

Sample Correlations of Current Population Survey Unemployment Statistics

Reid Rottach

Demographic Statistical Methods Division, US Census Bureau, Washington DC 20233

Abstract

Each month, the Current Population Survey (CPS) interviews housing units (HUs) from eight rotating panels (rotation groups). Each panel is interviewed for four consecutive months, followed by eight months of not being in sample, followed by four more months of interviewing. One of the eight panels is new to the survey in any given month, replacing an outgoing panel selected from the same second-stage cluster of HUs. All panels in all months within a design are selected from the same primary sampling units, which are groups of counties. This approach to sampling results in a complex pattern of correlations among panel totals within and across months. Furthermore, the approach we use to construct estimation weights also affects correlations. We model the effect of different stages of sampling and weighting adjustments on CPS correlations, and in turn, on variances and their decomposition.

Key Words: composite weighting, calibration, annual sampling, phase-in/phase-out, variance components, modeling

1. Introduction

We are currently researching possible design changes to the CPS, which could affect the statistical accuracy of tests; specifically, determining the significance of a month-to-month change in unemployment. This suggests the need to study their effects on both variances and correlations.

In particular, we look at two design changes, referred to as annual sampling and phase-in/phase-out. These will affect the *correlations* among rotation group totals at different time lags but will not affect the *variance* of a rotation group total. So, for each time lag, we will construct an 8×8 correlation matrix that represents the correlations between all possible pairings of the eight rotation groups. We begin with directly estimated correlation matrices for all lags and across 57 months of data from the 2000 sample design of the CPS. The months studied are from August 2005 through April 2010. These correlations are then summarized so they depend only on the lag. Following this, we hypothesize what effect the design changes will have on the cells in the correlation matrix and make those adjustments.

Any views expressed are those of the author and not necessarily those of the U.S. Census Bureau.

From the derived correlation matrices, we obtain:

- Correlations of the CPS calibration and composite estimators
- The ratios of the variances of the different estimators
- The ratio of the variance due to PSU sampling to the total variance

These should:

- Be consistent with direct estimates when they are available
- Lead to non-negative components of variance

2. Sample Designs Studied

2.1 Current Sample Design

The 2000 design of the CPS has approximately 55,000 interviewed HUs each month. The survey is used to gauge the current labor force status of the civilian non-institutional population of the country. Key national statistics are measures of unemployment, employment, and not-in-labor-force, which can be broken down by different demographic groups. State and substate-level data is also of interest.

Sample for the survey is essentially selected in two stages, although a third stage of subsampling occurs in the field when the interviewer encounters a cluster of HUs in sample much larger than expected. In the first stage, primary sampling units (PSUs) formed of groups of counties are selected from state-level strata. Many of the strata consist of just one PSU that is selected with certainty; these are self-representing (SR) PSUs. One non-self-representing, or NSR, PSU is selected from each of the remaining strata, with probability proportional to size in the stratum. Within PSUs, clusters of HUs formed on demographic variables, generally block-level proportions, are selected systematically. These clusters make up most of the sample for the entire 2000 design of the survey, the remainder coming from new construction that is continually added to the sample. In any given month, approximately four HUs from each of these clusters is in sample; these are referred to as hits or ultimate sampling units, and the cluster in its entirety is referred to as a hit string. From month to month, approximately three-fourths of the hits in sample will be the same, and one-fourth will be replaced with new hits in the hit string.

In particular, the pattern of hits entering and exiting sample across months is based on eight rotating panels of which six overlap month to month and two are replaced. As an illustration, consider the following Figure 1, which represents four consecutive months of sample.

July	1	2	3	4	5	6	7	8			
August	.	2	3	4	5	6	7	8	1		
September	.	.	3	4	5	6	7	8	1	2	
October	.	.	.	4	5	6	7	8	1	2	3

Figure 1: Rotation groups from four months in 2008

The numbers one through eight represent different rotation groups. Each hit string is assigned to only one rotation group, so if we assume the hit strings are sampled approximately independently within the PSUs, then each of the eight rotation groups are

approximately independently selected within the PSUs. The columns of Figure 1 identify overlapping hits between months of sample. For example, between July and August, rotation groups {2,3,4,6,7,8} consist of overlapping hits, while rotation groups {1,5} consist of overlapping hit strings, but not hits. Each hit will be in sample for four months, followed by eight months of not being in sample, followed in turn by four more months in sample, which is the rotation scheme referred to as 4-8-4. For example, in August, rotation group 1 is in sample for the first time, and it can be seen that it continues in sample through September and October. If we were to continue adding months to the chart through November 2009, the 4-8-4 pattern for the hits represented by that column would emerge.

2.2 Annual Sampling

As part of the 2010 Redesign of the CPS, we are considering a new approach to sampling in which HUs will be selected within PSUs each year, rather than every ten years as we do now. We refer to this as annual sampling, and refer to one year's worth of sample as an annual sample. One consequence of annual sampling is a loss of correlation due to overlapping hit strings; when an incoming rotation comes from a different annual sample than the outgoing rotation it replaces, we assume the correlation between the groups is zero since they were approximately independently selected. This is different from the current design in which the incoming and outgoing rotation groups are always selected from the same hit string, and we expect a positive correlation between those groups.

Consider the effect at a one month lag. We currently have two rotation groups with overlapping hit strings. One of these is permanently outgoing in the first month, and will be replaced with an incoming rotation selected from a different annual sample (e.g., rotation group 1 between July and August in Figure 1). We know they would be selected from a different annual sample because they first came into sample 16 months apart. The other rotation group is temporarily outgoing in the first month, and will be in the same annual sample as the replacement rotation one-third of the time (e.g., rotation group 5 between July and August in Figure 1). We infer they would be in a new annual sample one-third of the time because they first came into sample eight months apart, which suggests that eight times out of 12 a new annual sample would have started in that span of time.

Compared with the current design, we expect the hit string correlations to be reduced by one-half when just one of the outgoing rotation groups is replaced by hits in a different annual sample (one-third of the time), and reduced to zero when both are (two-thirds of the time). So, on average, this is a 5/6 reduction in hit string correlation at lag 1. Following similar arguments, we approximate the reduction in hit string correlations as 7/8 for lag 2, 11/12 for lag 3, 23/24 for lag 4, and a complete reduction for the rest.

2.3 Phase In/Phase Out

The phase in of new PSUs as old design PSUs are phased out adds a complication to between-month correlations during that time period. We consider the phase in/phase out (PIPO) at the interface of the 1990 and 2000 sample designs to study its effect. Beginning April 2004 through June 2005, the CPS sample had HUs from both designs. There were three geographic areas where PIPO was separately implemented: the geographic areas in common between the 1990 and 2000 PSUs (continuing), the areas only represented in the 1990 sample (outgoing), and the areas only represented in the 2000 sample (new). Between April and November 2004, the 2000 design sample in continuing areas was phased in; between August 2004 and June 2005, the 2000 design sample in new areas

assigns nonrespondents a zero weight and inflates the weights of respondents in a way to approximate the inverse probability of selecting a respondent. Following this, a first-stage ratio adjustment is made to units in NSR PSUs. This adjustment is designed to bring the weighted population total within each NSR stratum closer to the Decennial Census population for that stratum, which will generally reduce the variance in estimates due to first-stage selection.

The next three adjustments to the weights are each a type of calibration to independent controls. They are the national coverage adjustment, the state coverage adjustment, and the second-stage ratio adjustment. Each of these adjusts the weights from prior weighting steps to meet controls. The coverage adjustments are post-stratification, which is a single ratio adjustment applied to the weights, while the second-stage ratio adjustment is performed iteratively, since adjusting to one set of controls may lead to deviations from another set of controls. Convergence to all controls is generally obtained within ten iterations, so the number of iterations has been fixed at ten. This iterative process is referred to as raking, and the resulting weights can be examined using results in sources such as Deville, et al. (1993) for raking ratio weights.

The national coverage adjustment rakes each of four rotation group pairs (1 and 5, 2 and 6, 3 and 7, 4 and 8) to demographic control groups defined by age groups within race, sex, and ethnicity categories. Equivalently, we describe these pairs as month-in-sample (MIS) pairs, where the MIS in a given month is a numbering of one through eight of the newest (incoming) to the oldest (outgoing) sample rotation. The state coverage adjustment uses state-level control groups, where the numbers of cells differ by state, and in some states raking is done for MIS pairs. The second-stage ratio adjustment is performed separately for the MIS pairs and adjusts simultaneously to control groups at the state level and more detailed controls at the national level. These are defined on age, race, sex, and ethnicity.

3.2 Compositing

The final adjustment to weights is the compositing step. These weights are created by first producing composite estimates of labor force characteristics, and then ratio adjusting the calibration weights to these in the same manner as we do for independent controls. The composite controls are of employment and unemployment levels, cross-classified by state and demographic categories. The composite estimator is given by equation [3.1]. The calibration weights are raked for ten iterations to meet these values.

$$Y'_t = (1 - K)\hat{Y}_t + K(Y'_{t-1} + \Delta_t) + A\hat{\beta}_t, \tag{3.1}$$

$$\text{where } \hat{Y}_t = \sum_{i=1, \dots, 8} x_{t,i}; \Delta_t = \frac{4}{3} \sum_{i \in \{2,3,4,6,7,8\}} (x_{t,i} - x_{t-1,i-1})$$

$$\hat{\beta}_t = \sum_{i \in \{1,5\}} x_{t,i} - \frac{1}{3} \sum_{i \in \{2,3,4,6,7,8\}} x_{t,i}$$

$i \in \{1,2,3,4,5,6,7,8\}$ represents MIS

$x_{t,i}$ represents the calibration estimates at month t and MIS i

$K = \{0.4 \text{ for unemployed}; 0.7 \text{ for employed}\}$

$A = \{0.3 \text{ for unemployed}; 0.4 \text{ for employed}\}$

More details on the weighting adjustments can be found in *CPS Technical Paper 66* (US Census Bureau, 2006).

4. Correlations and Variance Ratios

4.1 Notation

The covariances we discuss depend on different stages of sampling, different levels of overlap, the rotation group in the MIS pair, and the lag. We will identify correlations by these four indexes, as in:

$$\rho_{stage,level,type,lag}$$

where $stage = \{tot, w, b\}$ for total, within, and between components, respectively; $level = \{h, hs, p\}$ to identify overlapping hits, hit-strings, or just PSUs, respectively; $type = \{d, i\}$ for directly overlapped or indirectly (the same MIS pair), respectively; $lag = \{0, 1, 2, \dots\}$

If an index is replaced by a dot in one of our expressions, it suggests that the correlations among all possible values of the index are theoretically equal. Details about covariance estimation and how we partition covariances into within- and between- components are in Attachment 1.

4.2 Within-PSU Correlations of Rotation Group Totals

Within a design, all rotation groups will have a positive correlation due to the first-stage selection of PSUs. In the second stage, the systematic selection of hit strings could lead theoretically to a correlation, although it is common to model this as though the clusters selected were independent. An example would be the paired selection model described in Wolter (1985). Under this model, the within-PSU sampling correlation among rotation groups within a given month would be zero. Many of the weighting adjustments, though, would likely have induced correlations among the rotation groups. We will consider only those adjustments after the first stage, in which case there are weighting adjustments performed on all rotation groups together, and within MIS pairs. These adjustments generally induce correlations between units that are negative on average, which has some intuition, since adjusting one of the weights up may generally require other weights to be adjusted down. So we expect correlations that are negative on average among all rotations, with the largest negative correlations between rotation groups in the same MIS pair.

The within-PSU correlation matrix for two vectors of rotation group totals will have the approximate structure:

$$\mathbf{R}_{w,lag} = \begin{bmatrix} \mathbf{M}_d & \mathbf{M}_i \\ \mathbf{M}_i & \mathbf{M}_d \end{bmatrix} \quad [4.1]$$

where \mathbf{M}_d and \mathbf{M}_i are diagonal matrices. The main diagonal has elements $\rho_{w,h,d,\ell}$ if the rotation group has overlapping hits at time lag ℓ , and $\rho_{w,hs,d,\ell}$ if it only has overlapping hit-strings; both off-diagonals have elements $\rho_{w,h,i,\ell}$ if the MIS pair of the rotation group has overlapping hits at that time lag, and $\rho_{w,hs,i,\ell}$ if its MIS pair has overlapping hit-strings. This ignores the correlation due to calibration across all MIS pairs, which would lead to non-zero values in the elements we approximated as zero. We found that for

national estimates of unemployment, those elements were close enough to zero to be ignored.

4.3 Between-PSU Correlations of Rotation Group Totals

Unlike the within-PSU correlations, we would expect all between-PSU correlations of rotation group totals to be nonzero (i.e., large enough that we would not approximate them as zero) since they were selected from the same set of PSUs. For the rotation group pairs that had a within-PSU correlation close to zero, the covariance can be expressed as either side of the equation $\sigma_{tot}^2 \rho_{tot,p,\bullet,\ell} = \sigma_b^2 \rho_{b,p,\bullet,\ell}$. This leads to $\rho_{tot,p,\bullet,\ell} = (\sigma_b^2 / \sigma_{tot}^2) \rho_{b,p,\bullet,\ell}$, in which the left-hand side is the correlation we estimate directly using the collapsed stratum estimator (see Attachment 1 for details). At lag 0, the between-PSU correlation is one, which provides an estimate of the ratio of between to total variance of the rotation group totals: $\rho_{tot,p,\bullet,0} = \sigma_b^2 / \sigma_{tot}^2$.

The correlation due to sampling PSUs is the same for all pairs of rotation groups. This can be inferred from our definition of between-PSU covariance along with the assumption that all rotation group totals have the same expected value (see Attachment 1). The between-PSU correlation matrix is given by equation [4.2] below. The total covariance and correlation matrices are shown in equations [4.3] and [4.4].

$$\mathbf{R}_{b,\ell} = \frac{\rho_{tot,p,\bullet,\ell}}{\rho_{tot,p,\bullet,0}} \mathbf{1}_{8 \times 8}, \text{ where } \mathbf{1}_{8 \times 8} \text{ is an } 8 \times 8 \text{ matrix of ones} \quad [4.2]$$

$$\text{Cov}_{tot}(\hat{\mathbf{y}}_t, \hat{\mathbf{y}}_{t+\ell}) = \sigma_w^2 \mathbf{R}_{w,\ell} + \sigma_b^2 \mathbf{R}_{b,\ell} \quad [4.3]$$

$$\mathbf{R}_{tot,\ell} = (1 - \rho_{tot,p,\bullet,0}) \mathbf{R}_{w,\ell} + \rho_{tot,p,\bullet,0} \mathbf{R}_{b,\ell} \quad [4.4]$$

4.4 Correlations and Variance Ratios for the Calibration Estimator

The calibration estimator is a sum across rotation groups of the rotation group totals, $\hat{Y}_t = \mathbf{1}' \tilde{\mathbf{y}}_t$, where $\tilde{\mathbf{y}}_t$ is the 8×1 vector of rotation group totals. The covariance of two monthly estimates at time lag l is given in equation [4.5], where σ_{stage}^2 is the variance of a single rotation group total, assumed to be constant, and $\mathbf{R}_{stage,l}$ is a matrix of rotation group correlations at lag l . The correlation is then given in equation [4.6]. The ratio of between to total variance follows from the expression for $\mathbf{R}_{tot,l}$, and is given in equation [4.7].

$$\text{Cov}_{stage}(\hat{Y}_t, \hat{Y}_{t+\ell}) = \sigma_{stage}^2 \mathbf{1}' \mathbf{R}_{stage,\ell} \mathbf{1} \quad [4.5]$$

$$\text{Corr}_{stage}(\hat{Y}_t, \hat{Y}_{t+\ell}) = \frac{\mathbf{1}' \mathbf{R}_{stage,\ell} \mathbf{1}}{\mathbf{1}' \mathbf{R}_{stage,0} \mathbf{1}} \quad [4.6]$$

$$\frac{V_b(\hat{Y}_t)}{V_{tot}(\hat{Y}_t)} = \frac{\mathbf{1}' \mathbf{R}_{b,0} \mathbf{1} \rho_{tot,p,\bullet,0}}{\mathbf{1}' \mathbf{R}_{tot,0} \mathbf{1}} = \frac{64 \rho_{tot,p,\bullet,0}}{\mathbf{1}' \mathbf{R}_{tot,0} \mathbf{1}} \quad [4.7]$$

4.5 Correlations and Variance Ratios for the Composite Estimator

From the composite estimator given in equation [3.1] we can derive equation [4.8], where \mathbf{c} is a 16×1 vector of parameters computed from A and K. The left-hand side of this

equation is close to Y'_t when $m \geq 5$, so we consider approximations to Y'_t and Y'_{t-1} of the form given in equations [4.9] based on $m=5$. The vectors \mathbf{d}_0 and \mathbf{d}_1 are 56×1 , and are computed from \mathbf{A} and \mathbf{K} . The first eight components of \mathbf{d}_1 are zero, and we can choose the last eight components of \mathbf{d}_0 to be zero to maintain an exact equality with the right-hand side of equation [4.8]. This leads to an estimated covariance and correlation at a one-month lag given by equations [4.10] and [4.11]. We can use this to approximate the ratio of the variance of the composite estimator to the variance of the calibration estimator, as in equation [4.12], as well as the ratio of the between to total variance of the composite estimator, as in equation [4.13].

$$Y'_t - K^m Y'_{t-m} = \mathbf{c}' \left(\begin{Bmatrix} \hat{\mathbf{y}}_t \\ \hat{\mathbf{y}}_{t-1} \end{Bmatrix} + K \begin{Bmatrix} \hat{\mathbf{y}}_{t-1} \\ \hat{\mathbf{y}}_{t-2} \end{Bmatrix} + K^2 \begin{Bmatrix} \hat{\mathbf{y}}_{t-2} \\ \hat{\mathbf{y}}_{t-3} \end{Bmatrix} + \dots + K^{m-1} \begin{Bmatrix} \hat{\mathbf{y}}_{t-(m-1)} \\ \hat{\mathbf{y}}_{t-m} \end{Bmatrix} \right) \quad [4.8]$$

$$Y'_t \cong \mathbf{d}'_0 \begin{Bmatrix} \hat{\mathbf{y}}_t \\ \hat{\mathbf{y}}_{t-1} \\ \dots \\ \hat{\mathbf{y}}_{t-6} \end{Bmatrix} \text{ and } Y'_{t-1} \cong \mathbf{d}'_1 \begin{Bmatrix} \hat{\mathbf{y}}_t \\ \hat{\mathbf{y}}_{t-1} \\ \dots \\ \hat{\mathbf{y}}_{t-6} \end{Bmatrix} \quad [4.9]$$

$$Cov_s(Y'_t, Y'_{t-1}) \cong \sigma_s^2 \mathbf{d}'_0 \tilde{\mathbf{R}}_{stage} \mathbf{d}_1 \quad [4.10]$$

$$\text{where } \tilde{\mathbf{R}}_{stage} = \begin{bmatrix} \mathbf{R}_{s,0} & \mathbf{R}_{s,1} & \mathbf{R}_{s,2} & \dots & \mathbf{R}_{s,6} \\ \mathbf{R}_{s,1} & \mathbf{R}_{s,0} & \mathbf{R}_{s,1} & \dots & \mathbf{R}_{s,5} \\ \mathbf{R}_{s,2} & \mathbf{R}_{s,1} & \mathbf{R}_{s,0} & \dots & \mathbf{R}_{s,4} \\ \dots & \dots & \dots & \dots & \dots \\ \mathbf{R}_{s,6} & \mathbf{R}_{s,5} & \mathbf{R}_{s,4} & \dots & \mathbf{R}_{s,0} \end{bmatrix}$$

$$Corr_s(Y'_t, Y'_{t-1}) \cong \frac{\mathbf{d}'_0 \tilde{\mathbf{R}}_s \mathbf{d}_1}{\mathbf{d}'_0 \tilde{\mathbf{R}}_s \mathbf{d}_0} \quad [4.11]$$

$$\frac{V_s(Y'_t)}{V_s(\hat{Y}_t)} \cong \frac{\mathbf{d}'_0 \tilde{\mathbf{R}}_s \mathbf{d}_0}{\mathbf{1}' \mathbf{R}_{s,0} \mathbf{1}} \quad [4.12]$$

$$\frac{V_b(Y'_t)}{V_{tot}(Y'_t)} \cong \frac{\mathbf{d}'_0 \tilde{\mathbf{R}}_b \mathbf{d}_0}{\mathbf{d}'_0 \tilde{\mathbf{R}}_{tot} \mathbf{d}_0} \quad [4.13]$$

5. Numerical Results for Total Unemployed

5.1 Direct Estimates of the Parameters

Our expressions for correlation have depended on the lag l and not the time t . As is shown in Figure 3, the true correlations are constantly changing. The correlations due to sampling and weighting have a trend as well as a seasonal component. So if we were to average the correlations to use in our models, the results would be highly dependent on what months were selected for the averaging. We would like to choose feasible values for these while being mindful of the survey requirements.

In particular, one of the requirements for the CPS is that a difference of 0.2% in the unemployment rate across two months must be statistically significant at the 10% confidence level. The monthly standard errors used to define this requirement are based on an unemployment rate of 6%. The values we used (Table 1) were obtained by regressing the correlation on the seasonally-adjustment unemployment, and finding a predicted value for an unemployment rate of 6%. In some cases, we found that the correlations do not change much for different lags, so we chose a single value to use for those.

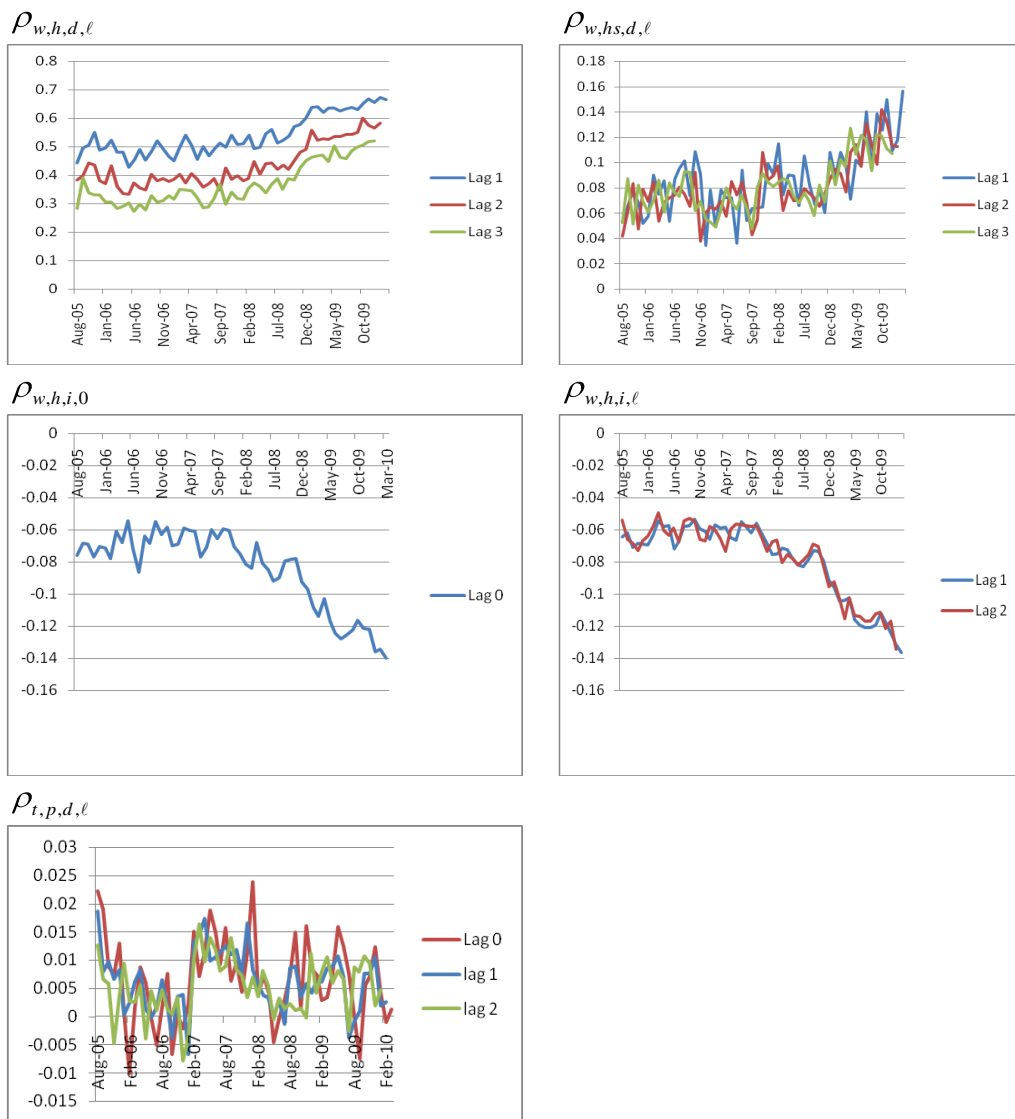


Figure 3: Correlation versus time

Table 1: Parameter Values Used for Our Model

<i>Lag</i>	$\rho_{w,h,d,\ell}$	$\rho_{w,hs,d,\ell}$	$\rho_{w,h,i,\ell}$	$\rho_{w,hs,i,\ell}$	$\rho_{tot,p,\bullet,\ell}$
0	1	.	-0.085	.	0.0065
1	0.536	0.082	-0.077	-0.053	0.0062
2	0.432	0.082	-0.077	-0.053	0.0053
3	0.368	0.082	-0.077	-0.053	0.0059
4, 5, 6	.	0.082	.	-0.053	0.0046

5.2 Adjusted Parameters and Output

The factors we applied to the rotation group correlations are given in Tables 2 and 3 below. For PIPO, we only computed factors for lags 0 and 1, and can therefore only present results for the calibration estimator.

In neither case was there a dramatic change in correlation or variance, as can be seen in Tables 4 and 5. The month-to-month correlation of the composite estimator is about 1.5% lower for the annual sampling design, according to this model. This would suggest that confidence intervals around a month-to-month change estimate would be slightly wider for annual sampling. This result is somewhat different from what was presented last year in Rottach, et al. (2009), where an even greater drop in correlation was estimated. A direct comparison of the results is included in Attachment 2.

During PIPO, there was a more noticeable drop when we compare to the lowest correlation, which was almost 6% below that of the current design. This is counteracted somewhat by slightly lower variances in those months due to sample being spread across PSUs in both designs. We present more results for PIPO in Lubich, et al. (2010), including lag 12 correlations and comparisons with direct estimates.

Table 2: Factors Applied to Certain Parameters for Annual Sampling

<i>Lag</i>	$\rho_{w,h,d,\ell}$	$\rho_{w,hs,d,\ell}$	$\rho_{w,h,i,\ell}$	$\rho_{w,hs,i,\ell}$	$\rho_{tot,p,\bullet,\ell}$
0	1	.	1	.	1
1	1	0.17	1	0.17	1
2	1	0.11	1	0.11	1
3	1	0.08	1	0.08	1
4	.	0.04	.	0.04	1
5,6	.	0	.	0	1

Table 3: Factors Applied to Certain Parameters for Lags 0 and 1 During PIPO

<i>Months</i>	$\rho_{w,hs,d,1}$	$\rho_{w,hs,i,1}$	$\rho_{tot,p,\bullet,1}$	$\rho_{tot,p,\bullet,0}$
March through June 2004	0.50	0.50	1	1
July	1	1	0.813	0.75
August	1	1	0.563	0.50
September	1	1	0.563	0.50
October	1	1	0.813	0.75
November through February 2005	0.10	0.10	1	1
March through June	0.55	0.55	1	1
July	1	1	1	1

Table 4: Lag 1 Correlations and Variance Ratios for the Current Design and Annual Sampling (AS)

	Correlation of \hat{Y}_t	Correlation of Y'_t	$\frac{V(Y'_t)}{V(\hat{Y}_t)}$	$\frac{V_b(\hat{Y}_t)}{V_{tot}(\hat{Y}_t)}$	$\frac{V_b(Y'_t)}{V_{tot}(Y'_t)}$
Current				0.054	0.056
<i>within-PSU</i>	0.384	0.399	0.954		
<i>total</i>	0.415	0.430	0.956		
AS				0.054	0.057
<i>within-PSU</i>	0.378	0.391	0.947		
<i>total</i>	0.409	0.423	0.950		

Table 5: Lag 1 Correlations and Variance Ratios During PIPO

<i>Months (t)</i>	Within Correlation of \hat{Y}_t	Total Correlation of \hat{Y}_t	$\frac{V_b(\hat{Y}_t)}{V_{tot}(\hat{Y}_t)}$	$\frac{V_{pip0}(\hat{Y}_t)}{V_{2000}(\hat{Y}_t)}$
March through June 2004	0.380	0.411	0.054	1
July	0.376	0.399	0.045	1
August	0.376	0.391	0.031	0.990
September	0.376	0.391	0.031	0.976
October	0.376	0.399	0.045	0.976
November through February 2005	0.377	0.408	0.054	0.990
March through June	0.381	0.412	0.054	1
July	0.384	0.415	0.054	1

References

- Deville, J. (1999), "Variance Estimation for Complex Statistics and Estimators: Linearization and the Residual Techniques," *Survey Methodology*, Vol. 25, No. 2, pp 193-203.
- Deville, J., Särndal, C.E., and Sautory, O. (1993), "Generalized Raking Procedures in Survey Sampling," *Journal of the American Statistical Association*, 88, 1013-1020.
- Ernst, L.R. (1986), "Maximizing the Overlap Between Surveys When Information is Incomplete," *European Journal of Operational Research*, 27, 192-200.
- Fay, R. E. and Train, G. (1995), "Aspects of Survey and Model-Based Postcensal Estimation of Income and Poverty Characteristics for States and Counties," *Proceedings of the Government Statistics Section, American Statistical Association*, pp. 154-159.
- Lubich, A., Rottach, R., Neiman, M., Coggins, B. (2010), "Current Population Survey Correlations Across and Within the 1990/2000 Phase-In/Phase-Out Period," *JSM Proceedings, Survey Research Methods Section, Alexandria, VA: American Statistical Association*.

- Rottach, R., Lubich, A., Reist, B. (2009), “Exploring Statistical Issues of Annual Sampling for the Current Population Survey,” Proceedings of the Section on Survey Research Methods, American Statistical Association, Washington, DC.
- U.S. Census Bureau (2006), “Current Population Survey: Design and Methodology, Technical Paper 66,” October 2006.
- Wolter, K. (1985), Introduction to Variance Estimation, New York: Springer-Verlag

Attachment 1. Covariance Estimation Using Weighted Residuals

We ignore the affect of weighting adjustments up to the first stage, essentially treating the weights up to that point as our design weights. We employ a variance estimator derived for the generalized regression estimator (GREG) and apply it to each of the three calibration adjustments used for the survey. The coverage adjustments are post-stratification, which is a particular case of the GREG. The final raking procedure is not generalized regression, but both approaches are calibration and have been shown to be close approximations of each other; see Deville, et al. (1993). Using the results for the GREG and applying it to other calibration estimators is a technique discussed in that paper. The variance estimator uses calibration-weighted residuals and can be viewed as an approximate linearization to our weighting adjustments. In the case of multiple complex adjustments (in our case, three successive calibration steps), we follow advice from Deville (1999).

For each calibration step, a vector of residuals is created by a linear projection matrix, \mathbf{P} . This is the complement of the hat matrix for a weighted least squares regression. For the post-stratification adjustments, the residuals can also be expressed by a vector minus a vector of means within the post-stratification cells. The raking adjustment is linearized by calibration-weighted least squares regression. The residual vector we use is $\mathbf{e}_t = \mathbf{P}_{nat} \mathbf{P}_{sc} \mathbf{P}_{ss} \mathbf{y}_t$, where \mathbf{P}_{nat} , \mathbf{P}_{sc} , and \mathbf{P}_{ss} represent the matrices corresponding to the second-stage, state coverage, and national coverage adjustment linearizations applied to a vector of survey responses, \mathbf{y}_t . There are two distinct covariance estimators that are applied to the residuals, motivated by a partition of the total covariance.

We partition the covariances into components due to sampling PSUs (between) and sampling within PSUs. This relationship can be expressed in a form consistent with the Law of Total Covariance, involving conditional expectations. In our case, we condition on the set of PSUs selected:

$$Cov(\hat{Y}_t, \hat{Y}_{t+\ell}) = Cov(E[\hat{Y}_t | \{PSUs\}], E[\hat{Y}_{t+\ell} | \{PSUs\}]) + E[Cov(\hat{Y}_t, \hat{Y}_{t+\ell} | \{PSUs\})],$$

where \hat{Y}_t is an estimator at time t ; $\{PSUs\}$ represents the set of PSUs selected; $E[\cdot]$ is expectation

We rewrite this expression with new notation, using subscripts to indicate total (*tot*), between (*b*), and within-PSU (*w*) covariance:

$$Cov_{tot}(\hat{Y}_t, \hat{Y}_{t+\ell}) = Cov_b(\hat{Y}_t, \hat{Y}_{t+\ell}) + Cov_w(\hat{Y}_t, \hat{Y}_{t+\ell})$$

The two covariance estimators we use are for total covariance and the covariance due to just within-PSU sampling. For within-PSU sampling, we use successive difference replication (*sdr*), as described in Fay and Train (1995). This is slightly different from the successive difference estimator found in sources such as Wolter (1985), and was intended for use as a replication estimator. It has a linearization form, though, and is a close approximation to the usual successive difference estimator. The estimator of total covariance uses successive differencing in SR PSUs, and a collapsed stratum estimator (*cs*) for the NSR PSUs. A description of *cs* can be found in Wolter (1985). This estimator involves forming collapsed strata, which in our case will consist of either two or three

sampling strata. For both *sdr* and *cs*, we first sum the residuals \mathbf{e}_t to the hit level, weighting by calibration weights. The estimators *sdr* and *cs* are given by equations [A.1] and [A.2].

$$\hat{C}_{sdr} = \frac{1}{2} \mathbf{d}'_t \mathbf{d}_{t+\ell} \quad [\text{A.1}]$$

where $\mathbf{d}_t = \begin{Bmatrix} \hat{\mathbf{e}}_t \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ \hat{\mathbf{e}}_t \end{Bmatrix}$; $\hat{\mathbf{e}}_t$ is the vector of totals of \mathbf{e}_t weighted by the calibration weights and summed to the hit level

$$\hat{C}_{cs}(\hat{Y}_t, \hat{Y}_{t+\ell}) = (\hat{\mathbf{e}}_{t,cs,+} - \bar{\mathbf{e}}_{t,+,+})' \mathbf{M}(\hat{\mathbf{e}}_{t+\ell,cs,+} - \bar{\mathbf{e}}_{t+\ell,+,+}) \quad [\text{A.2}]$$

where $\hat{\mathbf{e}}_{t,cs,+}$ is the vector of totals of $\hat{\mathbf{e}}_t$ summed to the sampling stratum level; $\bar{\mathbf{e}}_{t,+,+}$ is the vector of collapsed-stratum level means of the sampling stratum totals, where the ratio of the size (Census 2000 HU counts) of the sampling stratum to the average size of these within the collapsed stratum is then applied to each element; \mathbf{M} is a diagonal matrix with elements $(\# \text{ of strata collapsed})/(\# \text{ of strata collapsed} - 1)$

Attachment 2. Comparing the Correlation Model with the Simulation Results Presented in Rottach, Lubich, and Reist (2009)

These results are somewhat different from what were presented last year based on a simulation. To be able to make a more direct comparison of the two approaches, we estimated new parameters for the model with the same months used in the simulation and took a straight average rather than finding predicted values. The dates used were June 2006 through July 2008.

Table A1: Parameter Values for a Direct Comparison with Rottach, Lubich, and Reist (2009)

<i>Lag</i>	$\rho_{w,h,d,\ell}$	$\rho_{w,hs,d,\ell}$	$\rho_{w,h,i,\ell}$	$\rho_{w,hs,i,\ell}$	$\rho_{tot,p,\bullet,\ell}$
0	1	.	-0.070	.	0.0065
1	0.499	0.079	-0.065	-0.044	0.0062

Table A2: Within-PSU Correlations of the Calibration and Composite Estimators for the Direct Comparison

	$Corr_{w, model}(\hat{Y}_t, \hat{Y}_{t-1})$	$Corr_{w, simulation}(\hat{Y}_t, \hat{Y}_{t-1})$
Current design	0.359	0.361
Annual sampling	0.352	0.330
Difference	0.007	0.031

Our estimate of the difference in correlations presented last year was 0.031 ± 0.027 at the 10% confidence level, and our new estimate of this difference is 0.007. This value is within the confidence bounds from the simulation, so what may seem like a substantial change in estimates may be more a reflection of the lack of precision in our prior estimate.