Improving Efficiency of Ratio-Type Estimators Through Order Statistics

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Abstract
In simple random sampling, the ratio method of estimation is a well-known technique for estimating the population mean of a study variable when the population mean of the auxiliary variable is known. In this study, a novel ratio estimator based on order statistics is introduced to increase the efficiency of the traditional ratio estimator. It is shown that the proposed estimator is considerably more efficient than the traditional ratio estimator over a very wide family of symmetric distributions. The robustness properties of the proposed estimator are also studied via simulations.

Key Words: survey sampling, ratio-type estimators, order statistics, modified maximum likelihood, robustness, symmetric distributions

1. Introduction

In simple random sampling setting, the traditional ratio estimator is defined by

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}}\bar{X}\ , \tag{1.1}$$

where $\bar{y}$ and $\bar{x}$ are the sample means of the study variable *Y* and the auxiliary variable *X*. The traditional ratio estimator (1.1) is commonly used for estimating the population mean of the study variable when the population mean of the auxiliary variable is known. The mean square error (MSE) of the ratio estimator is given by

$$\mathrm{MSE}(\bar{y}_r) \cong \frac{1-f}{n}\left(R^2 S_x^2 - 2RS_{xy} + S_y^2\right), \tag{1.2}$$

where $f = n/N$, *n* is the sample size and *N* is the population size. In equation (1.2), S_{xy} is the population covariance between the auxiliary and the study variables, S_y^2 and S_x^2 are the population variances of *Y* and *X*, respectively, and *R* is the ratio of the population means which is given by $R = \bar{Y}/\bar{X}$. When the population coefficient of variation or the population kurtosis of the auxiliary variable is known, Sisodio and Dwivedi (1981), Singh and Kakran (1993) and Upadhyaya and Singh (1999) suggested to use this information to reduce the efficiency of the traditional ratio estimator (1). Ray and Singh (1981), suggested ratio estimators of the type

$$\bar{y}_{RS} = \frac{\bar{y} + \tilde{b}_L\left(\bar{x}^{\alpha} - \bar{X}^{\alpha}\right)}{\bar{x}^{\gamma}}\bar{X}^{\gamma} \tag{1.3}$$

where α and γ are scalars, $\tilde{b}_L = s_{xy}/s_x^2$ is the regression coefficient obtained by the least square (LS) estimation, s_{xy} is the sample covariance between the auxiliary and the study variables and s_x^2 is the sample variance of *X*.

Kadilar and Cingi (2004) considered the case where the population coefficient of variation and the population kurtosis of *X* is known, and they combined the estimators of Sisodio and Dwivedi (1981), Singh and Kakran (1993) and Upadhyaya and Singh (1999) with Ray and Singh's estimator given in (1.3) by taking $\alpha = 1$ and $\gamma = 1$. They suggested the following five ratio-type estimators:

$$\bar{y}_{KC1} = \frac{\bar{y} + \tilde{b}_L\left(\bar{X} - \bar{x}\right)}{\bar{x}}\bar{X}, \ \bar{y}_{KC2} = \frac{\bar{y} + \tilde{b}_L\left(\bar{X} - \bar{x}\right)}{\bar{x} + C_x}\left(\bar{X} + C_x\right), \ \bar{y}_{KC3} = \frac{\bar{y} + \tilde{b}_L\left(\bar{X} - \bar{x}\right)}{\bar{x} + \beta_2(x)}\left(\bar{X} + \beta_2(x)\right),$$

$$\bar{y}_{KC4} = \frac{\bar{y} + \tilde{b}_L\left(\bar{X} - \bar{x}\right)}{\bar{x}\beta_2(x) + C_x}\left(\bar{X}\beta_2(x) + C_x\right), \ \bar{y}_{KC5} = \frac{\bar{y} + \tilde{b}_L\left(\bar{X} - \bar{x}\right)}{\bar{x}C_x + \beta_2(x)}\left(\bar{X}C_x + \beta_2(x)\right), \tag{1.4}$$

where $C_x = S_x/\bar{X}$ is the population coefficient of variation of *X* and $\beta_2(x)$ is the kurtosis of *X*. By using the first order Taylor approximation, Kadilar and Cingi (2004) obtained the MSE equation of these estimators, which are called Kadilar-Cingi estimators (KCEs), as

$$\text{MSE}\left(\bar{y}_{KCi}\right) \cong \frac{1-f}{n}\left(R_{KCi}^2 S_x^2 + 2B_L R_{KCi} S_x^2 + B_L^2 S_x^2 - 2R_{KCi} S_{xy} - 2B_L S_{xy} + S_y^2\right) \tag{1.5}$$

where $i = 1,\ldots,5$ and $B_L = S_{xy}/S_x^2$. The population ratios R_{KCi} in (1.5) are given by

$$R_{KC1} = R = \frac{\bar{Y}}{\bar{X}}, \ R_{KC2} = \frac{\bar{Y}}{\bar{X} + C_x}, \ R_{KC3} = \frac{\bar{Y}}{\bar{X} + \beta_2(x)},$$

$$R_{KC4} = \frac{\bar{Y}\beta_2(x)}{\bar{X}\beta_2(x) + C_x}, \ R_{KC5} = \frac{\bar{Y}C_x}{\bar{X}C_x + \beta_2(x)}. \tag{1.6}$$

Although the ratio-type estimators are very valuable in estimating the mean of a finite population when the information on the auxiliary variable is known, they are very sensitive to data anomalies, especially to outliers; see Chambers (1986), Gwet and Rivest (1992) and Farrell and Barrera (2007). Kadilar et al. (2007) attempted to robustify the estimators (1.4), by replacing the least square estimator (LSE) $\tilde{b}_L$ with Huber's M-estimator. By using a real life example, they showed that these new estimators are more efficient than the ones given in (1.4) when there are outliers in the data. Motivated from

Kadilar et al. (2007), Oral and Kadilar (2010) suggested utilizing modified maximum likelihood (MML) methodology in the KCEs instead of Huber's M estimation. They showed that the MML-integrated KCEs can improve the efficiencies of the ratio-type estimators under data anomalies. However, in all of these studies, the performances of the estimators depend on some conditions that need to be satisfied in order to give better estimates.

In this study, we propose a new ratio-type estimator, which is based on MML estimation, and show that when the underlying error distribution is not normal, the proposed estimator has always less MSE than the traditional ratio estimator (1.1) via simulations. The method of MML estimation was originated by Tiku (1967, 1978, 1980) and has been used extensively in literature (Puthenpura and Sinha, 1986; Tan and Tabatabai, 1988; Tiku et al., 1986; Tiku and Suresh 1992; Oral, 2006). The methodology of MML is employed in situations where the maximum likelihood (ML) estimation is intractable. The MML estimators (MMLEs) have closed forms, and they are easy to compute. Vaughan and Tiku (2000) proved that under some regularity conditions, MMLEs have exactly the same asymptotic properties as ML estimators (MLEs), and for small n values they are known to be essentially as efficient as MLEs.

This study is organized as follows: In the next section, we briefly discuss MMLEs in long-tailed symmetric (LTS) distributions. In the third section, we propose a new estimator, which takes the advantage of order statistics. Then, with the help of simulations we show that the MSE of the proposed estimator is always smaller than the traditional ratio estimator unless the underlying distribution is normal. In Section 4, we search the robustness properties of the proposed estimator.

2. Long-tailed Symmetric Family

Let $y_1, y_2, ..., y_n$ be a simple random sample from a super-population f. In the framework of the super-population model, let f be a very wide symmetric family of distributions, that is, from the LTS family

$$f(y) = \frac{c}{\sigma\sqrt{k\pi}}\left\{1 + \frac{(y-\mu)^2}{k\sigma^2}\right\}^{-p}, -\infty < y < \infty, \tag{2.1}$$

where $c = \Gamma(p)/\Gamma(p-0.5)$, $\mu = \bar{X}b$, b is the regression coefficient, p is the shape parameter, $k = 2p-3$ and $p \geq 2$. Throughout this study, we will assume that the shape parameter p is known; when the shape parameter is not known, it is easy to determine a plausible one from Q-Q plots. Realize that when $p = \infty$, the distribution (2.1) reduces to a normal distribution $N(\mu, \sigma^2)$. The likelihood function obtained from (2.1) is given by

$$\ln L \propto -n \ln \sigma - p \sum_{i=1}^{n} \ln\left\{1 + \frac{1}{k} z_i^{\,2}\right\}, \quad z_i = (y_i - \mu)/\sigma .$$

Assuming that σ is known, the MLE of μ is the solution of the likelihood equation

$$\frac{d\ln L}{d\mu}=\frac{2p}{k\sigma}\sum_{i=1}^{n}g(z_i)=0\ ,\ \ g(z)=z\big/\left\{1+(1/k)z^2\right\}. \tag{2.2}$$

The equation (2.2) is known to have multiple roots for all $p<\infty$ (Vaughan, 1992a). It is not known how many roots equation (2.2) has, but their number increases with *n*. The methodology of MML proceeds in three steps as follows:

(a) express the likelihood equation in terms of the order statistics

$$y_{(1)}\le y_{(2)}\le \ldots \le y_{(n)}$$

(b) linearize the intractable term by using the first two terms of a Taylor series expansion around

$$t_{(i)}=E(z_{(i)}),\ z_{(i)}=(y_{(i)}-\mu)\big/\sigma \qquad (1\le i\le n)$$

(c) incorporate the linear function in the likelihood equation, and solve the resulting equation which has a unique solution and is the MMLE.

The values of $t_{(i)}$ $(1\le i\le n)$ are given in Tiku and Kumra (1981) for *p*=2 (.5) 10 and in Vaughan (1992b) for p=1.5 when $n\le 20$. For $n>20$, their approximate values can be used which are obtained from the equations

$$\frac{c}{\sqrt{k\pi}}\int_{-\infty}^{t_{(i)}}\left\{1+\frac{z^2}{k}\right\}^{-p}dz=\frac{i}{n+1}\ ,\ 1\le i\le n \tag{2.3}$$

Note that, $t=\sqrt{(\nu/k)}\,z$ has a Student's t-distribution with $\nu=2p-1$ degrees of freedom. Since complete sums are invariant to ordering, i.e.,

$$\sum_{i=1}^{n}y_i=\sum_{i=1}^{n}y_{(i)}\ ,$$

it immediately follows that

$$\frac{d\ln L}{d\mu}=\frac{2p}{k\sigma}\sum_{i=1}^{n}g(z_{(i)})=0\,. \tag{2.4}$$

Since $z_{(i)}$ is located in the vicinity of $t_{(i)}$ at any rate for large *n*, and $g(z)$ is linear in any small interval $a<z<b$, a Taylor series expansion of $g(z_{(i)})$ around $t_{(i)}$ is pertinent. The first two terms of this expansion yield

$$\begin{aligned} g(z_{(i)}) &\cong g(t_{(i)})+\left[z_{(i)}-t_{(i)}\right]\left\{\frac{d}{dz}g(z)\,\Big|_{z=t_{(i)}}\right\} \\ &=\alpha_i+\beta_i\,z_{(i)}\ ,\ 1\le i\le n \end{aligned} \tag{2.5}$$

where

$$\alpha_i = \frac{(2/k)t_{(i)}^3}{\left\{1+(1/k)t_{(i)}^2\right\}^2} \quad \text{and} \quad \beta_i = \frac{1-(1/k)t^2_{(i)}}{\left\{1+(1/k)t^2_{(i)}\right\}^2}. \tag{2.6}$$

It may be noted that for symmetric distributions, $t_{(i)} = -t_{(n-i+1)}$. Consequently,

$$\alpha_i = -\alpha_{n-i+1}\ ,\ \sum_{i=1}^{n}\alpha_i = 0\text{, and } \beta_i = \beta_{n-i+1}. \tag{2.7}$$

Incorporating (2.5)-(2.6) in (2.4) gives the modified likelihood equation:

$$\frac{d\ln L}{d\mu} \cong \frac{d\ln L^*}{d\mu} = \frac{2p}{k\sigma}\sum_{i=1}^{n}(\alpha_i + \beta_i z_{(i)}) = 0. \tag{2.8}$$

The solution of (2.8) is the MMLE $\hat{\mu}$:

$$\hat{\mu} = \sum_{i=1}^{n}\beta_i y_{(i)} \Big/ m\ ,\ m = \sum_{i=1}^{n}\beta_i\ ; \tag{2.9}$$

$\hat{\mu}$ is unbiased for μ and its variance is given by $V(\hat{\mu}) = (\boldsymbol{\beta}'\boldsymbol{\Omega}\boldsymbol{\beta})\sigma^2 / m^2$, where $\boldsymbol{\beta}' = (\beta_1, \beta_2 ..., \beta_n)$ and $\boldsymbol{\Omega}$ is the variance-covariance matrix of the standardized variates $z_{(i)}$, $(1 \le i \le n)$. The elements of $\boldsymbol{\Omega}$ are tabulated in Tiku and Kumra (1981) and in Vaughan (1992b). Vaughan and Tiku (2000) showed that, under some very general regularity conditions, a modified likelihood equation obtained as above is asymptotically equivalent to the corresponding likelihood equation, that is,

$$\lim_{n\to\infty}\frac{1}{n}\left\{\frac{d\ln L}{d\mu} - \frac{d\ln L^*}{d\mu}\right\} = 0. \tag{2.11}$$

It follows, therefore, that asymptotically the MML estimator $\hat{\mu}$ is the MVB estimator.

When σ is not known, the MMLE $\hat{\sigma}$ can be obtained similarly as (see Tiku and Suresh, 1992; Tiku and Vellaisamy, 1996):

$$\hat{\sigma} = \left(B + \sqrt{B^2 + 4nC}\right) \Big/ 2\sqrt{n(n-1)}\ , \tag{2.12}$$

where

$$B = (2p/k)\textstyle\sum_{i=1}^{n}\alpha_i y_{(i)}\text{ , and } C = (2p/k)\sum_{i=1}^{n}\beta_i\left(y_{(i)} - \hat{\mu}\right)^2. \tag{2.13}$$

Like $\hat{\mu}$, $\hat{\sigma}$ is asymptotically fully efficient, i.e., $\hat{\sigma}$ is asymptotically unbiased and its variance is equal to the MVB(σ); see Tiku and Suresh (1992), and Vaughan (1992). For small n, the bias in $\hat{\sigma}$ is negligible and its variance is only marginally bigger than the MVB. In other words, $\hat{\mu}$ and $\hat{\sigma}$ are both highly efficient for all n (Tiku and Suresh 1992, Vaughan 1992a). In fact, they are asymptotically fully efficient. It has been rigorously established (Bhattacharyya 1985, Tan 1985, Tiku and Suresh 1992, Vaughan 1992a, Vaughan and Tiku 2000, Akkaya and Tiku 2001) that the MMLEs have the following properties under some very general regularity conditions:

(1) asymptotically, they are fully efficient.

(2) for small samples, they are highly efficient.

(3) since the MMLEs are explicit functions of sample observations, they are easy to compute.

In this study, we take advantage of this method and open up the ratio-type estimation to data anomalies. We also discuss the robustness issues with respect to outliers.

3. Proposed Estimator and MSE Comparisons

Tiku and Bhasin (1982) and Tiku and Vellaisamy (1996) adapted the MMLE (2.9) in the context of sample survey and showed that adapting MMLEs lead to improvements in efficiencies in estimating the mean of a finite population. Motivated from this result, we suggest using

$$\bar{y}_{pr} = \frac{\hat{\mu}}{\bar{x}} \bar{X} , \tag{3.1}$$

when the population mean of the auxiliary variable is known. Realize that $\hat{\mu}$ in (3.1) is the MMLE given by (2.9). To compare the efficiencies of the proposed estimator (3.1) with the traditional estimator (1.1), we assume that the population mean of the auxiliary variable is known and the relationship between Y and X is given as $y_i = b x_i + e_i$, $1 \le i \le N$, so that a ratio-type estimator is intended to use. Let $y_1, y_2, ..., y_n$ be a simple random sample from a super-population f from (2.1) and let Π_N denotes the corresponding finite population consisting of N pairs $(x_1, y_1), ..., (x_N, y_N)$. To calculate the MSE of the proposed estimator (3.1), one has to calculate $\bar{y}_{pr}$ for all possible $C = C_n^N$ simple random samples of size n from Π_N. Since C is extremely large, we conduct a Monte Carlo study as follows:

We take N=100 in each simulation and generate Π_{100} pairs from the assumed super-population. From the generated finite population Π_{100}, we choose at random T=50000 of all the possible C_n^{100} simple random samples of size n (n=5, 10, 15) which gives 50000 values of $\bar{y}_{pr}$. To compare the efficiencies of the proposed estimator for a given n value, we calculate the following values of the MSEs

$$MSE(\bar{y}_r) = \sum_{k=1}^{T} \left(\bar{y}_r - \bar{Y}\right)^2 \Big/ T \text{ and } MSE(\bar{y}_{pr}) = \sum_{k=1}^{T} \left(\bar{y}_{pr} - \bar{Y}\right)^2 \Big/ T \tag{3.2}$$

where $\bar{Y} = \sum_{i=1}^{N} y_i \Big/ N$. To assure that the population correlation ρ_{xy} is sufficiently large, we choose the values of the parameter b in the model $y = bx + e$ such that the correlation coefficient of all populations is 0.75. In all the simulations, we generate y_i from (2.1) and x_i from $U(0,1)$ independently and calculate e_i for $1 \le i \le N$. Note that we take $\sigma^2 = 1$ in all simulations without loss of generality. The simulated values of the relative efficiencies

$$RE = 100\left\{ MSE(\bar{y}_{pr}) \big/ MSE(\bar{y}_r) \right\} \tag{3.3}$$

are given in Table 1. From Table 1, it is obviously seen that the proposed estimator is always more efficient than the traditional ratio estimator unless the distribution is normally distributed; when the distribution is normal, the two estimators give exactly the same results.

Table 1: The relative efficiencies $RE = 100\left\{ MSE(\bar{y}_{pr}) \big/ MSE(\bar{y}_r) \right\}$

n	p=2	p=3	p=5	$p=\infty$
5	64.755	82.373	96.358	100.00
10	49.684	64.576	94.226	100.00
15	45.105	54.880	93.741	100.00

4. Robustness

Outliers in sample data are a frequently encountered problem for survey statisticians (Chambers, 1986). In this section, we take N=100 and $\sigma^2 = 1$ without loss of generality and we search the robustness properties of the proposed estimator under two different outlier models as follows.

We assume that X is from $U(0,1)$ and the true model to be $LTS(p = 3, \sigma = 1)$. We determine our super-population models as:

(1) Outlier Model 1: $N - N_o$ observations from $LTS(p = 3, \sigma = 1)$ and N_o (we do not know which) from $LTS(p = 3, \sigma = 1.5)$,

(2) Outlier Model 2: $N - N_o$ observations from $LTS(p = 3, \sigma = 1)$ and N_o (we do not know which) from $LTS(p = 3, \sigma = 2.5)$,

We calculate N_o in the models (1)-(2) from the formula $[| 0.5 + 0.1N |]$ which is equal to 10 for N=100. Realize that the true model is $LTS(p = 3, \sigma = 1.5)$ and the models (1)-(2) are chosen as its plausible alternatives. The simulated values of the MSEs of the proposed estimator and the traditional ratio estimator are calculated from (3.2) as explained in the previous section. The MSE of the proposed estimator and the relative efficiencies obtained from (3.3) are given in Table 2. It is clear from Table 2 that the proposed estimator is more efficient than the traditional ratio estimator under both outlier models.

Realize that the MSE of the proposed estimator does not change much for a given n. Thus, the proposed estimator is more efficient and robust compared to the traditional ratio estimator.

Table 2: Relative efficiencies under super-populations (1)-(2)

	True Model		Model (1)		Model (2)	
n	$MSE(\bar{y}_{pr})$	RE	$MSE(\bar{y}_{pr})$	RE	$MSE(\bar{y}_{pr})$	RE
5	0.3217	82.373	0.4648	75.088	1.0481	65.595
10	0.1187	64.576	0.1790	52.813	0.3790	39.732
15	0.0705	54.880	0.1084	42.521	0.2186	28.999

5. Concluding Remark

From the results presented in this paper we conclude that the utilization of MML methodology can give considerably more precise estimates than those based on the classical assumption of normality, which hardly ever holds true. In this study we show that MMLE integrated ratio-type estimators can lead to more efficient estimators than the traditional ratio estimator over a very wide family of symmetric distributions. The results presented here will be extended to the skewed distributions in a future paper.

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