Applying Alternative Variance Estimation Methods for Totals Under Raking in SOI's Corporate Sample

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Abstract: SOI's Tax Year 2006 Corporate sample is a stratified Bernoulli sample of approximately 110,000 corporate income tax returns. Raking adjustments are performed using the sample's design strata, related to the return's size of assets and income, and the primary industry, based on collapsed categories of the six-digit NAICS code. We apply several alternatives to estimate the variance of national- and domain-level totals of several key variables of interest: ignoring raking, post-stratification, a Taylor series approximation, and the delete-a-group jackknife replication estimator (with 100 and 200 groups). Results demonstrate that the poststratified total had the highest variance estimates, while the linearization and Jackknife when implemented incorrectly produced variance estimates that were too small, despite large sample sizes.

Key words: administrative data, survey sampling, raking, Taylor series approximation

1. Introduction

1.1. SOI's 2006 Corporate Sample Design and Selection Method

The stratified Bernoulli sample design is used by most of SOI's cross-sectional studies (IRS Winter 2010). In each study's frame population, every unit has a unique identifier; the Employer Identification Number (EIN) is used for corporations. Each return's EIN is used to produce a permanent random number between 0 and 1, denoted r_i, for all units in the population ($i \in U$). Unit i is then selected for SOI's sample if $r_i < \pi_h$, where π_h is the pre-assigned sampling rate for stratum h that tax return i belongs to.

The Tax Year 2006 frame population includes all corporations organized for profit that filed a Form 1120 (Corporation Income Tax Return), Form 1120-A (Short-form Corporation Income Tax Return), Form 1120-F (Income Tax Return of a Foreign Corporation), Form 1120-L (Life Insurance Company Income Tax Return), Form 1120-PC (Property and Casualty Insurance Company Income Tax Return), Form 1120-REIT (Income Tax Return for Real Estate Investment Trusts), Form 1120-RIC (Regulated Investment Companies) or Form 1120S (Income Tax Return for an S Corporation) and posted to the IRS Business Master File (BMF) over the period of July 2006 through June 2008. Changes in the returns' information due to tax auditing are not included in the frame population. Table 1 shows the population and sample sizes for the 2006 sample:

Table 1. Tax Year 2006 Population and Sample Counts

Form Type	Population Size	Sample Size
Form 1120S	4,162,484	31,492
Form 1120 (without PTXC) and Form 1120-A	2,213,433	49,720
Form 1120-F	30,932	4,353
Form 1120-RIC	11,060	8,636
Form 1120-PC	6,127	1,589
Form 1120-REIT	1,214	979
Form 1120-L	911	484
Special Studies (incl. PTXC)	14,134	14,102

PTXC=Possessions Tax Credit

A Bernoulli sample was selected independently from each stratum, with rates ranging from 0.25% to 100%. Stratification for SOI's corporate sample first uses 1120 form type. Within the form type, the population is further stratified using either size of the return's assets alone, or both size of assets and a measure of income. Forms 1120 (with neither Form 5735 nor Form 8844 attached) and 1120-A are stratified by size of assets and size of "proceeds". The asset value is the largest of the absolute value of the three asset fields (asset value from the front page and beginning and ending asset values from the Balance Sheet). The "proceeds" measure is the larger of the absolute value of net income (total income minus total deductions) or the absolute value of "cash flow" (net income plus net depreciation plus depletion). Form 1120S is stratified by absolute value of total assets and size of ordinary income. Forms 1120 (with Form 5735 attached), 1120-F, 1120-L, 1120-PC, 1120-REIT, and 1120-RIC are stratified only using the absolute value of total assets.

The raking adjustments to improve the sample's industry-level estimates are performed only for the 1120/1120-A and 1120S non-certainty strata, i.e., returns that were selected for the sample with rates lower than 100%. Returns in these 20 strata constitute a large portion of the total corporate sample. Table 2 shows the sample sizes for the 2004-2006 samples.

Table 2. Raking Strata Sample Counts

Form Type	2004	2005	2006
1120/1120-A	23,037	34,349	35,341
1120S	13,133	24,737	24,596
Subtotal	36,170	59,086	59,937
Total sample size	146,269	116,150	111,355

The total sample size decreased in 2005. Four strata were changed from certainty strata to non-certainty strata which were then included in the raking.

2.2. Variables and Estimation Domains of Interest

We consider eleven variables of interest, all of whose variance estimates are published in the form of coefficients of variation (IRS 2006): Gross Receipts, Net Depreciation, Net Income, Cost of Goods Sold, Depreciable Assets, Total Assets, Net Worth, Total Taxes Computed After Credits, Total Receipts, Positive Income, and Deficit. These variables are more/less correlated with the stratification variables. The estimated correlations between the variables and total assets (which is highly correlated with the total assets used in stratification) are shown in Table 3:

Table 3. Estimated Correlation of Variables of Interest with Stratification Variables

Variable	Total Assets	Proceeds
Gross Receipts	0.51	0.57
Net Depreciation	0.43	0.43
Net Income	0.18	0.30
Cost of Goods Sold	0.44	0.30
Total Assets	0.96	0.48
Depreciable Assets	0.53	0.42
Net Worth	0.49	0.22
Taxes After Credits	0.27	0.26
Total Receipts	0.53	0.58
Positive Income	0.37	0.43
Deficit	0.17	0.04

In addition to national-level totals of these variables of interest, we are also interested in the estimated totals for twenty-one major industrial groupings. The major industry totals, and the number of SOI 2006 sample units and number of units in each industry in the entire sample and the raking strata, are shown in Table 4 (industries 8 and 21 were removed for small sample size disclosure):

Table 4. Major Industries and Sample Sizes

Industry Major #	Major Industry Name	# Sample Units	# Units in Raking Strata
1	Agriculture, forestry, fishing, and hunting	1,949	1,506
2	Mining	1,518	737
3	Utilities	414	138
4	Construction	9,397	7,483
5	Manufacturing	13,070	6,367
6	Wholesale trade	9,669	6,408
7	Retail trade	8,529	6,429
9	Transportation and warehousing	2,424	1,650
10	Information	3,065	1,617
11	Finance and insurance	20,211	2,686
12	Real Estate and rental and leasing	8,645	6,437
13	Professional, scientific, and technical services	7,315	5,229
14	Management of companies (holding companies)	6,589	1,241
15	Administrative and support and waste management & remediation services	2,253	1,592
16	Educational services	371	262
17	Health care and social assistance	2,837	2,194
18	Arts, entertainment, and recreation	1,086	798
19	Accommodation and food services	2,426	1,755
20	Other Services	2,138	1,837

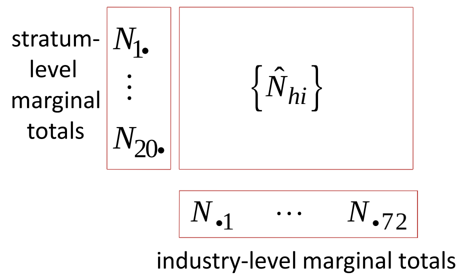
The number of sample units in each major industry varies widely, since the industry is not in the sample design. Most of the major industries have a large number of sample units, with the exception of utilities (3) and educational services (16). As the variables are highly skewed toward zero, we will see that this leads to higher estimated CVs. In addition, most of the industry majors are further broken into minor-level industries for the Complete Report publication (particularly manufacturing).

Several alternative variance estimators for totals estimated with raking adjustments have been proposed in the literature. In this paper, we apply some alternatives to national- and domain-level totals, then compare the empirical results.

2. Raking Algorithm

SOI's Corporate sample uses a bounded raking procedure (Oh and Scheuren 1987). The stratum-level weights are adjusted to also match marginal totals by 72 industrial groupings created by collapsing the 6-digit North American Industrial Classification System (NAICS) codes. Thus we have the setup for raking by stratum and industry that is shown in Figure 1 on the following page.

Figure 1. Raking Setup



The weights $w_h = \dfrac{N_{h\bullet}}{n_{h\bullet}}$ are adjusted such that they add up to the 72 group totals, taken from the BMF (the frame). SOI uses a bounded raking ratio method to produce these industry-level weights. The algorithm is summarized in the following steps:

(1) The initial weight (at iteration 0) is the poststratification weight in each matrix cell defined by (stratum ID) x (industry ID): $w_{hi}^{(0)} = \dfrac{N_{hi}^{(0)}}{n_{hi}} = \dfrac{N_{hi}}{n_{hi}}$.

The weights $w_{hi}^{(0)}$ add up across strata to the industry totals, the $N_{\bullet i}$'s, but they do not add to the strata totals, the $N_{h\bullet}$'s, so we adjust them.

(2) Use poststratification to adjust the $N_{hi}^{(0)}$ counts such that they add up to the strata totals, the $N_{h\bullet}$'s: $N_{hi}^{(1)} = \dfrac{N_{hi}^{(0)}}{\sum_{i=1}^{I} N_{hi}^{(0)}} \times N_{h\bullet}$.

The corresponding weights are $w_{hi}^{(1)} = \dfrac{N_{hi}^{(1)}}{n_{hi}}$. These weights add up across the industries to the strata totals, the $N_{i\bullet}$'s, but they do not add up to the marginal industry totals, the $N_{\bullet i}$'s, so we adjust the population counts again.

(3) Use poststratification to adjust the $N_{hi}^{(1)}$ counts such that they add up to the industry totals, the $N_{\bullet i}$'s: $N_{hi}^{(2)} = \dfrac{N_{hi}^{(1)}}{\sum_{i=1}^{20} N_{hi}^{(1)}} \times N_{\bullet i}$.

The corresponding weights are $w_{hi}^{(2)} = \dfrac{N_{hi}^{(2)}}{n_{hi}}$. These weights add up across the strata to the industry totals, the $N_{\bullet j}$'s, but they do not add up to the strata totals, the $N_{h\bullet}$'s. However, the weights $w_{hi}^{(2)}$ are closer to adding up to the $N_{h\bullet}$'s than the weights $w_{hi}^{(0)}$ from step 2. We repeat steps 2 and 3 until the sum of the raked weights are "close enough" to adding up to both the strata totals, $N_{h\bullet}$, and industry totals, $N_{\bullet i}$ (both are within 0.0001). This usually occurs in 15-20 iterations. During SOI production, the raking-based weights are further smoothed due to small (h,i) sample sizes and reduce the variance estimates. We exclude this step from our evaluation.

3. Alternative Estimation Methods for Totals and Their Variances

Method 1: *Before raking*. We denote $\hat{T} = \sum_{h=1}^{H} \frac{N_h}{n_h} \sum_{k \in h} y_k$ as the national-level total of the variable of interest y estimated using the stratum-level weights $\frac{N_h}{n_h}$. These are the conditional weights obtained after post-stratifying the base weights (based on the inverse probability of selection) to the frame population count within each stratum (Brewer 1979). This removes variability from the stratum sample sizes being random variables, which occurs using Bernoulli sampling. We can then use the stratified simple random sampling variance estimator

$$var\left(\hat{T}\right) = \sum_{h=1}^{H} \left(1 - \frac{n_h}{N_h}\right) \frac{N_h^2}{n_h\left(n_h - 1\right)} \sum_{k \in h} \left(y_k - \bar{y}_h\right)^2 . \tag{4.1}$$

We can modify this formula for domain (major industry)-level estimation by replacing all y_k 's with $z_k = y_k$ if $k \in j$ and 0 otherwise. SUDAAN does this with the "subpopn" statement, so the standard SUDAAN code for stratified simple random sampling was used to produce (4.1) estimates (RTI 2008).

Method 2: *post-stratification (PS)*. We use poststratification to adjust the stratum-level weights in Method 1 to also match the frame population counts of the 72 industry groups. The estimated total resembles the estimated total after one iteration of the raking algorithm, $\hat{T}_{PS} = \sum_{i=1}^{72} \frac{N_i}{\hat{N}_i} \hat{T}_i$, where $\hat{T}_i, \hat{N}_i$ are the estimated total of all y_k and estimated population size of post-stratum i , respectively. To estimate the variance, we simply use SUDAAN's proc descript (which uses a linearization variance estimator, p. 407 of RTI 2008). The raking industry ID in the "postvar" statement and put the associated 72 group population counts in the "postwgt" statement.

Method 3: *Taylor series linearization estimator (Binder)*. The final weight for unit k in cell hi is the ratio of the cell population count (from raking) to the cell sample count (a random number, since it is not in the sample design): $w_k = \frac{N_{hi}^{\text{final}}}{n_{hi}}$. The raking-based estimator for the total is

$$\begin{aligned} \hat{T}_{rake} &= \sum_{h=1(\text{stratum})}^{H} \sum_{i=1(\text{industry})}^{I} \sum_{k \in s_{hi}} w_k y_k \\ &= \sum_{h=1}^{H} \sum_{i=1}^{I} N_{hi}^{\text{final}} \bar{y}_{hi} \end{aligned} , \tag{4.2}$$

where $\bar{y}_{hi} = \frac{1}{n_{hi}} \sum_{k \in U_{hi}} \delta_k y_{hik}$ and the sample inclusion indictor for unit k is $\delta_k = 1$ if $k \in s$ and 0 otherwise. Binder and Theberge (1988) propose a linearization form for (4.2):

$$\hat{T}_{linear} = \sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{k \in s_{hi}} w_k d_k \ , \tag{4.3}$$

where $d_k = \alpha_{(k)} \beta_{(k)} \left[\sum_{h=1}^{H} \sum_{i=1}^{I} \left(lrc_{(k)} Z_{hi} \right) x_k + y_k \right]$, $\alpha_{(k)} \beta_{(k)}$ is the product of the row/column raking adjustments (i.e., $\alpha_{(k)} \beta_{(k)} = N_{hi}^{(0)} \big/ N_{hi}^{\text{final}}$), $lrc_{(k)} = 1$ if $k \in \left(h, i\right)$ and 0 otherwise, $x_k = 1$, and Z_{hi} is the unweighted sample total of y_k in the cell (h,i). Since (4.3) contains all linear terms, its variance under stratified random sampling is

$$\begin{aligned} var_L\left(\hat{T}_{linear}\right) &= Var\left[\sum_{i=1}^{I}\sum_{k\in s_{hi}} w_k d_k\right] \\ &= \sum_{h=1}^{H}\left(1-\frac{n_h}{N_h}\right)\frac{N_h^2}{n_h\left(n_h-1\right)}\sum_{k\in h}\left(\hat{Z}_k-\hat{\bar{Z}}_h\right)^2 . \end{aligned} \tag{4.4}$$

Method 5: *Jackknife replication (JKn)*. We also consider using a jackknife replication variance estimator (e.g., Ch. 4 in Wolter 1985). The jackknife is advertised as being simple to implement without the complicated analytic decompositions required for method 4. However here it requires that the base weights are recomputed for each replicate, then each replicate group weights are raked independently to the marginal totals (Section 2) to produce raking-based replicate weights to fully capture all the variability under raking. This is not equivalent to using the raking weights originally output from the algorithm to create replicate weights. Jackknife variance estimation for stratified sampling (Rao and Shao 1992; Yung and Rao 1996; 2000) involves the variance estimator

$$var_{Jk}\left(\hat{T}_{rake}\right)=\sum_{h=1}^{H}\left(\frac{n_h-1}{n_h}\right)\sum_{k\in h}\left(\hat{T}_{rake(k)}-\hat{T}_{rake}\right)^2 , \tag{4.5}$$

where $\hat{T}_{rake(k)}=\frac{n_h}{n_h-1}\sum_{h=1}^{H}\sum_{i=1}^{I}N_{hi(k)}^{\text{final}}\bar{y}_{hi(k)}$ is the estimate of (4.2) obtained when deleting unit k within stratum h. To avoid producing 56,396 sets of replicate weights using the delete-one-unit jackknife, we randomly assigned units to replicate groups and use the delete-a-group jackknife (variance estimator 4 on p. 179 in Wolter 1985), dropping an entire group of sample units within a stratum rather than one at a time. Since a relatively large number of groups is needed for unbiased variance estimation, we use 100 and 200 groups. Since Valliant *et al.* (2008) demonstrated that this variance estimator performed best using equal-sized groups within strata, we formed 5 groups/stratum for 100 groups and 10 groups/stratum for 200 groups. To do this, we randomly excluded a few returns within each stratum from calculating the (4.5) variance estimates: 46 (or 0.08% of the number of raking units) for the 100 groups and 96 (0.17%) for the 200 groups.

For the Jkn variance estimator to correctly capture all the variability incurred under the raking algorithm, it is important to apply the raking adjustments to all the replicate weights. However, the raking algorithm, as implemented by SOI, would not converge for the replicates, we therefore employed two versions of the jackknife. First, we formed the replicates using the stratum-level weights, which were then raked using WesVar's less restrictive raking algorithm to rake each set of replicate weights (Wesvar 2010), then used the raked replicate weights to calculate (4.5). Since this is the correct method, we call this "JKn right." We also formed the replicates after the SOI raking algorithm was used on the full sample, which we call "JKn wrong.". Theoretically (Valliant 1993), doing the JKn wrong method should produce variance estimates that are too large, however if they are relatively close to the JKn right variances, this is acceptable.

5. Results

5.1. CV Estimates of National-Level Totals

Here we compare the coefficients of variation (CVs) the estimated standard error of the total to the total itself: $CV\left(\hat{T}\right)=\hat{T}\Big/SE\left(\hat{T}\right)$ of the estimated totals before and after the raking adjustments are applied (plus the associated amounts from the non-raking strata). The totals produced using the alternative methods involving are shown in Table 5, while the associated CV's are in Table 6.

Table 5. Alternative Estimates of National-Level Totals (in '000's)

Variable	Before Raking	PS	Raking	JKn wrong*	JKn right*
Gross Receipts	23,316,050,615	24,071,677,303	23,237,955,489	23,305,363,739	23,631,535,973
Net Depreciation	564,066,591	574,796,168	563,052,332	563,588,783	568,458,799
Net Income	1,933,386,215	1,956,283,519	1,931,313,601	1,932,031,246	1,939,098,498
Cost of Goods Sold	14,803,061,967	15,272,232,347	14,786,820,104	14,808,444,052	14,988,307,543
Depreciable Assets	8,818,499,087	8,999,063,220	8,820,105,138	8,813,906,676	8,89,028,304
Total Assets	73,084,041,882	73,436,454,730	73,037,539,862	73,081,357,938	73,220,148,635
Net Worth	25,997,111,327	26,097,315,237	25,986,087,530	25,995,394,790	26,030,600,002
Taxes After Credits	353,141,737	355,336,193	353,573,395	353,146,160	354,205,839
Total Receipts	27,408,021,944	28,180,368,344	27,324,846,225	27,396,481,502	27,400,408,336
Positive Income	2,239,855,737	2,276,834,896	2,234,567,447	2,238,079,821	2,238,282,373
Deficit	306,469,522	320,551,377	303,253,846	306,048,576	306,178,587

* estimates from 200 groups shown.

The Table 5 totals are all close to the before raking, which means that the industry-level weighting adjustments do not have a large impact on the national-level totals of our variables of interest. The estimated Raking, JKn wrong, and JKn right totals in Table 5 are difference because we randomly deleted units to create the equal sized JKn groups. But, as Table 6 demonstrates, the resulting difference is negligible.

Table 6. CVs (as %'s) of National-Level Totals, Under Alternative Variance Estimation Methods

			Variance methods accounting for raking				
	Before Raking Total	**PS Total**	**Raking Total**	**JKn wrong Total**		**JKn right Total**	
Variable	*Direct Variance*	*SUDAAN*	*Binder*	*JK100*	*JK200*	*JK100*	*JK200*
Gross Receipts	0.23	0.30	0.23	0.23	0.22	0.24	0.24
Net Depreciation	0.17	0.21	0.17	0.19	0.18	0.22	0.20
Net Income	0.59	0.44	0.44	0.44	0.43	0.44	0.44
Cost of Goods Sold	0.29	0.39	0.29	0.32	0.29	0.33	0.31
Depreciable Assets	0.13	0.17	0.13	0.15	0.13	0.17	0.15
Total Assets	0.01	0.03	0.01	0.01	0.01	0.01	0.01
Net Worth	0.04	0.05	0.04	0.04	0.04	0.04	0.04
Taxes After Credits	0.13	0.14	0.12	0.13	0.13	0.16	0.16
Total Receipts	0.20	0.26	0.20	0.21	0.19	0.21	0.21
Positive Income	0.50	0.37	0.36	0.36	0.36	0.36	0.36
Deficit	0.56	0.60	0.53	0.56	0.51	0.62	0.56

Like the Table 5 totals, the coefficients of variation in Table 6 are also very close. Generally the post-stratification totals have the largest coefficients of variation across the alternatives. In addition, the Binder and JKn wrong CVs are generally too small, when compared to both of the the JKn right CVs for the Net Depreciation, Cost of Goods Sold, Depreciable Assets, Taxes After Credits, and Deficit variables. The CVs being smaller when the replicate weights are formed incorrectly is the opposite of those in Valliant *et al.* (2008), where the incorrect jackknife replicate groups lead to more conservative variance estimates.

5.2. Variance Estimates of Major Industry-Level Raking Totals

To compare the variance estimates for the domain-level raking-based totals, Figure 2 on the following pages shows plots of the ratio of the four raking alternatives for each variable of interest to the variance of the JKn right with 200 groups. In each plot, the 19 majors are sorted by descending total sample size (see Table 4).

Figure 2. Ratio of Alternative Variance Estimates for Raking Totals to Jkn Right With 200 Groups, by Major Industry

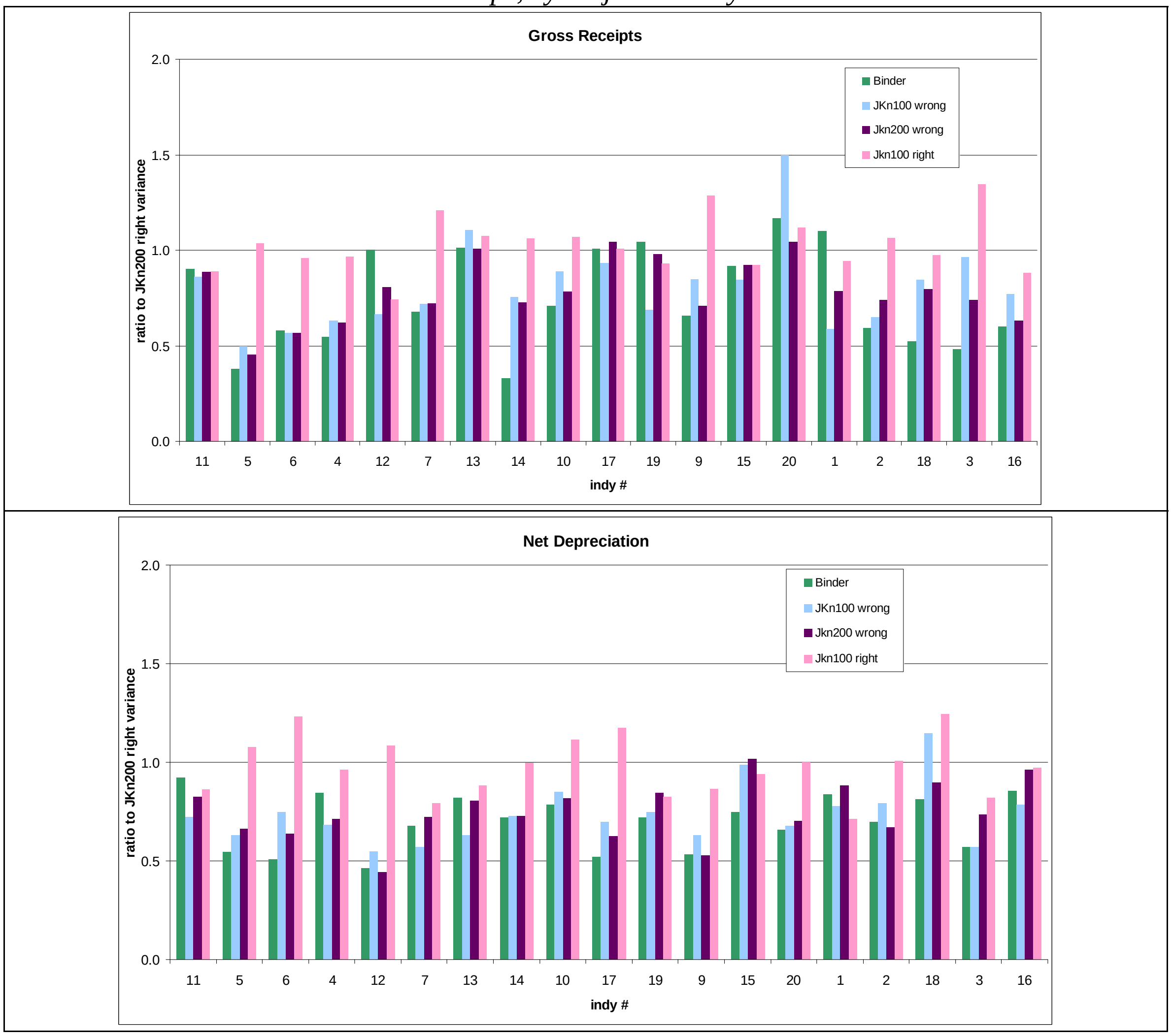
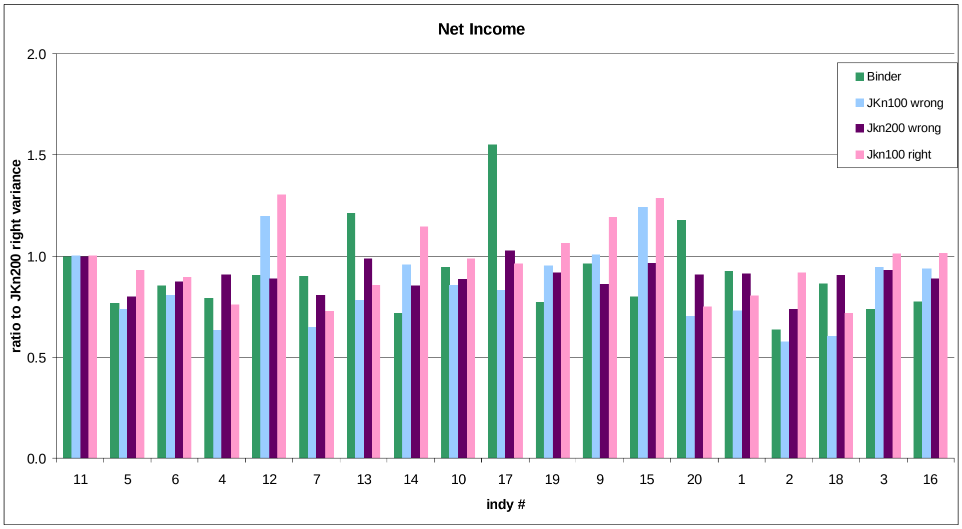
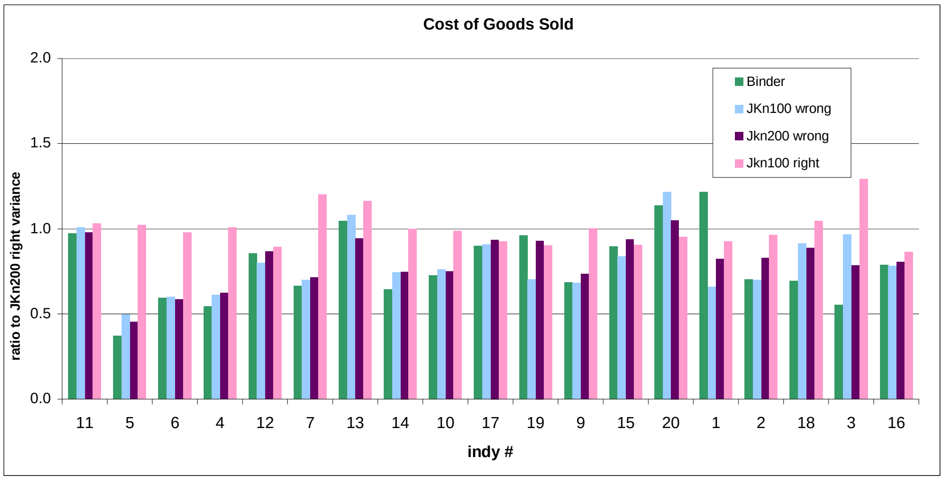


Figure 2. Alternative Estimated Coefficients of Variation of Major Industry-Level Totals – cont'd





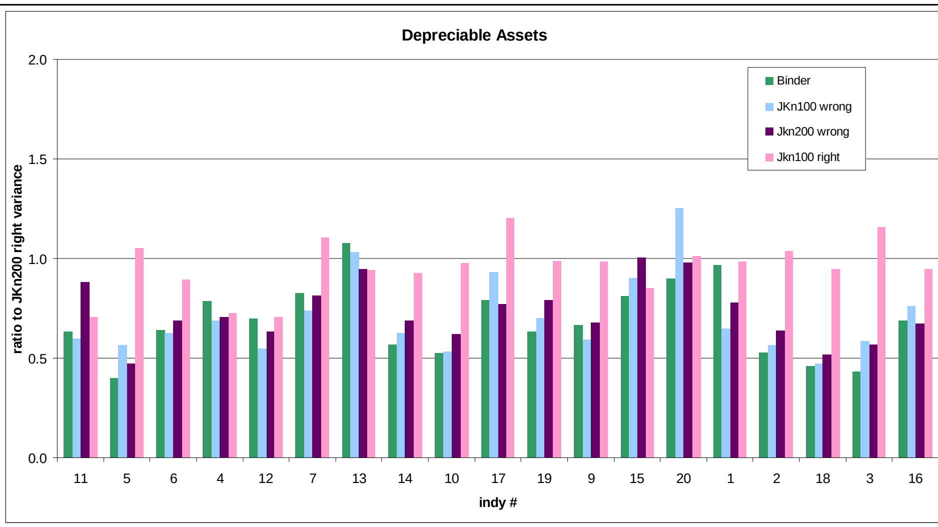
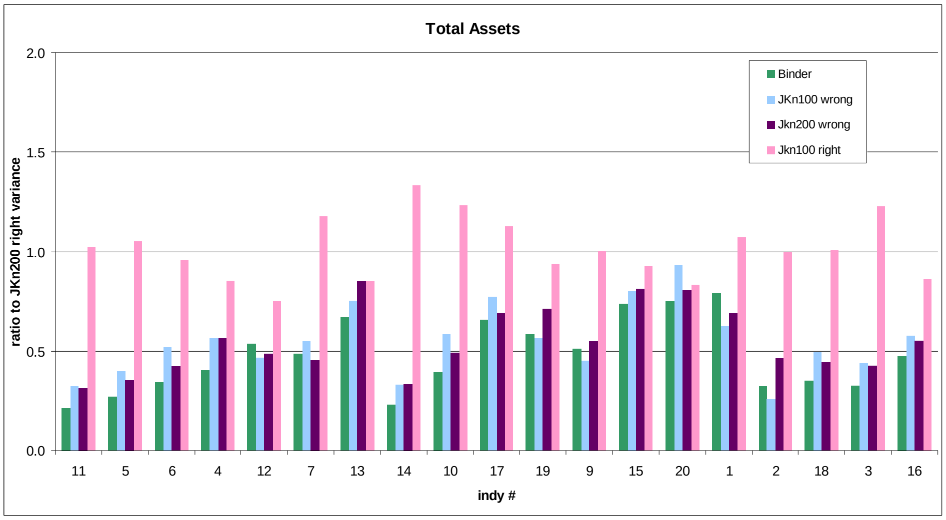


Figure 2. Alternative Estimated Coefficients of Variation of Major Industry-Level Totals – cont'd



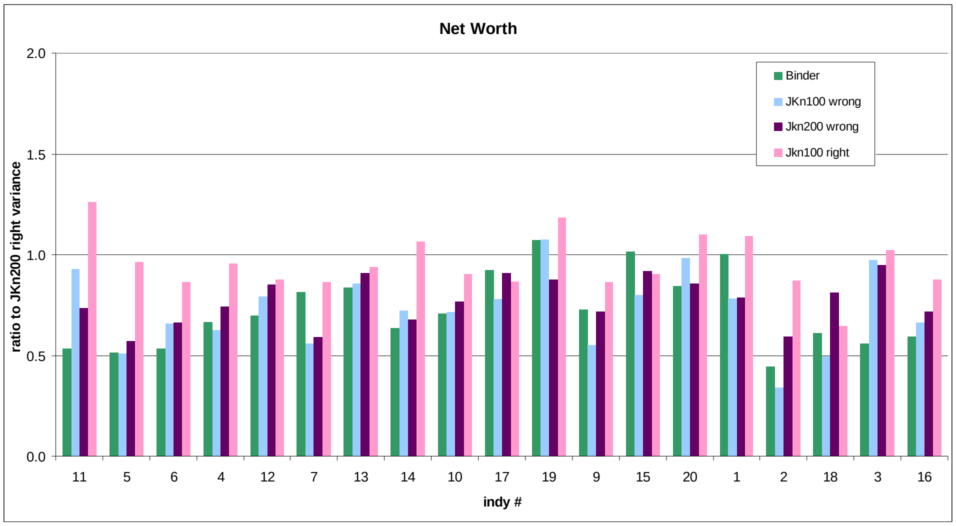
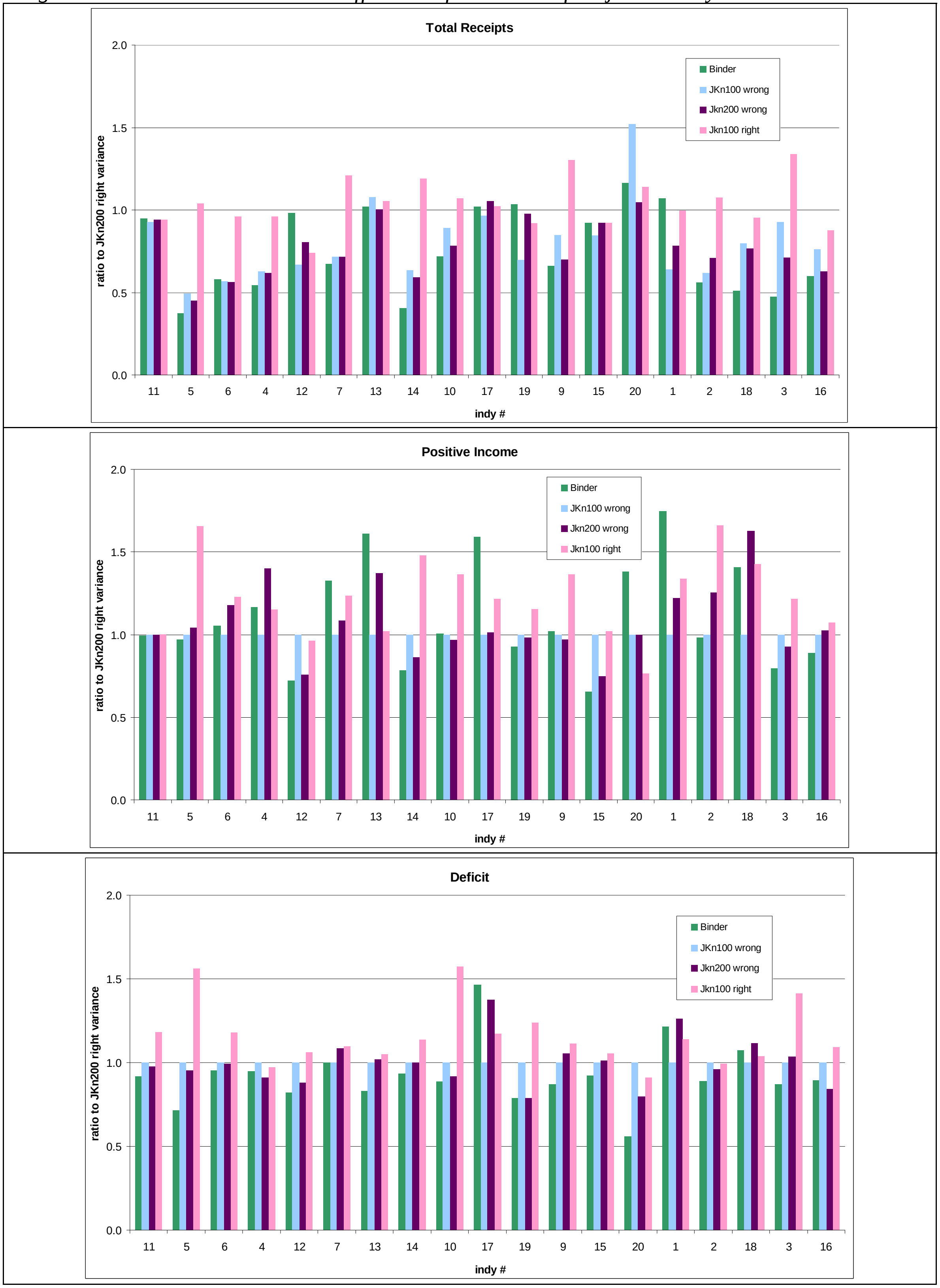


Figure 2. Alternative Estimated Coefficients of Variation of Major Industry-Level Totals – cont'd



In all plots, ratios equal to one indicate that a variance estimate is equivalent to the JKn right results with 200 replicate groups. We see that the alternative industry-level variance estimates are generally smaller than the JKn 200 right variance estimates, indicated by ratios less than one. This indicates that the Binder linearization method and implementing the jackknife incorrectly lead to smaller variance estimates. There generally is less of a difference for the JKn 100 right variance estimates. It is also difficult to discern any patterns related to the sample size, from larger industries on the left of each plot to the smaller industries on the right.

6. Conclusions

We applied some alternative estimators of totals and their variances to data collected in SOI's 2006 corporate sample. For our application, the post-stratification estimated totals (with poststrata defined by 72 industry groups) had larger variances than either the stratified estimator (with no poststratification or raking) or the raking estimator (with margins defined by design stratum and industry). For alternatives used to estimate the variance of totals under raking adjustments, generally the Binder Linearization and group jackknife with incorrectly formed replicate groups methods produced variance estimates that were both too small, despite having large sample sizes.

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