxiliary variables such as demographic and socioeconomic characteristics are used to create weighting classes. For such a weighting class estimator, the reweighting estimator uses $\hat{f}_{ci} = 1/\hat{\theta}_{ci}$ so (1) can be written as

$$\hat{Y}^R = \sum_c \sum_r \frac{y_i}{\pi_i \hat{\theta}_{ci}} = \sum_c \sum_r \hat{f}_{ci} w_i y_i \,. \tag{2}$$

Little and Vartivarian (2003) (referred to as LV in this paper) raised the question of whether the reweighting adjustment factors should be estimated using the selection weights or not. The weighting classes described in the literature primarily uses simple random samples where the weights are constant, and thus, the two approaches are identical. Theoretically, we know from the double sampling theory that using the sampling weights is appropriate if the assumed response model gives unbiased estimates. In practice, the model is almost always wrong, and there is little guidance on whether using the weights improves the quality of the estimates. While LV raised the question about using the weights, this was not the focus of their study and more research on this is needed.

This paper expands upon the initial work of LV in a number of ways, and more directly assesses the effect of using weights for the nonresponse adjustment factor. We explore estimators of totals and ratios in addition to the means studied by LV. In doing this, we also point out some reasons for the observed performance of the means as defined in the LV study. In addition, we assess the effect of different sampling rates while LV restricted their work to designs in which the sampling rates in the strata were fixed. These extensions provide some interesting insights into the effect of using weights for computing reweighting factors.

2. Previous Study

Little and Vartivarian (2003) evaluated the performance of nine reweighted estimators of the population mean through simulation. They drew stratified samples from an artificial population with 10,000 elements. The population is classified into two strata (Z) and has two nonresponse adjustment cells (X) that cross strata. Table 1 shows the distribution of the population. In our study we use the same population.

Table 1: Population Counts by Strata Z and Nonresponse Adjustment Cell X

	Nonresponse adjustment cell	
Sampling strata	*X=0*	*X=1*
Z=0	3,064	3,931
Z=1	2,079	926

Source: Little and Vartivarian (2003).

The variable of interest, Y, is a binary variable with a Bernoulli distribution where the probability of $Y = 1$ is defined by a logistic model with $\text{logit}(Y = 1|X, Z) = 0.5 + \gamma_X(X - \overline{X}) + \gamma_Z(Z - \overline{Z}) + \gamma_{XZ}(X - \overline{X})(Z - \overline{Z})$. The response propensity R is also

Bernoulli where the probability of $R=1$ follows a logistic model with $\text{logit}(R|X,Z)=0.5+\beta_X(X-\overline{X})+\beta_Z(Z-\overline{Z})+\beta_{XZ}(X-\overline{X})(Z-\overline{Z})$. Different populations and response propensities can be generated depending on the values of $\gamma_X,\gamma_Z,\gamma_{XZ},\beta_X,\beta_Z$ and β_{XZ} as shown in Table 2. In the simulation, a fixed sample size of 262 is selected for stratum *Z=0* and 50 in *Z=1* are drawn. The LV study evaluates the performance of the estimator through the root mean squared error (RMSE) of the estimates for different populations.

Table 2: Models for Y and R

Model for population *Y*	Model for response propensity *R*	γ_X,β_X	γ_Z,β_Z	γ_{XZ},β_{XZ}
$[XZ]^Y$	$[XZ]^R$	2	2	2
$[X+Z]^Y$	$[X+Z]^R$	2	2	0
$[X]^Y$	$[X]^R$	2	0	0
$[Z]^Y$	$[Z]^R$	0	2	0
$[\phi]^Y$	$[\phi]^R$	0	0	0

Source: Little and Vartivarian (2003).

3. The Estimators

We consider three reweighted estimators described in the LV paper. These estimators differ in the way the adjustment factor $\hat{f}_i$ is computed. The estimators are maximum likelihood ML(*xz*) estimator, weighted response estimator, and unweighted response estimator.

Definitions

Consider a stratified design for *Y* with elements *y* where the strata are denoted by *z* = 1,.., *Z*; a sample of size n_{+z} is drawn from stratum *z* with a probability of selection π_z. Because of nonresponse, we only observe r_{+z} respondents in stratum *z*. The adjustment for nonresponse is done within weighting classes defined by *x*=1,…, *X*. In this notation, n_{xz} and r_{xz} are the sample size and number of respondents in the cell created by the intersection of the nonresponse cell *x* and stratum *z* respectively (i.e., $x\cap z$). The estimators to be evaluated in this study are shown in Table 3.

Table 3: Reweighted Estimators

Estimator	*Expression*	*Weight*	*Response rate*
Weighted response rate	$\hat{\bar{Y}}_{wrr}=\dfrac{\sum_x\sum_z w^*_{xz}r_{xz}\bar{y}_{xz}}{\sum_x\sum_z w^*_{xz}r_{xz}}$	$w^*_{xz}=\dfrac{(N_{+z}/n_{+z})(r/N)}{\hat{\phi}^*_x}$	$\hat{\phi}^*_x=\dfrac{\sum_z(r_{xz}/\pi_z)}{\sum_z(n_{xz}/\pi_z)}$
Unweighted response rate	$\hat{\bar{Y}}_{urr}=\dfrac{\sum_x\sum_z w''_{xz}r_{xz}\bar{y}_{xz}}{\sum_x\sum_z w''_{xz}r_{xz}}$	$w''_{xz}=\dfrac{(N_{+z}/n_{+z})(r/N)}{\hat{\phi}_x}$	$\hat{\phi}_x=\dfrac{r_{x+}}{n_{x+}}$
ML(*xz*) estimator	$\hat{\bar{Y}}_{ml}=\dfrac{\sum_x\sum_z w'_{xz}r_{xz}\bar{y}_{xz}}{\sum_x\sum_z w'_{xz}r_{xz}}$	$w'_{xz}=\dfrac{(N_{+z}/n_{+z})(r/N)}{\hat{\phi}_{xz}}$	$\hat{\phi}_{xz}=\dfrac{r_{xz}}{n_{xz}}$

Source: Little and Vartivarian (2003).

Since we are evaluating reweighted estimators, we begin by examining the differences in the way the adjustment factor $\hat{f}_{ci}$ is computed. The formulas in Table 1 do not isolate the expression of $\hat{f}_{ci}$ from the sampling weight. To make the extension to estimating totals such as $\hat{Y}$ such a re-expression is useful. Substitute $\pi_z = n_{+z}/N_{z+}$ and define the sampling weight as $w_z = N_{+z}/n_{+z}$ where N_{+z} is the total size in stratum *z*. This way of expressing the estimators of totals are presented in Table 4.

Table 4: Alternative Expressions for the Reweighted Estimators for Totals

Estimator	*Expression*	*Adjustment factor*
Weighted response rate	$\hat{Y}_{wrr} = \sum_z \sum_x \sum_r \hat{f}_x^{wrr} w_z y_{xzr}$	$\hat{f}_x^{wrr} = \dfrac{\sum_z (n_{xz} N_{+z}/n_{+z})}{\sum_z (r_{xz} N_{+z}/n_{+z})}$
Unweighted response rate	$\hat{Y}_{urr} = \sum_z \sum_x \sum_r \hat{f}_x^{urr} w_z y_{xzr}$	$\hat{f}_x^{urr} = \dfrac{\sum_z n_{xz}}{\sum_z r_{xz}} = \dfrac{n_{x+}}{r_{x+}}$
ML(*xz*) estimator	$\hat{Y}_{ml} = \sum_z \sum_x \sum_r \hat{f}_{xz}^{ml} w_z y_{xzr}$	$\hat{f}_{xz}^{ml} = \dfrac{n_{xz}}{r_{xz}}$

The expressions of the adjustment factors in Table 4 are also helpful for understanding the LV nomenclature. The estimator called the weighted response rate uses an adjustment factor computed as the inverse of the weighted response rate within the nonresponse cell using the stratum sampling weight N_{+z}/n_{+z}. The estimator called unweighted response rate uses an adjustment factor computed as the inverse of the unweighted response rate within the nonresponse cell. In other words, the adjustment factor is the ratio of the count of the sample drawn to the observed count of respondents within the nonresponse cell. The "unweighted" estimator does not mean that the estimator is not weighted by the inverse of the selection probability, but rather that the estimator includes an adjustment that uses the unweighted response rate.

An estimator of the mean $\hat{\bar{\bar{Y}}} = \hat{Y}/\hat{N}$ using the expression in Table 4 can be easily derived by defining $\hat{N}$ as $y_{xzr} = 1$. One could incorrectly assume that the numerators and denominators of the estimators in Table 3 are ratios of totals $\hat{Y}$ and $\hat{N}$ respectively. In particular, notice that the weighted mean has a fixed denominator of N in the estimators of Table 3. Nevertheless, the expressions for the mean $\hat{\bar{\bar{Y}}}$ in Table 3 are valid estimators of the mean. We return to this issue shortly.

4. Simple Cases

Särndal et al. (1992) use the response homogeneity group (RHG) model to describe the properties of this estimator for simple random samples (SRS). It is shown that the estimator is unbiased and the adjustment factor is computed as $\hat{f}_x = n_x/r_x$ for adjustment cell x when the response propensities within cells are homogenous. When the response propensities within the group are not homogeneous, the estimator is biased. Note that in SRS the weights are constant, so the weighted and unweighted adjustments are identical.

To generalize to a stratified design, think of the SRS sample as a stratified design with one stratum. The estimator is unbiased in the stratified design as long as the nonresponse adjustment cells are created within stratum. The form of the adjustment factor in this case is $\hat{f}_{xz} = n_{xz}/r_{xz}$. This adjustment factor corresponds to the ML(*xz*) estimator in Table 4. Furthermore, since the cells do not cross strata, the sampling weights within the cells are the same and the adjustment factors for the weighted and unweighted estimators have the same expression. In other words, when the nonresponse adjustment cells do not cross strata, the weighted, unweighted, and ML(*xz*) estimators are identical.

Looking at the expression of the factor for the ML(*xz*) estimator in Table 4, we see that *by definition*, the adjustment factor of the ML(*xz*) estimator is always computed within the cell created by the intersection of the sampling strata *z* and the nonresponse cell *x* (i.e., cell $x \cap z$). Since the response propensities in the cells $x \cap z$ are always homogenous for all nonresponse models evaluated in the LV study, theory says the estimator is unbiased. The simulations in Table V of the LV paper confirm this theoretical result.

The weighted and unweighted estimators in the same simulations use nonresponse adjustment cells that cross strata (i.e., cell *x* instead of cell $x \cap z$) and in most of the simulation conditions, the response propensities are not homogeneous; therefore, the weighted and unweighted estimators are generally biased. If the weighted and unweighted estimators were based on the same information (the intersection cells), they would give results identical to the ML(*xz*) estimator. We are examining the effect of the adjustments when the models are not perfectly specified (which is almost always the case in practice), the ML(*xz*) estimator is not of interest to us in our study.

There are several situations where the ML(*xz*), weighted, and unweighted estimators are similar or identical in expectation. One situation corresponds to the response model $[\phi]^R = (\beta_x = 0, \beta_z = 0, \beta_{xz} = 0)$where the propensity to respond is the same in all cells. In this situation, although the unweighted and weighted estimators use *z* as nonresponse cells, in expectation, the adjustment factors are the same and unbiased. This is confirmed by the simulation results in Table V of the LV paper rows 5, 10, 15, 20, and 25. This result can also be shown algebraically noticing that in expectation the ratios n_{xz}/r_{xz} are constant. The simulation is consistent with a theoretical result that states when the model holds then the estimators are unbiased.

Another situation where the response propensity is the same across strata *z* corresponds to the response model $[X]^R = (\beta_x = 2, \beta_z = 0, \beta_{xz} = 0)$. In this case, the response propensities are also homogeneous within the cells *z*, which are used in the weighted and unweighted estimator and within the cell $x \cap z$ used as adjustment cell in the ML(*xz*) estimator. In this case too, the estimators are the same in expectation and this result is confirmed by the simulation in Table V of the LV paper rows 3, 8, 13, 18, and 23. These results can also be shown algebraically. Finally, there is a situation where the weighted and unweighted estimators are equal in expectation but not necessarily the same as the ML(*xz*) estimator. This situation arises when the sample is proportionally allocated to the strata. In this case, the sampling weights and adjustment factors are the same for the weighted and unweighted estimators (i.e., $\hat{f}_x^{wrr} = \hat{f}_x^{urr}$).

In summary, when the nonresponse cells have a homogeneous response propensity, the estimators ML(*xz*), weighted and unweighted, have the same form and are unbiased. On the other hand, comparisons where the ML(*xz*) estimator uses homogeneous response groups as nonresponse cells while the weighted and unweighted estimators use non-homogeneous response rates give unfair advantage to the ML(*xz*) estimator because it is always unbiased while the weighted and weighted estimator are always biased because the model fails.

The LV paper advocates the use of design (strata) information in creating the nonresponse cells. While we do not disagree with this suggestion, we would rephrase it to say that their results show the importance of correctly specifying the model. When the same model is used in the simulation situations, the weighted and unweighted estimators are essentially the same as the ML(*xz*) estimator, and comparisons are uninformative.

5. The Estimators of the Mean

In this section we examine the comparison between the weighted and unweighted estimator of the mean. The weighted estimator of the mean for the population of the LV paper can be written as $\hat{\bar{Y}}_{wrr} = \hat{Y}_{wrr}/\hat{N}_{wrr}$ where $\hat{N}_{wrr} = \sum_z \sum_x \sum_r \hat{f}_x^{wrr} w_z$. Substituting $\hat{f}_x^{wrr} = \sum_z(\frac{n_{xz}N_{+z}}{n_{+z}})/\sum_z(\frac{r_{xz}N_{+z}}{n_{+z}})$ and $w_z = N_{+z}/n_{+z}$ we observe $\hat{N}_{wrr} = \sum_z \sum_x \sum_r \hat{f}_x^{wrr} N_{+z}/n_{+z} = N$. In other words, the weighted estimator can be written as $\hat{\bar{Y}}_{wrr} = \hat{Y}_{wrr}/N$.

Clearly, the weighted estimator of the mean $\hat{\bar{Y}}_{wrr}$ is not a ratio estimator (the denominator is not a random variable); the weighted estimator of the mean in the LV paper is linear estimator with a constant denominator. This is the direct result of the way the nonresponse factor adjusts the weights to always recover the total *N*.

On the other hand, the unweighted estimator is a ratio estimator because the estimate of *N* in the unweighted estimator is a random variable. As a result, the unweighted mean $\hat{\bar{Y}}_{urr}$ and the weighted mean $\hat{\bar{Y}}_{wrr}$ have different statistical properties. The former takes advantage of the correlation between $\hat{Y}_{urr}$ and $\hat{N}_{urr}$. When the two are positively correlated, the bias for the unweighted estimator is reduced (as $\hat{Y}_{urr}$ increases so does $\hat{N}_{urr}$ so the bias is reduced); this is one of the attractive features of ratio estimators. The weighted estimator does not benefit from the correlation because the denominator is fixed. We believe a better comparison for evaluating the utility of using the weights for computing the nonresponse factor is to use a weighted estimator that is a ratio.

To do this, we evaluated the MSE for two ratios. In the first ratio we computed another variable Q with same distribution as *Y*. The second ratio is the mean of a domain of *Y*, where the domain is randomly determined within *Y*. As the domain begins to approach the full population, the denominator of the ratio has less variability and goes to *N* and the situation is similar to that investigated in LV.

6. Findings

We repeated the LV simulation but expanded it in some ways. We included estimators of the total and ratios as discussed above. The ratios include domain means that we believe are more reflective of estimates used in practice. We also varied the sampling rates while holding the total sample fixed; LV had fixed sample sizes by stratum. The simulation was written in *R* (R Development Core Team 2011) using the package *survey* (Lumley 2011) with 10,000 runs. We evaluated the estimator examining the root mean squared error (RMSE) and its components (bias and variance). Better estimators are those with a lower

RMSE. We note that for those cases that matched those in LV, our results confirm those they reported.

6.1 Estimators of Totals

Figure 1 shows the RMSE of the weighted and unweighted estimates for the total for the population defined by $[XZ]^Y = (\gamma_x = 2, \gamma_z = 2, \gamma_{xz} = 2)$ and response propensity $[X + Z]^R = (\beta_x = 2, \beta_z = 2, \beta_{xz} = 0)$. We chose this combination because the LV paper shows that the weighted estimator of the mean underperforms compared to the unweighted estimator in this situation. The horizontal axis represents the relative sampling rate defined as the ratio of the sampling rates computed as $rsmp = (n_o/N_o)/(n_1/N_1)$. The simulation in the LV paper corresponds to the relative sampling rate of 2.25.

As expected, the RMSE of the estimates are the same when the sample is proportionally allocated (the lines cross when the relative sampling rate = 1). However, while the RMSE of the unweighted mean is a function of the relative sampling rate, the RMSE of the weighted estimator is almost constant regardless of the sampling rate. Notice that in this case, if the unweighted total is used for the LV fixed rate of 2.25, then the RMSE is almost twice that for a weighted total. When the second stratum is oversampled with respect to the first stratum (relative sampling rate less than one), the unweighted total can be better, but its performance is not uniform.

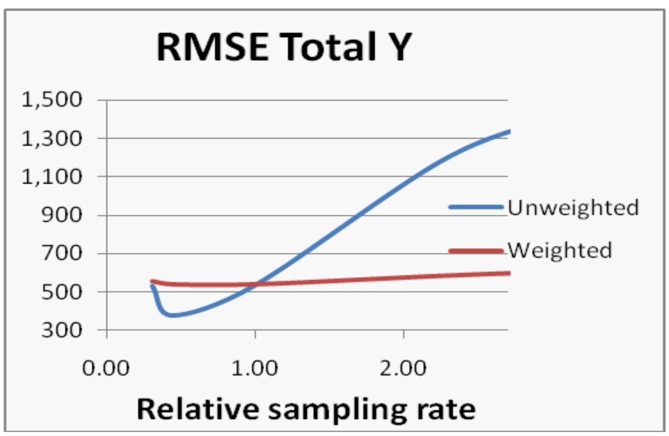

Figure 1: RMSE x 10,000 of Total Y for 10,000 runs

Figures 2 and 3 show the bias and variance of the estimates of the total *Y*. The bias of the weighted estimator is constant and does not depend on the sampling rate. There is an increase of the variance at high sampling rates, but this effect is the result of the small sample size in one stratum and one nonresponse adjustment cell. If we restrict the nonresponse adjustment cells to include a minimum of 35 respondents, we expect the contribution of the variance to the RMSE to be approximately constant. In contrast, for the unweighted estimator the bias is highly variable and is the main contributor to the RMSE. It is interesting that in these simulations, using the weighted estimator does not increase the variability of the estimator greatly, which is a common concern raised about weighting the rates.

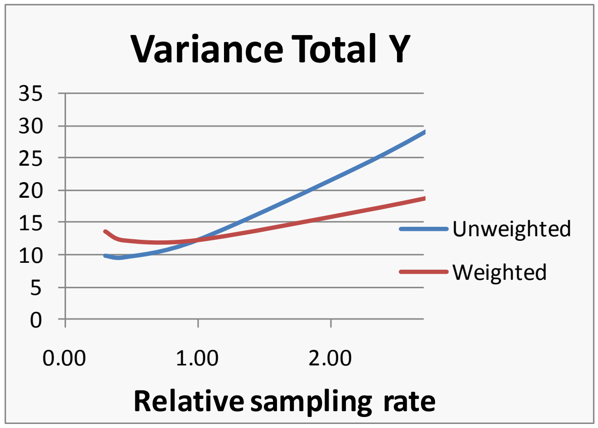

Figure 2: Variance of Total Y for 10,000 runs

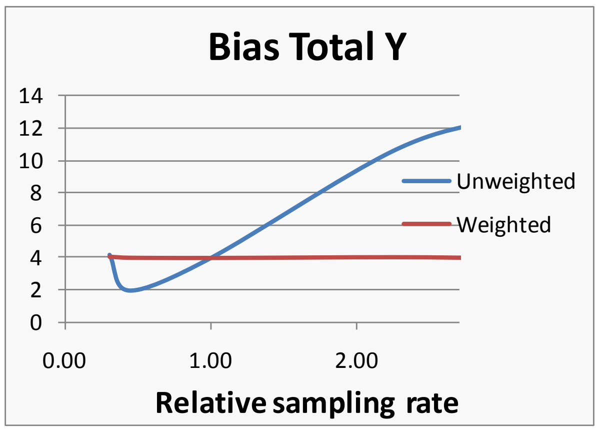

Figure 3: Bias of Total Y for 10,000 runs

This suggests to us that weighting the rates is important. We think it is desirable to use estimators where the bias is not highly dependent on how the sample is allocated. In most surveys, the sample is allocated to achieve a predetermined precision and not to reduce the bias of a particular estimator by oversampling or undersampling nonresponse adjustment cells.

6.2 Estimators of Ratios

Before examining the comparisons of ratios, we review the comparison of the means as presented in the LV paper. Figure 4 shows the RMSE of the weighted and unweighted means from the LV paper for the population $[XZ]^Y = (\gamma_x = 2, \gamma_z = 2, \gamma_{xz} = 2)$ and response propensity $[X + Z]^R = (\beta_x = 2, \beta_z = 2, \beta_{xz} = 0)$. These results confirm the findings in the LV paper that the unweighted estimator performs better than the weighted estimator for a relative sampling rate =2.25 (the only rate they used). Figure 4 shows this conclusion does not hold for other rates; if the second stratum is oversampled, we reach the opposite conclusion. This highlights a generic problem of making generalizations from simulations.

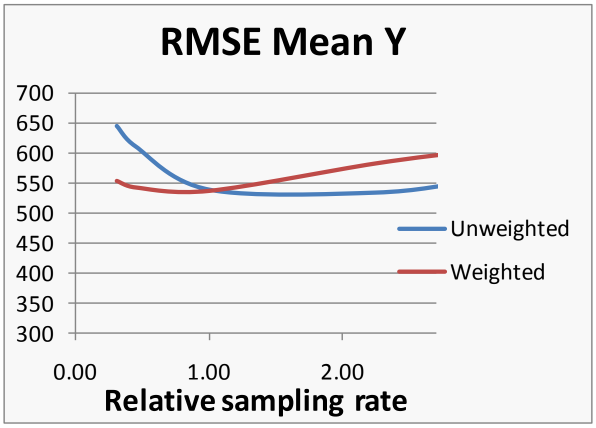

Figure 4: RMSE x 10,000 of Mean Y for 10,000 runs

Figures 5 and 6 show the components of the RMSE for the estimates of the means. As shown in the table, the main component of the RMSE is the bias because both estimates have comparable variances independent of the sampling rate.

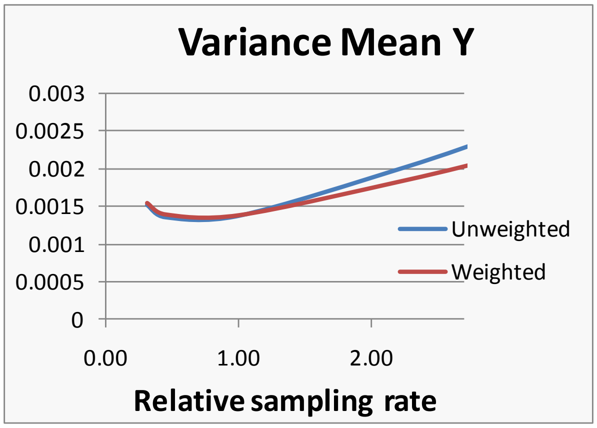

Figure 5: Variance of Mean Y for 10,000 runs

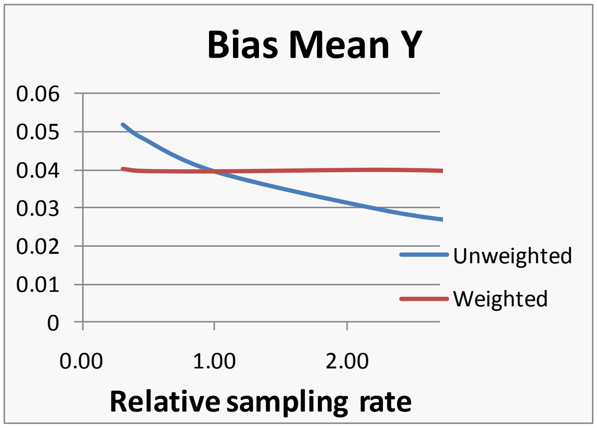

Figure 6: Bias of Mean Y for 10,000 runs

As we mentioned before, the LV mean comparisons are between two different types of estimators (one is a ratio and the other is not). We examine the performance of comparable ratio estimators. We created another variable *Q* that has the same distribution of *Y* and computed the ratio estimate of *Q/Y*. We note that this example is not realistic because the model fails in the same way on both variables of the ratio. In practice, it is unlikely to have homogeneous response propensity within the cells for different variables unless they are highly correlated.
Figure 7 shows the RMSE of the ratio *Q/Y*. The figure shows that the reweighted ratio performs better when the relative sampling rate is greater than one, while it slightly underperforms when the rate is less than one.

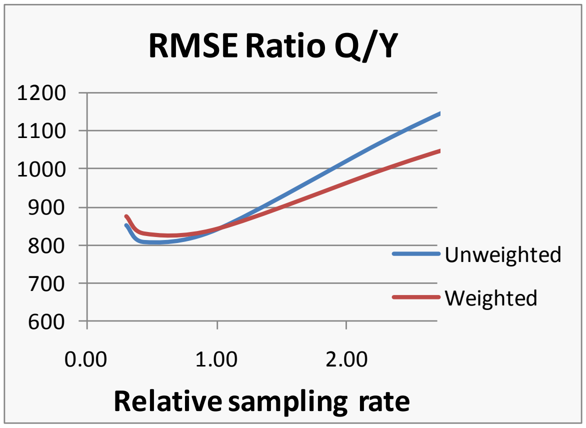

Figure 7: RMSE x 10,000 of Ratio Q/Y for 10,000 runs

Figures 8 and 9 show the components of the RMSE of the estimators of the ratio *Q/Y*. Similar to the previous findings, the bias of the reweighted estimator when the model fails is constant and it does not depend on the sampling rate. Furthermore, the variance of the reweighted estimator is lower than the variance of the unweighted estimator. This is the opposite to the conclusion of the LV paper.

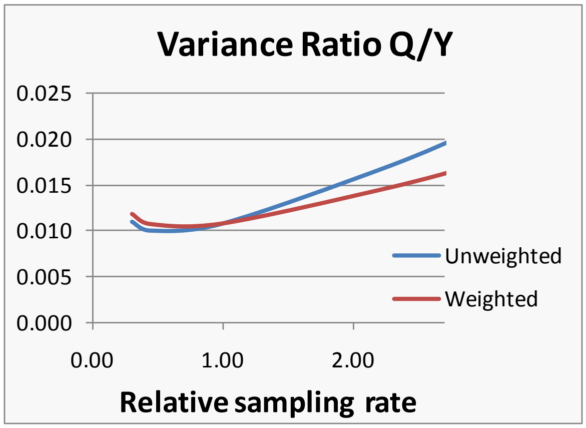

Figure 8: Variance of Ratio Q/Y for 10,000 runs

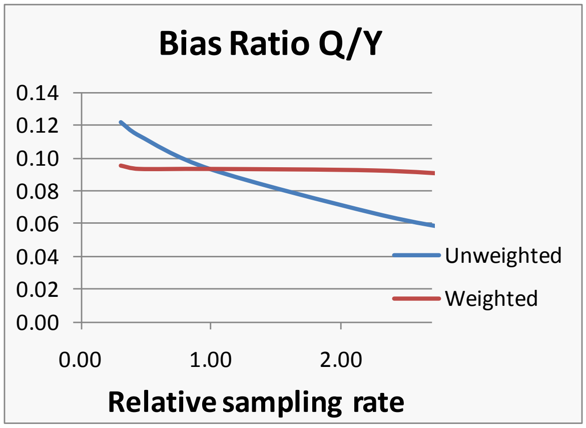

Figure 9: Bias of Ratio Q/Y for 10,000 runs

We simulated two domain means of *Y*. Choosing a domain of *Y* ensures that the estimators compared are ratio estimators. The domains are a random one half and one fourth of *Y*. The first domain was randomly selected from *Y* independently of the strata and nonresponse adjustment cell. The second domain was randomly selected from the first domain. Figures 10 and 11 show the RMSE of the domain means of *Y* when the domains are 25 percent and 50 percent.

Figure 10 shows that the weighted and unweighted estimators for the 25 percent domain have the same performance for a relative sampling rate greater than one. The weighted estimator also performs better when the relative sampling rates are less than one. The situation is different for the mean of the 50 percent domain. For a relative sampling rate greater than one, the unweighted estimator performs better. As in the 25 percent domain, the weighted estimator performs better when the relative sampling rate is less than one.

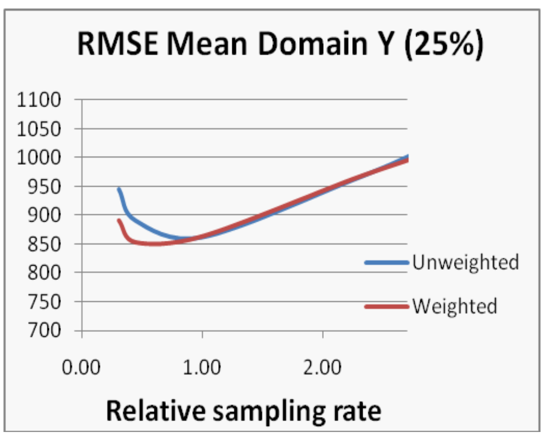

Figure 10: RMSE x 10,000 of Mean Domain Y (25%) for 10,000 runs

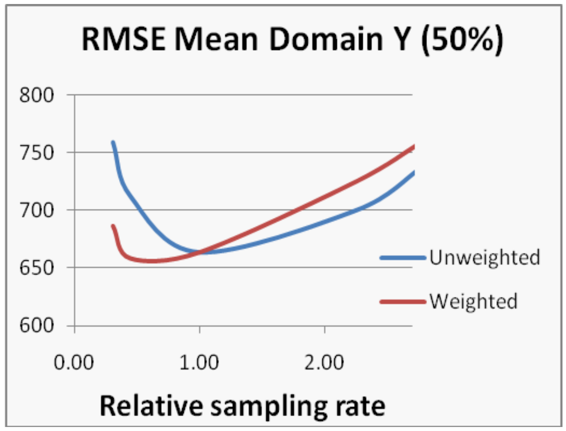

Figure 11: RMSE x 10,000 of Mean Domain Y (50%) for 10,000 runs

7. Comments and Conclusions

We believe these simulations help understand the difference between using a weighted and unweighted nonresponse adjustment factor in several ways that were not addressed in LV. The LV study did not include any situation where the model failed for all estimators (the ML model was always satisfied, so in our context, it only provides a measure of the deviation from the ideal).

With respect to the findings, we observed that the bias (and consequently the RMSE) of the unweighted estimator was highly affected by the sampling rates, while the weighted estimator was not. In fact, the bias of the weighted estimators in the cases we explored was constant across sampling rates, while the bias of the unweighted estimator could be very large for some rates. The variances of the estimates using the weighted rates were nearly the same as those of the unweighted estimators, showing that using weights does not always increase the variance of the estimates substantially.

Another important extension was to other types of estimators. LV considered only means, but our extension showed that some of their findings did not apply to totals and ratios. In fact, the findings for totals were very different. In this vein, we also compared means in a way that we believe is more typical in practice by treating both as ratio estimators.

Our research is driven by the belief that bias is the main problem to be addressed by nonresponse adjustment. While weighting the rates does not eliminate bias if the model is wrong (we are not aware of claims that it should), we have seen that in our limited simulations the bias is constant when the rates are weighted. This implies the bias is not a function of the sample allocation, which we consider to be an important and reassuring finding. While there are situations where the unweighted estimator has a lower bias than the weighted, these situations are difficult to predict in practice and to this would make it difficult to take advantage of this better performance. We find the highly variable bias of the unweighted estimator troubling.

Finally, as we noted above, generalizations based on simulations are always tentative because other conditions that are not simulated may give very different findings. Our findings are based on simulations so we plan to explore some of these findings analytically so that we can better predict when the results might hold more generally.

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