# An Empirical Evaluation of Various Direct, Synthetic, and Traditional Composite Small Area Estimators 

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#### Abstract

Currently, the Statistics of Income (SOI) Division of the Internal Revenue Service uses the Individual Returns Transaction File, administrative data for the population of Form 1040 tax returns, to produce totals of various tax return variables at the state level. Previous research using SOI's Form 1040 sample, a large national sample of cleaned administrative tax records, suggests that the IRS data are subject to various kinds of errors that do not affect tax liability. For this reason, alternative approaches to state-level estimation using the edited sample file are desirable. This paper compares several variables’ different direct, synthetic, and traditional composite estimates of state-level totals and compares the alternatives to the IRS-based totals.


Key words: Administrative Records, General Regression Estimator, Indirect Estimators, Survey sampling

## 1. Introduction: Small Area Estimation Using Administrative IRS Data and Its Associated Nonsampling Errors

The approximately 133 million tax records on the Internal Revenue Service’s (IRS) Individual Returns Transaction File have several uses to multiple government agencies. In particular, these data serve as the sampling frame for the Statistics of Income (SOI) Division of IRS, as well as a source of population data for other tabulations. For example, SOI publishes tabulated monetary amounts and the associated number of returns by state and Adjusted Gross Income (AGI) categories using these data (Table 2 in each Spring issue of the SOI Bulletin).

These population data, based on administrative tax records for the U.S. tax filing population, are not errorfree. While estimates from these data are free from sampling error, the data contain various nonsampling errors, as discovered in prior SOI research comparing return records in the transaction file to records for the same returns in SOI's augmented and edited Form 1040 sample. Only items necessary for computer processing of a tax return are retained on the transaction file, as opposed to items that might be needed for other purposes, such as producing statistical estimates. Measurement errors exist between the IRS and SOI data values due to different data editing rules. For revenue processing purposes, IRS does not spend scarce resources correcting errors that do not affect tax liability in the approximately 130 million tax
return records it processes each year. Since tax liability is correct, this approach does no harm to IRS's tax collection mission or to taxpayers, but it can adversely affect the usability of the data for statistical purposes. SOI's transcription and editing staff receive extensive training, and the sample of approximately 230,000 returns is augmented with additional items from the return, and more closely monitored and checked for data consistency. Errors occur particularly for variables that are indirectly related to tax liability, such as State and Local Income Taxes deducted on Schedule A. They were also discovered for variables such as Taxable Interest and Business Income/Loss from Sole Proprietors (as reported on Schedule C) in the Tax Year 2003 IRS data. To correct these errors, SOI had to delay its publication of Table 2 for several months. Other limitations in the IRS data include a smaller amount of information being available, compared to SOI's sample, and data are often provided to SOI in tabular form, with monetary amounts rounded to thousands and certain high income taxpayers are omitted.

## 2. Data Description

## 2.1: The SOI Sample

SOI draws annual samples of the Form 1040 tax returns to produce richer and cleaner data for population estimation and tax modeling purposes. Stratification for the finite population of tax returns for SOI’s Tax Year 2004 (i.e., income earned in 2004 and reported in 2005) Individual sample used the following categories:

1. Nontaxable returns with adjusted gross income or expanded income of $\$ 200,000$ or more.
2. High combined business and farm total receipts of $\$ 50,000,000$ or more.
3. Presence/absence of special forms or schedules (Form 2555, Form 1116, Form 1040 Schedule C, and Form 1040 Schedule F).

Stratum assignment priority was based on the order in which a return met one of these categories. For example, if a return met (1) and (2), it fell into strata based on (1). Within category (3), further stratification used size of total gross positive or negative income and an indicator of
the return's "usefulness" for tax policy modeling purposes (Scali and Testa 2006). The positive/negative income values in strata boundaries were indexed for inflation between 1991 and the current tax year (Hostetter et al. 1990). This resulted in 216 strata. While the sample was designed for tax modeling and produces reliable nationallevel estimates, it is not large enough to produce statelevel estimates.

Each tax return in the target population was assigned to a stratum based on these criteria, then subjected to sampling in a two-step procedure. Within each stratum, a .05 percent stratified simple random sample, called the Continuous Work History Sample (CWHS), was selected (Weber 2004). For returns not selected for this sample, a Bernoulli sample was independently selected from each stratum, with sampling rates from 0.05 to 100 percent.

SOI's data capture and cleaning procedures resulted in a sample of 200,778 (including 65,948 CWHS returns) returns from an estimated population of 133,189,982 returns. We placed the 34,484 tax returns that SOI sampled with certainty into one certainty stratum, since they represented a census of tax returns. Thus, without loss of generality, we exclude this stratum from the population and develop our estimation method to estimate totals from all other strata. In this way, all errors in the certainty units are isolated and accounted for; only the portion of the total produced from the non-certainty units needs to be estimated. To estimate the entire population
total, we simply add the total from the certainty strata to our estimate for the remaining population.

## 2.2: Small Areas and Variables of Interest

The reduced dataset for this analysis was created by first separating SOI’s Tax Year 2004 sample into the certainty and non-certainty units. For both, the weighted sample data were tabulated to the state by Adjusted Gross Income (AGI) category level, where "state" included the 50 U.S. states, Washington DC, and an "other" category that included returns filed by civilians and military individuals living abroad, such as U.S. possessions and territories, Puerto Rico, etc. We also considered eight categories of AGI: Negative; $\$ 0$ under $\$ 20,000 ;$ 20,000 under \$30,000; \$30,000 under \$50,000; \$50,000 under \$75,000; $\$ 75,000$ under $\$ 100,000$; $\$ 100,000$ under $\$ 200,000$; and $\$ 200,000$ and higher. These 52 states combined with the AGI categories resulted in 416 small areas. We consider estimates for the 52 states in this paper, utilizing the fact that there are differences in our variables of interest at the AGI category-level data.

The IRS data, prior to cleaning by SOI staff, were also compiled to this level. The ten variables we selected for this study can be grouped into two categories: variables that are more or less susceptible to errors in the IRS data. They are as listed, with their location on the Form 1040 tax form and a brief description, in Table 1 below.

Table 1: Variable Names, Tax Form Location, and Description, by Variable of Interest

| Susceptible to <br> Error | Variable | Location on <br> 2004 Tax Form | Description ${ }^{\text {a }}$ |
| :---: | :--- | :---: | :--- |

a: page numbers from IRS 2005.

Since SOI's sample does not use state in the stratification, the number of sample returns by state varies considerably. Six of the states we considered large enough, i.e., more than 5,000 noncertainty returns within each one, such that the associated direct estimates are reasonable. The remaining states were collapsed into groups based on whether or not the state had state income taxes, geographic region, and whether the state had a relatively large or small size of income. This resulted in 21 groups. They are listed, with the associated number of certainty and noncertainty sample units, in Table 2:

Table 2: States and Number of Certainty (c) and Noncertainty Sample Units (nc), by Collapsed Group

| States Within Group | $c$ | $n c$ |
| :--- | ---: | ---: |
| Alaska, Washington | 811 | 4,024 |
| Arkansas, Alabama, Mississippi, Louisiana | 620 | 5,927 |
| Arizona, New Mexico, Utah, Colorado | 1,432 | 7,415 |
| California | 6,539 | 23,990 |
| Connecticut, Rhode Island, Massachusetts | 2,211 | 7,952 |
| Washington DC, Maryland, Delaware | 777 | 4,180 |
| Florida, Tennessee | 4,052 | 14,566 |
| Georgia, North Carolina, South Carolina | 1,265 | 10,108 |
| Hawaii, Other | 790 | 1,815 |
| Iowa, Nebraska, Kansas, Missouri, Oklahoma | 997 | 8,061 |
| Illinois | 1,539 | 7,451 |
| Indiana, Ohio, Kentucky | 1,135 | 9,908 |
| Maine, Vermont, New Hampshire | 215 | 1,770 |
| Michigan, Wisconsin, Minnesota | 1,447 | 10,379 |
| Montana, North Dakota, Idaho, Oregon | 435 | 3,364 |
| New Jersey | 1,273 | 6,138 |
| Nevada, Wyoming, South Dakota | 934 | 2,450 |
| New York | 4,527 | 13,101 |
| Pennsylvania | 931 | 6,480 |
| Texas | 2,318 | 11,427 |
| Virginia, West Virginia | 731 | 4,798 |

## 3. Direct Estimators

Let $y_{k}$ be the value of the characteristic of interest for the $k t h$ tax return, $k \in U$, the finite population of tax returns. We are interested in estimating the finite population total:

$$
Y=\sum_{k \in U} y_{k}
$$

Let $s$ denote the sample of tax returns drawn from the population of tax returns using the stratified Bernouli sampling design. Let $s_{d} \subset s$ denote the part of the sample that belongs to the domain $d$ of interest. Let $w_{k}$ denote the sampling weight for the $k$ th sampled tax return, $k \in s$. The sampling weight represents a certain number of population units in the finite population. With Bernoulli sampling within each stratum, we have epsem sampling within each stratum, i.e., the sampling weights are the same for all the sampled units belonging to the same stratum. The weights vary across strata, due to
disproportionate allocation of the sample into different strata. Our domain cuts across the design strata, so weights of sampled units inside a domain are generally different.

Let

$$
Y_{d}=\sum_{k \in U_{d}} y_{k}
$$

denote the population total for the $d$ th domain (excluding the tax returns belonging to the centainty stratum) and $y_{k}$ is the value of the study variable for the $k$ th population unit. In order to understand the extent and cause of errors in the IRS file, we consider the estimation of

$$
R=Y / X
$$

where $Y[X]$ denotes the AGI population total that the SOI [IRS] file corresponds to. We know $X$, but not $Y$. We estimate R for all the $D \times G$ cells [ $R_{d g}$ ], $D$ domains [ $R_{d}$ ], $G$ groups [ $R_{g}$ ] and for the nation [ $R_{N}$ ].

Let $s_{d}, s_{g}$ and $s_{d g}$ denote the set of sampled units belonging to domain $d$, group $g$ and cell formed by $d$ th domain and $g$ th group formed by a categorized size of AGI. Let $S_{A ; c}$ denote the set of sampled units in an arbitrary set of sampled units $A$ that are common between the SOI and IRS files. For example, $s_{d g ; c}$ denotes the set of samples in domain $d$ and group $g$ that are common between the SOI and IRS files. The notations $s_{d ; c}, s_{g ; c}$ and $s_{N ; c}$ denote similar sets for the domain $d$, group g, and the nation. Note that we may not introduce the new symbols $s_{d g ; c}, s_{d ; c}, s_{g ; c}$, and $s_{N ; c}$ if there is a one-to-one correspondence between the SOI sample and IRTF. We estimate $R_{d g}, R_{d}, R_{g}$ and $R_{N}$ by:

$$
\begin{aligned}
& \hat{R}_{d g}=\hat{Y}_{d g ; c} / \hat{X}_{d g ; c} \\
& \hat{R}_{d}=\hat{Y}_{d ; c} / \hat{X}_{d ; c} \\
& \hat{R}_{g}=\hat{Y}_{g ; c} / \hat{X}_{g ; c} \\
& \hat{R}_{N}=\hat{Y}_{N ; c} / \hat{X}_{N ; c}
\end{aligned}
$$

where the numerator and denominator components are the weighted sum of $y_{k}$ and $x_{k}$ over the appropriate summation, respectively. For example, with $\hat{R}_{d g}$, we have

$$
\hat{Y}_{d g ; c}=\sum_{k \in s_{d g ; c}} w_{k} y_{k}, \hat{X}_{d g ; c}=\sum_{k \in s_{d g ; c}} w_{k} x_{k}
$$

If the IRS file is error free, we would expect the above ratios to be exactly 1 . But, since there are errors in the IRS data, we expect them to vary around 1 . For example, the figure in A. 1 at the end of this paper contains $\hat{R}_{d}$ for each variable of interest. A vertical reference line of one is drawn and the states are sorted by their number of noncertainty units in the sample. The $\hat{R}_{d}$ 's fluctuate
around one for all variables, particularly when the state sample size decreases (and sampling variance increases). They also fluctuate more from one for variables that are more susceptible to the errors: that scale is 0.80 to 1.20 (compared to 0.99 to 1.04 for the "less" susceptible ones). We consider seven direct estimators:

$$
\begin{align*}
& \hat{Y}_{d D 1}=\sum_{k \in s_{d}} w_{k} y_{k},  \tag{1}\\
& \hat{Y}_{d D 2}=N_{d} \times \sum_{k \in S_{d}} w_{k} y_{k} / \sum_{k \in s_{d}} w_{k},  \tag{2}\\
& \hat{Y}_{d D 3}=\hat{R}_{d} X_{d},  \tag{3}\\
& \hat{Y}_{d D 4}=\sum_{g}\left[N_{d g} \times \sum_{k \in s_{d g}} w_{k} y_{k} / \sum_{k \in s_{d g}} w_{k}\right],  \tag{4}\\
& \hat{Y}_{d D 5}=\sum_{g} \hat{R}_{d g} X_{d g},  \tag{5}\\
& \hat{Y}_{d D 6}=\hat{Y}_{d D 1}+\hat{R}_{N}\left(X_{d}-\hat{X}_{d}\right),  \tag{6}\\
& \hat{Y}_{d D 7}=\hat{Y}_{d D 1}+\sum_{g} \hat{R}_{g}\left(X_{d g}-\hat{X}_{d g}\right) . \tag{7}
\end{align*}
$$

These are equal to or are various forms of the expansion estimator, weighted survey mean estimator, combined ratio estimator, poststratification estimator, separate ratio estimator, combined survey regression estimator, and separate regression estimator, respectively. They are "direct" estimators since all involve sample-based components at the small areas levels. The benefit of direct estimators is that they are completely or nearly designunbiased estimators for the population total. However, they are subject to higher sampling variability, since they are based on the number of returns within each state (or state crossed with AGI group), which can be small.

## 4. Synthetic Estimators

We consider five synthetic estimators:

$$
\begin{align*}
& \hat{Y}_{d S 1}=\sum_{g} N_{d g} \times \sum_{k \in s_{g}} w_{k} y_{k} / \sum_{k \in s_{g}} w_{k}  \tag{8}\\
& \hat{Y}_{d S 2}=\hat{R}_{N} X_{d}  \tag{9}\\
& \hat{Y}_{d S 3}=\sum_{g} \hat{R}_{g} X_{d g}  \tag{10}\\
& \hat{Y}_{d S 4}=\sum_{g} \hat{Y}_{g 1} \times X_{d g} / X_{g}  \tag{11}\\
& \hat{Y}_{d S 5}=\sum_{g} \hat{Y}_{g 1} \times N_{d g} / N_{g} \tag{12}
\end{align*}
$$

These estimators involve combining information across states and/or AGI groups to estimate the state-level totals. Estimators (8) and (12) are a form of (4), (9) of (3), and (10) and (11) are a form of (5). Due to implicit assumptions with each (see, e.g., section 4.2.1 in Rao
2003), they may not necessarily be design-unbiased. However, they may have lower variances, resulting in overall lower total error.

## 5. Composite Estimators

To overcome the problems separately associated with the direct and synthetic estimators, we also examine composite estimators. They have the following general form:

$$
\hat{Y}_{d C}=\hat{\phi}_{d} \hat{Y}_{d D}+\left(1-\hat{\phi}_{d}\right) \hat{Y}_{d S}
$$

where $\hat{Y}_{d D}$ is a direct estimator for the state- total, $\hat{Y}_{d S}$ is a synthetic estimator, and $\hat{\phi}_{d}$ is a "suitably chosen weight" on the direct estimator (expression 4.3.1 in Rao 2003). We present results using two composite estimator weights:

$$
\hat{\phi}_{d}= \begin{cases}1 & \text { if } \hat{N}_{d} / N_{d} \geq 1  \tag{13}\\ {\left[\hat{N}_{d} / N_{d}\right]^{2}} & \text { if } \hat{N}_{d} / N_{d}<1\end{cases}
$$

$$
\hat{\phi}=\left\{\begin{array}{l}
0 \quad \text { if } \sum_{d} \operatorname{var}\left(\hat{Y}_{d D}\right) / \sum_{d}\left(\hat{Y}_{d D}-\hat{Y}_{d S}\right)^{2} \geq 1  \tag{14}\\
1-\sum_{d} \operatorname{var}\left(\hat{Y}_{d D}\right) / \sum_{d}\left(\hat{Y}_{d D}-\hat{Y}_{d S}\right)^{2} \text { otherwise }
\end{array},\right.
$$

with various combinations of direct estimators (1)-(7) combined with synthetic estimators (8)-(12). The weight in (13) was proposed by Sarndal and Hidiroglou (1989), while (14) is a form of the James-Stein estimator (expression 4.4.3 in Rao 2003) with a common weight. They are different in that (13) depends on the state but not variable of interest, while (14) depends on the variable but not state.

## 6. Results

Table A. 2 at the end of this paper contains the IRS totals $X_{d}$ for each variable and collapsed state group. This allows for a useful comparison between estimates similar to those published by SOI and our alternatives.

Figure A. 3 contains plots of the relative differences of the direct estimates in (1), shown in A.2, to various alternatives, for variables that are less susceptible to error. The estimates are referenced with the subscript in each plot. Three combinations of direct and synthetic estimators are considered: (1) and (8); (1) and (11); and (2) and (12). Combined with the two weight choices (13) and (14), we have six composite estimators. These are labeled "C" and "JS," with the direct and synthetic number, resp. For example, "C 1,1" refers to a composite estimator with (1) as the direct, (8) as the synthetic, and weight (13). Relative differences outside ( $-10 \%, 10 \%$ )
were truncated and the states are sorted on each horizontal axis by ascending size of the CV of $\hat{Y}_{d D 1}$ in (1).

For AGI, the relative differences to the IRS totals were within 2 percent for all groups except NV/WY/SD and HI/Other, as this variable had lower amounts of both sampling error in the direct estimates and nonsampling error in the IRS totals. Salaries and Wages and Total Tax Liability had similar patterns as noted in AGI, but somewhat larger relative differences. The Earned Income Tax Credit plot showed even larger relative differences. This was caused by larger sampling errors (e.g., the highest CV of $\hat{Y}_{d D 1}$ was $18 \%$, compared to $4 \%$ for AGI). Since this credit was also claimed only by lower income taxpayers, there were several zero values in both the SOI and IRS data given the sample design described in section 2.1. Differences between the AGI-category level ratios and one resulted in poorer synthetic estimates (with extremely high relative differences) using (9) and (12) for both larger and smaller states. Direct estimates (3) and (4) looked stable.

Figure A. 4 contains the same ratio plots the direct estimates from Table A.1, for variables that are more susceptible to error. Relative errors outside ( $-100 \%$, 100\%) were truncated and again the states were sorted by the CV of $\hat{Y}_{d D 1}$. These variables had much different results for the different estimators, particularly for the smaller state groups - there was not a clear pattern due to the sampling error, as noted in Figure A.3. The relative differences were most often highest for the HI/Other group, where the SOI sample estimates are very far from the IRS totals. This also caused differences in the direct or synthetic estimates that used the estimated group population size form the SOI sample (about 790,000 returns) and the IRS total (about 1.5 million). The same instability with estimators (9) and (12) occurred with all of these variables. However, the IRS totals here are considered less reliable due to nonsampling error, resulting in larger relative differences.

## 7. Conclusions, Limitations, and Future Considerations

In general, the direct estimates are further from the IRS totals (particularly for smaller states), while the synthetic are closer, and the composite are a compromise between the two. Starting in Tax Year 2005, the CWHS will become a ten percent stratified simple random sample. This means that approximately 65,000 non-certainty units will be added to SOI's sample, which will increase the reliability of the direct estimates.

Our comparisons were between various estimated totals and the corresponding IRS ones. We should also compare the direct estimates' sampling error to the mean square error in the synthetic and composite ones. These
are more difficult to compute, particularly since more sample units are required for reliable estimates. Another alternative to consider is using composite estimatorsfrom (1), (3) and (4) as the direct estimates and the IRS totals as the synthetic ones. A natural extension of the composite estimates is small area modeling, which is also currently under consideration. Ultimately we are also interested in the state-level estimates, but the collapsing of states into groups allowed for a useful comparison between the alternatives, it also demonstrated that the direct estimates were affected by sampling error in smaller states. Thus, adjustments are needed when applying them simply at the state level.

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A.1. Estimated $\hat{R}_{d}$ 's, by Variable Type, Variable of Interest, and State (sorted by ascending state noncertainty sample size)

Less Susceptible to Error


More Susceptible to Error

A.2. Table of IRS Totals $X_{d}$ (in Thousands of Dollars), by Collapsed State Group

|  | Less Susceptible to Error |  |  |  | More Susceptible to Error |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N State Group | Adjusted Gross Income | Salaries and Wages | Total Tax Liability | Earned Income Tax Credit | Business Profit / Loss | Schedule <br> D Capital <br> Gains/Loss | Total Contributions | Taxes Paid | Interest Paid Deduction | Total Itemized Deductions |
| AK, WA | 169,450,168 | 121,048,669 | 22,235,015 | 663,339 | 6,219,819 | 11,796,324 | 3,695,776 | 6,195,165 | 11,176,349 | 3,695,776 |
| AR, AL, MS, LA | 240,278,162 | 181,097,537 | 26,373,363 | 3,522,206 | 8,066,558 | 9,685,873 | 6,321,664 | 9,344,287 | 10,274,109 | 6,321,664 |
| AZ, NM, UT, CO | 309,176,619 | 222,883,909 | 37,248,137 | 1,792,852 | 10,050,014 | 22,747,510 | 8,439,119 | 14,321,997 | 21,963,900 | 8,439,119 |
| CA | 881,752,963 | 624,514,425 | 121,339,747 | 4,449,344 | 44,266,526 | 73,195,955 | 21,867,927 | 67,399,138 | 71,443,413 | 21,867,927 |
| CT, RI, MA | 339,501,588 | 238,252,628 | 51,940,323 | 879,239 | 14,452,095 | 27,862,641 | 7,083,498 | 23,445,637 | 17,701,780 | 7,083,498 |
| DC, MD, DE | 197,805,604 | 145,614,770 | 26,519,027 | 785,361 | 6,133,845 | 11,962,072 | 6,034,002 | 13,795,287 | 12,607,868 | 6,034,002 |
| FL, TN | 529,878,356 | 349,964,046 | 72,055,545 | 3,955,877 | 18,799,220 | 53,912,196 | 12,509,213 | 16,999,688 | 26,594,987 | 12,509,213 |
| GA, NC, SC | 428,830,827 | 322,440,428 | 49,915,635 | 3,990,066 | 13,042,714 | 22,351,530 | 13,201,226 | 24,522,210 | 28,768,405 | 13,201,226 |
| HI, Other | 77,696,909 | 73,436,533 | 9,644,685 | 196,176 | 2,709,878 | 7,738,281 | 1,019,693 | 2,824,833 | 2,926,349 | 1,019,693 |
| IA, NE, KS, MO, OK | 325,128,782 | 240,695,352 | 37,419,546 | 2,140,592 | 9,668,713 | 14,985,751 | 7,800,656 | 16,992,219 | 15,056,976 | 7,800,656 |
| IL | 312,951,784 | 228,115,769 | 42,656,588 | 1,576,538 | 9,333,379 | 21,421,047 | 7,054,523 | 15,938,756 | 16,575,098 | 7,054,523 |
| IN, OH, KT | 441,711,523 | 336,208,255 | 50,887,852 | 2,768,805 | 13,651,937 | 16,774,468 | 9,202,408 | 23,364,434 | 23,091,120 | 9,202,408 |
| ME, VT, NH | 74,966,026 | 54,891,225 | 9,186,096 | 292,609 | 3,739,824 | 4,999,477 | 1,181,216 | 3,904,904 | 3,482,376 | 1,181,216 |
| MI, WI, MN | 471,487,557 | 354,296,826 | 57,030,989 | 2,067,922 | 13,387,287 | 20,990,791 | 10,832,989 | 27,737,408 | 25,623,815 | 10,832,989 |
| MT, ND, ID, OR | 127,237,758 | 89,502,052 | 14,287,382 | 742,205 | 5,008,009 | 8,444,990 | 3,070,032 | 8,569,985 | 7,509,054 | 3,070,032 |
| NJ | 264,917,673 | 199,028,894 | 39,188,251 | 857,954 | 9,598,198 | 14,729,732 | 5,533,706 | 22,336,098 | 13,915,865 | 5,533,706 |
| NV, WY, SD | 90,163,667 | 57,805,634 | 12,298,955 | 423,539 | 2,938,308 | 12,274,045 | 2,085,551 | 2,929,571 | 5,025,388 | 2,085,551 |
| NY | 509,011,438 | 359,825,754 | 75,885,191 | 2,672,975 | 18,993,061 | 45,111,257 | 14,454,792 | 44,903,606 | 21,255,412 | 14,454,792 |
| PA | 278,531,309 | 207,054,076 | 35,026,827 | 1,304,085 | 9,707,338 | 13,124,940 | 5,687,268 | 14,473,936 | 11,985,528 | 5,687,268 |
| TX | 448,956,879 | 338,710,156 | 59,941,678 | 4,509,906 | 18,836,804 | 26,138,888 | 9,927,578 | 15,421,149 | 17,658,406 | 9,927,578 |
| VA, WV | 225,665,995 | 168,339,288 | 28,857,291 | 1,122,083 | 7,402,708 | 11,953,120 | 5,195,825 | 11,621,403 | 13,660,405 | 5,195,825 |

## Section on Survey Research Methods

A.3. Comparison Plots of Relative Differences of Alternative Estimator to IRS Totals, by Variable, Collapsed State Group, and Estimator (Variables Less Susceptible to Error, sorted by size of the CV of the Direct)


## Section on Survey Research Methods

A.4. Comparison Plots of Relative Differences of Alternative Estimator to Direct Estimates, by Variable, Collapsed State Group, and Estimator (Variables More Susceptible to Error, sorted by size of the CV of the Direct)


Net Schedule D Capital Gains/Loss


Total Taxes Paid Deduction


Total Itemized Deductions


