To Replicate (A Weight Adjustment Procedure) Or Not To Replicate? An Analysis of the Variance Estimation Effects of a Shortcut Procedure Using the Stratified Jackknife

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Abstract

Many surveys employ weight adjustment procedures to compensate for unit non-response. This paper frames such procedures as second-phase sampling adjustments (where the second phase refers to a Bernoulli sample of respondents from sampled units). We compare the effect of replicating the unit non-response weight adjustment procedure on stratified jackknife variance estimates to those obtained using the corresponding “shortcut procedure” variance estimates on the same data. Two weight adjustment procedures are examined.

Key words: unit non-response, Bernoulli sample, linearization variance estimator

1 Introduction

Many surveys employ weight adjustment procedures to compensate for unit non-response. Such weight adjustment procedures can be viewed as bias corrections accounting for underestimation or as second-phase sampling adjustments (where the second phase refers to a random sample of respondents from the initial sample). The latter view couches the issue of unit non-response as a variance estimation problem. If we assume a uniform response mechanism in each weighting class, then the second phase sample can be viewed as a Bernoulli sample with a random sample size of respondents -- each with response probability $\pi_p$ -- from the first-phase sampled units in each weighting class (Särndal, C., Swensson, B., and Wretman, J., 1992, pp. 62-65 and Kott, 1994). This two-phase sample interpretation is equally valid with a missing-at-random (MAR) model, which assumes probability of unit non-response depends on an auxiliary variable or set of auxiliary variables, not the characteristic(s) of interest. In the two-phase framework, we compare the effect of replicating the unit non-response weight adjustment procedure on stratified jackknife variance estimates to those obtained using the corresponding “shortcut procedure” stratified jackknife variance estimates on the same data.

Shortcut procedure variance estimates use replicate weights constructed from the full-sample unit non-response adjusted weights instead of repeating the weighting procedure in each replicate. This approach was originally presented (not recommended) in Wolter (1985, pp. 82-84). Note that the shortcut procedure variance estimator is not a naïve variance estimator, which treats imputed values as though they were reported values and which has been repeatedly shown to underestimate the true variance. The naïve variance estimator replaces the missing item responses with imputed values, yielding a dataset with no (visible) item non-response, and the replicate values do not contain any missing values. Our replicate assignment procedures assign sample units to replicates. Consequently, these replicate estimates contain both responding and non-responding units, regardless of the replicate reweighting procedure.

We consider two different weight adjustment procedures, both of which are adjustment-to-sample models described in Kalton and Flores-Cervantes (2003), i.e., all sampling weights in a weighting class $p$ are multiplied by a factor derived from data corresponding to sample units. The first procedure (the ratio adjustment procedure) controls the respondent estimates to full-sample estimates of an auxiliary variable. The second procedure (the count adjustment) yields estimates that are controlled to sample counts; when weighted counts (population estimates) are used, then the count adjustment procedure is the “quasi-randomization” estimator (Oh and Scheuren, 1983); when weighting cells are sample strata, then the count procedure is the inverse unweighted response rate adjustment recommended by Vartivarian and Little (2002).

2 The Jackknife Linearization Estimator

Given a Two-Phase Sample with Bernoulli Sampling at the Second Phase

1 This report is released to inform interested parties of ongoing research and to encourage discussion of work in progress. The views expression on statistical and methodological issues are those of the authors and do not necessarily reflect those of the U.S. Census Bureau or Statistics Canada.
Suppose we have a stratified SRS-WOR design with $n_h$ sampled units in the $h$th stratum ($h = 1, 2, \ldots, L$). Let $y_{hi}$ ($i=1, \ldots, n_h$) be the value of the variable of interest for the $i$th unit in the $h$th stratum (obtained from survey respondents) and let $x_{hi}$ be the corresponding auxiliary variable (available for all sampled units). For simplicity, assume that sampling fractions in all strata are negligible, and that there are at least two sampled units in all strata. In addition, we assume that there exist $P$ disjoint weighting classes, which can cut across the design strata. With unit non-response, we divide the sampled units, $s$, into respondents, $s_r$, and non-respondents, $s_n$. The following derivations assume that each weighting class contains at least two respondents, although in practice, surveys generally collapse weighting cells with insufficient respondents.

### 2.1 Ratio Adjustment Procedure

Since $x_{hi}$ is known for all sampled units in $s$, it can be used in the unit non-response weight adjustment. The non-response weight adjustment for the $p$th weighting class is

$$
\delta_{sp}^w = \frac{\sum_{i=1}^{n_h} w_{hi} x_{hi} \delta_{hi}^p}{\sum_{i=1}^{n_h} w_{hi} x_{hi} \delta_{hi}^p x_{hi}^{\prime}}
$$

where $\sum_{i=1}^{n_h}$ denotes the summation over all sampled units, $I_{hi}$ is the response indicator variable ($I_{hi} = 1$ if $(hi) \in s$, and =0 otherwise), and $\delta_{hi}^p$ is the weighting class indicator variable ($\delta_{hi}^p = 1$ if $(hi) \in$ weighting class $p$ and =0 otherwise), and $w_{hi}$ is the design weight (= $N_h/n_h$ for a stratified SRS-WOR sample). The non-response adjusted weight in each weighting cell $p$ is then given by $\tilde{w}_{hi}^p = d_{sp}^w w_{hi}$ and the non-response adjusted estimator is

$$
\hat{Y} = \sum_p \sum_s \tilde{w}_{hi}^p y_{hi} I_{hi} \delta_{hi}^p = \sum_p \frac{\hat{X}_p}{X_p^\prime} \hat{Y}_p^p,
$$

where $\hat{Y}_p^p$ is the respondent-based estimate of characteristic $Y$. This expression is exactly equal to the ratio imputation estimator presented in Deville and Särndal (1994).

We obtain the variance of $\hat{Y}$ by decomposing it within weighting cell $p$ (see Cochran, 1977, p. 343)
second model is assumed when all survey variables of interest are highly correlated with auxiliary stratification variables.

As shown below, the fully reweighted stratified jackknife variance estimator implicitly simulates all error components of the two-phase sample MSE \( \hat{Y} \). To see this, we first define the stratified jackknife weights when the \( j \)th unit in the \( g \)th stratum is removed as

\[
 w_{gij} = \begin{cases} 
 0 & \text{if } (hi) = (gj) \\
 \frac{n_g}{n_g - 1} w_{ij} & \text{if } h = g, i \neq j \\
 w_{ij} & \text{otherwise} 
\end{cases}
\]

where \( n_g \geq 2 \) for all strata \( g \). The non-response adjustment when the \( (gj) \)th unit is deleted is then

\[
 d_{gij} = \frac{\sum w_{hij} X_{hi} \delta_{hi} I_{hi}}{\sum w_{hij} X_{hi} \delta_{hi}} = \frac{\hat{X}_g}{X_{r}(g)}
\]

The non-response adjusted jackknife weight is

\[
 \hat{w}_{gij} = d_{gij} w_{gij}.
\]

The replicate estimate when the \( j \)th unit in the \( g \)th stratum is removed is as

\[
 \hat{Y}(g) = \sum_p \sum_{(hi)} \hat{w}_{gij} y_{hi} I_{hi} \delta_{hi} = \sum_p \frac{\hat{X}_g}{X_{r}(g)} \hat{Y}_{r}(g),
\]

and the jackknife variance estimator of \( \hat{Y} \) is

\[
 v_j(\hat{Y}) = \sum_g \left(\frac{n_g}{n_g - 1}\right) \left(\sum_p \left(\frac{\hat{Y}_g}{X_{r}(g)} - \hat{Y}_r\right)\right)^2.
\]

To obtain a linearization type variance estimator, note that from a Taylor Series expansion we have (ignoring second order and higher derivatives)

\[
 \frac{\hat{X}_g}{X_{r}(g)} \hat{Y}_r - \frac{\hat{X}_g}{X_{r}} \hat{Y}_r \approx \hat{X}_g \left(\frac{\hat{Y}_r(\hat{X}_g)}{X_{r}(g)} - \hat{Y}_r\right) - \frac{\hat{X}_g}{X_{r}} \left(\frac{\hat{Y}_r}{X_{r}(g)} - \hat{Y}_r\right) + \frac{\hat{X}_g}{X_{r}} \left(\hat{X}_g - \hat{X}_r\right)
\]

\[
 = \hat{B}_p + \hat{A}_p = (\hat{B}_{py} - \hat{B}_{px}) + \hat{A}_p
\]

which yields the reweighted jackknife linearization variance estimator

\[
 v_j^R(\hat{Y}) = \sum_g \left(\frac{n_g}{n_g - 1}\right) \sum_p \left(\frac{\varepsilon_g^p - \hat{w}_{gij} e_{gij} \delta_{gij}}{X_{r}^p} \hat{Y}_r^p\right)^2
\]

\[
 \text{and } \hat{w}_{gij} = \left(\frac{1}{n_g}\right) \sum_j \hat{w}_{gij} e_{gij} \delta_{gij}.
\]

(Derivation available upon demand from the authors. It is not a coincidence that the bracketed term of \( v_j^R(\hat{Y}) \) has exactly the same form as the MSE expression of a two-phase ratio estimator. Moreover, the first term in this linearization estimate – the \( \hat{B}_p \) term – contains two explicit error components: one for the characteristic of interest and one for the auxiliary variable (both within the response set). Our derived linearization estimator is slightly different from the comparable estimator derived in both Rao and Sitter (1995) and Shao and Steele (1999), following from different assumptions about the second phase sample of respondents.

Turning to the shortcut jackknife variance estimator, where the non-response adjustment is not recalculated in each replicate, we express the non-response adjusted jackknife weight as

\[
 \hat{w}_{gij} = d_{gij} w_{gij}.
\]

The replicate estimate is given as

\[
 \hat{Y}^{(2)} = \sum_p \sum_{(hi)} \hat{w}_{gij} y_{hi} I_{hi} \delta_{hi} = \sum_p \frac{\hat{X}_g}{X_{r}(g)} \hat{Y}_{r}(g),
\]

and the shortcut jackknife variance estimator is

\[
 v_j^{(2)} = \sum_g \left(\frac{n_g}{n_g - 1}\right) \left(\sum_p \left(\frac{\hat{Y}_g}{X_{r}(g)} - \hat{Y}_r\right)\right)^2.
\]

Choosing the fully reweighted and shortcut linearization estimators, we see that the difference between the reweighted and the shortcut jackknife variance estimators is the term

\[
 \sum_p \left(\frac{\hat{Y}_g}{X_{r}(g)} - \hat{Y}_r\right) \left(\frac{\hat{X}_g}{X_{r}(g)} - \hat{X}_r\right) = \sum_p \left(\hat{A}_p - \hat{B}_p\right)
\]

\[
 = \sum_p \hat{A}_p - \hat{B}_p
\]

\[
 = \sum_p \hat{A}_p = \hat{A}_p
\]

\[
 = \sum_p \hat{A}_p = \hat{A}_p
\]
Within the squared term. The shortcut variance estimator is a partial approximation of the ordinary ratio estimate error: it does not include the error contribution of the auxiliary variable to the ratio estimate or its covariance with the characteristic. Thus, the $A_p$ term approximates the difference between the error contribution of the auxiliary variable from 1st (sample units) to 2nd phase (respondent units) and the error contribution from the auxiliary variable in the ordinary ratio estimate, both missing from the shortcut jackknife variance estimator. In a given weighting cell, the $A_p$ term should be small when the unit response rate is high (say greater than 70%) since $(\hat{X}_p / \hat{X}_{p(g)})$ is a non-response adjusted estimator of $\hat{X}_p$ when the $(g)_i$th term is deleted. Finally

$$v_p^2(\hat{Y}) - v_p^2(\hat{Y}) = \sum_{g} n_g^{-1} \sum_{j} \left( A_j^2 + 2A_j B_j \right)$$

where

$$B_j = \sum_{g} \frac{\hat{X}_p}{\hat{X}_{p(g)}} (\hat{Y}_{p(g)} - \hat{Y}_p)$$

$$= \sum_{g} n_g^{-1} \sum_{j} \left( \sum_{i} w_{g_i} y_{g_i} \delta_{g_i} - \sum_{i} w_{g_i} y_{g_i} \delta_{g_i} \right)$$

With a stratified SRS sampling plan where the weighting classes correspond to strata, we can show that $\hat{Y}_p = (N_p r_p / n_p) \hat{y}_p$, $\hat{X}_p = (n_p x_p / r_p x_p)$, $(\hat{Y}_p / \hat{X}_p)$ and $\hat{X}_p / \hat{X}_{p(g)} = (n_p x_p / r_p x_p)$ where $r_p = \sum_{i} I_{h_i}$.

$$\hat{y}_p = (r_p)^{-1} \sum_{i} y_{p(i)} I_{p(i)}$$

and

$$\hat{x}_p = (n_p)^{-1} \sum_{i} x_{p(i)}$$

Using these expressions, we have

$$\hat{A}_j = \left( \frac{\hat{y}_{p(g)}}{\hat{x}_{p(g)}} \right)^2 \left( \frac{n_g}{n_p} \right) \left( \frac{N_g}{n_p} x_p \right) \left( \frac{n_p}{r_p x_p} \right) \left( \frac{I_{p(g)}}{I_{p}} \right) - 1$$

and

$$\hat{B}_j = \sum_{g} n_g^{-1} \sum_{j} \hat{A}_j^2$$

$$= \sum_{g} n_g^{-1} \left( \frac{\hat{y}_{p(g)}}{\hat{x}_{p(g)}} \right)^2 \left( \frac{n_g}{n_p} \right) \sum_{j} x_{p(g)}^2 \left( \frac{n_p}{r_p x_p} \right) \left( \frac{I_{p(g)}}{I_{p}} \right) - 1.$$
Thus, Substituting we get, (1

\[
\tilde{w}_{(g)} = \begin{cases} 
0 & \text{if } (hi) = (gj) \\
\frac{N_g n_g n_g - 1}{n_g n_g - 1} \frac{r_{gj}}{r_{(gj)}} & \text{if } h = g, i \neq j \\
\text{otherwise} 
\end{cases}
\]

where \( r_{(g)} \) are the number of respondent units in the jackknife replicate. These replicate weights have exactly the same form as the full-sample adjusted weights, and the replicate estimates are also Horvitz-Thompson estimates based on respondents.

The reweighted jackknife linearization variance estimator is given as

\[
v^2_j(\hat{Y}) = \sum_g n_g^{-1} \sum_i \left[ \sum_{p} \tilde{N}_p \left( \frac{1}{n_p} \sum_{r_p \in g} w_{pG} I_{g}(y_{pG} - \hat{y}_p) \right)^2 \right]
\]

\[
= \hat{A} \hat{B} - \hat{B}_{\text{ps}} = \hat{A}^* \]

(\text{Derivation available upon demand from the authors})

and the shortcut jackknife linearization variance estimator is given by

\[
v^2_j(\hat{Y}_{ace}) = \sum_g n_g^{-1} \sum_i \left[ \sum_{p} \tilde{N}_p \left( \frac{1}{n_p} \sum_{r_p \in g} w_{pG} I_{g}(y_{pG} - \hat{y}_p) \right)^2 \right]
\]

The difference between \( v^2_j(\hat{Y}) \) and \( v^2_j(\hat{Y}_{ace}) \) is approximately

\[
\frac{\hat{y}_p}{\hat{N}_p} \left( \tilde{N}_p(\hat{y}_p) - \tilde{N}_p \right) \frac{\hat{y}_p}{\hat{N}_p} \left( \tilde{N}_p(\hat{y}_p) - \tilde{N}_p \right)
\]

\[
= \hat{A} \hat{B} - \hat{B}_{\text{ps}} = \hat{A}^* \]

When the weighting cells correspond to the sample strata, then \( \tilde{N}_p = (N_p/r_p, n_p), (\hat{Y}_p, \hat{N}_p) = \bar{y}_p \)

and \( \tilde{N}_p/\hat{N}_p = n_p/r_p \). Substituting we get,

\[
\hat{A}^* = \bar{y}_p \frac{n_p}{n_p - 1} \frac{N_p}{r_p} \frac{n_p}{r_p} I_{g} (y_{g} - \hat{y}_p)
\]

and

\[
(\hat{A}^*)^2 = \sum_g n_g^{-1} \sum_i \left( \hat{A}^* \right)^2 + \sum_g n_g^{-1} \sum_i \left( \hat{B}^* \right)^2
\]

Turning to the remaining error term, we have

\[
\hat{B}_{\text{ps}} = \frac{n_p}{r_p} \frac{n_p}{r_p} \frac{N_p}{n_p} \frac{r_p}{r_p} \left( \bar{y}_p - y_p \right)^2
\]

and

\[
2\hat{A}^* \hat{B}_{\text{ps}} - 2 \sum_g n_g^{-1} \sum_i \left( \hat{A}^* \right)^2
\]

Thus,

\[
v^2_j(\hat{Y}) - v^2_j(\hat{Y}) = \sum_g n_g^{-1} \sum_i \left( (\hat{A}^*)^2 + 2\hat{A}^* \hat{B}_{\text{ps}} \right)
\]

\[
= -\sum_g n_g^{-1} \sum_i \left( (\hat{A}^*)^2 \right) = -(\hat{A}^*)^2
\]

That is, the shortcut procedure variance estimator is always larger than the reweighted estimator. As \( r_p \to n_p \) the contribution from the \( g^j \) group to this term approaches zero, i.e., the higher the response rate, the closer the procedure is to the usual textbook variance estimator. As with the ratio adjustment procedure, the relative contribution of the missing \( (\hat{A}^*)^2 \) term to the total MSE depends on the unit response rate and the magnitude of the characteristic within each stratum.

### 3 Empirical Results

Section 2 addresses the question of what are the variance estimation effects of using a shortcut procedure compared to a reweighted procedure, demonstrating overestimation with the count adjustment procedure and expected overestimation with the ratio adjustment procedure. The question then becomes what is the degree of overestimation and what factors might cause the overestimation to be severe enough to bias survey conclusions.

To evaluate these questions empirically, we used survey data from the U.S. Census Bureau’s Annual Capital Expenditures Survey (ACES). The ACES survey is a mail-out/mail-back that collects data about the nature and level of capital expenditures in non-farm businesses operating within the United States. Respondents report capital expenditures for the calendar year in all subsidiaries and divisions for all operations within the United States. ACES respondents report total capital expenditures, broken down by type (expenditures on Structures and expenditures on Equipment). Hereafter, we refer to these characteristics as Total, Structures, and Equipment. Tables 1 through 3 compare standard error estimates of capital expenditures statistics from three years’ of ACES data: the first two data sets (from survey years 2002 and 2003) are the full collection of final tabulated ACES data and the third data set (survey year 2003) contains a mid-survey collection of data.

The ACES universe contains two sub-populations: employer companies and non-employer companies. Different forms are mailed to sample units depending on whether they are employer (ACE-1) companies or non-employer (ACE-2) companies. New ACE-1 and ACE-2 samples are selected each year, both with stratified SRS-WOR designs. The ACE-1 sample comprises approximately seventy-five percent of the ACES sample (roughly 45,000 companies selected per year for ACE-1, and 15,000 selected per year for ACE-2). The ACE-survey strata are defined by four company size class categories within industry (denoted 2A through 2D, ranked from largest to smallest within industry), with
approximately 500 non-certainty strata each year. Sampling fractions in the large-size class-within-industry strata (2A) can be fairly high (approximately 55% of the sample in these strata are sampled at rates between 1 and 2); sampling fractions in the other three size class-within-industry strata are usually less than 0.20 and sampling weights range from 5 to 1000, depending on industry and size-class strata. The ACE-2 component is much less highly stratified, with between a total of six to eight size-class strata used each year, and sampling fractions less than 0.01 in all strata. Because the response rates in certainty strata are generally close to 100%, we exclude certainty units from the variance estimates discussed below. We do not otherwise incorporate fpc’s into our calculations. The ACE-1 component has a fairly high expected unit response rate (approximately 80% in most strata), whereas the corresponding rate for ACE-2 tends to be somewhat lower (ranging from 60 to 80%).

The ACE-1 component uses the ratio adjustment procedure with administrative data payroll as auxiliary variable to account for unit non-response, whereas the ACE-2 component uses the count procedure. In almost all cases, ACES uses design strata as weighting cells: under complete unit non-response in an ACE-1 industry’s certainty stratum or in the large company (2A) stratum, the two strata are combined into one weighting cell (within the sample industry). Presently, there is no collapsing procedure in place for complete non-response in the three remaining ACE-1 (within-industry) non-certainty strata. In general, stratum collapsing for weight adjustment is a very rare occurrence and is hereafter ignored in this paper. To assess the effect of unit non-response weight adjustment procedure on the ACE-1 standard errors, we compute standard errors from ACE-1 data using both weight adjustment procedures (ratio and count). Since payroll data are not available for the ACE-2 component, we only present results using the count adjustment for that sub-population.

Capital expenditures data are fairly atypical business data, in that they often are characterized by low year-to-year correlation for the same reporting unit: for example, a company that spends a large amount of capital expenditures on structural (building) improvements one year is unlikely to invest much in structural improvements in the following year. Moreover, a reported value of zero for an expenditures item is quite legitimate and is often the response value for most items reported by a small company.

Table 1 presents standard error estimates for reweighted stratified jackknife (SJR) standard errors for the ACE-1 and ACE-2 data sets, along with the jackknife linearization standard errors obtained from reweighted procedures (LinR) and shortcut procedures (LinS). The shortcut procedure stratified jackknife and shortcut linearized jackknife standard errors are equivalent.

The reweighted jackknife and linearized reweighted jackknife standard error estimates are all within 1-percent of each other. Regardless of weight adjustment procedure, the linearized shortcut standard errors are larger than corresponding linearized reweighted standard errors. The degree of “overestimation” is, however, quite small: in most cases, the shortcut procedure standard errors are less than 2-percent larger than their fully reweighted procedure counterparts.

<table>
<thead>
<tr>
<th>Table 1: Comparison of Fully Reweighted and Shortcut Procedure Standard Errors (in Millions)</th>
<th>Ratio Adjustment</th>
<th>Count Adjustment</th>
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<tbody>
<tr>
<td></td>
<td>SJR</td>
<td>LinR</td>
</tr>
<tr>
<td>ACE-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>8.4</td>
<td>8.3</td>
</tr>
<tr>
<td>Structures</td>
<td>6.1</td>
<td>6.1</td>
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<tr>
<td>Equipment</td>
<td>4.6</td>
<td>4.6</td>
</tr>
<tr>
<td>ACE-2</td>
<td></td>
<td></td>
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<td>Total</td>
<td>3.0</td>
<td>3.0</td>
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<td>Structures</td>
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<td>1.9</td>
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<tr>
<td>Equipment</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>ACE-3</td>
<td></td>
<td></td>
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<tr>
<td>Total</td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td>Structures</td>
<td>8.3</td>
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<td>4.2</td>
<td>4.2</td>
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<tr>
<td>ACE-4</td>
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<tr>
<td>Total</td>
<td>5.7</td>
<td>5.7</td>
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<tr>
<td>Structures</td>
<td>4.9</td>
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<td>ACE-5</td>
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<td>Total</td>
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<td>17.1</td>
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<td>ACE-6</td>
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<tr>
<td>Total</td>
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<td>15.8</td>
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<tr>
<td>Structures</td>
<td>25.0</td>
<td>25.0</td>
</tr>
<tr>
<td>Equipment</td>
<td>89.0</td>
<td>89.0</td>
</tr>
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</table>

For the ratio adjustment procedure estimates, the difference between shortcut and reweighted procedure standard error estimates is the sum of two separate terms, $(\hat{\alpha}^*)^2$ (the auxiliary variable MSE contribution from both sampling phases) and $2\hat{\alpha}^*\hat{\beta}_i$. Table 2 presents the linearized reweighted variance estimates (LinR V) along with these estimated components. Notice that the relative contribution of the $(\hat{\alpha}^*)^2$ term is very small compared to the $2\hat{\alpha}^*\hat{\beta}_i$ covariance term and
to the total estimated variance. The relative magnitude of the \((\hat{A}^2)^2\) term indicates a “canceling” effect of the auxiliary variable error contributions from both sample phases, i.e., the error contribution from the 1\(^{st}\) to 2\(^{nd}\) phase is only slightly larger than the auxiliary variable error contribution to the ordinary ratio estimate. This is not unreasonable, given the consistently high unit response rates in the ACE-1 strata. The correlation between payroll and capital expenditures data is quite low for small companies, accounting for the proximity of corresponding reweighted and shortcut variance estimates.

Table 2: Variance Components of Fully Reweighted Linearized Jackknife with Ratio Adjustment (ACE-1) (x 10\(^2\))

<table>
<thead>
<tr>
<th>Item</th>
<th>LinR_V</th>
<th>((\hat{A}^2)^2)</th>
<th>(2\hat{A}^2\hat{B}_Y)</th>
<th>((\hat{A}^2)^2 + 2\hat{A}^2\hat{B}_Y)</th>
</tr>
</thead>
<tbody>
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<td>2001</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>69.7</td>
<td>2.3</td>
<td>-4.7</td>
<td>-2.3</td>
</tr>
<tr>
<td>Structures</td>
<td>37.5</td>
<td>0.7</td>
<td>-1.5</td>
<td>-0.8</td>
</tr>
<tr>
<td>Equipment</td>
<td>21.5</td>
<td>1.0</td>
<td>-1.9</td>
<td>-0.9</td>
</tr>
<tr>
<td>Total</td>
<td>89.8</td>
<td>1.7</td>
<td>-3.8</td>
<td>-2.1</td>
</tr>
<tr>
<td>Structures</td>
<td>68.4</td>
<td>0.66</td>
<td>-1.6</td>
<td>-0.9</td>
</tr>
<tr>
<td>Equipment</td>
<td>17.3</td>
<td>0.6</td>
<td>-1.3</td>
<td>-0.6</td>
</tr>
<tr>
<td>Total</td>
<td>381</td>
<td>9.3</td>
<td>-23.2</td>
<td>-14</td>
</tr>
<tr>
<td>Structures</td>
<td>18.1</td>
<td>0.6</td>
<td>-1.1</td>
<td>-0.4</td>
</tr>
<tr>
<td>Equipment</td>
<td>293</td>
<td>4.7</td>
<td>-13.3</td>
<td>-8.6</td>
</tr>
</tbody>
</table>

Table 3 presents the linearized reweighted variance estimates using the count adjustment procedure from the ACE-1 and ACE-2 data and the \((\hat{A}^2)^2\) component. Again, we see canceling in the auxiliary variable error contributions, with the ordinary ratio estimate error increase being offset by an approximately equally large error component due to the random sample size (1\(^{st}\) to 2\(^{nd}\) stage error component).

Table 3: Variance Components of Fully Reweighted Linearized Jackknife with Count Adjustment (x 10\(^2\))

<table>
<thead>
<tr>
<th>Item</th>
<th>LinR_V</th>
<th>((\hat{A}^2)^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACE-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>71.0</td>
<td>1.6</td>
</tr>
<tr>
<td>Structures</td>
<td>38.2</td>
<td>0.5</td>
</tr>
<tr>
<td>Equipment</td>
<td>21.5</td>
<td>0.7</td>
</tr>
<tr>
<td>ACE-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>9.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Structures</td>
<td>3.7</td>
<td>0.01</td>
</tr>
<tr>
<td>Equipment</td>
<td>4.0</td>
<td>0.08</td>
</tr>
<tr>
<td>ACE-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>94.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Structures</td>
<td>72.2</td>
<td>0.6</td>
</tr>
<tr>
<td>Equipment</td>
<td>18.0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The ACES data are characterized by high unit response rates and high reported zero rates. The results presented in Tables 2 and 3 are completely in line with a MAR model assumption: that is, there is a very small contribution to the overall error from unit non-response. The variance estimation results alone are not, however, sufficient for assuming ignorable non-response, since our derivations assume a uniform or MAR response mechanism.

4 Conclusion

This paper presents research undertaken to investigate the variance estimation effects of not replicating a unit non-response weight adjustment procedure. We assume that respondents comprise a Bernoulli sample of sampled units within a weighting class, so that the realized sample has a random sample size and this random element is reflected in the fully reweighted jackknife variance estimates.

Given an ignorable response mechanism and a two-phase sample design (stratified SRS-WOR at 1\(^{st}\) phase, Bernoulli at 2\(^{nd}\) phase), we show that using a shortcut procedure yields overly large MSE estimates with the count procedure adjustment and generally overestimates the MSE with a ratio adjustment procedure. The degree of overestimation is, however, a function of the weighting cell sample size, the weighting cell respondent rate, and in the case of the ratio adjustment procedure, the covariance between the characteristic of interest and the auxiliary variable. In a highly stratified survey with varying survey weights and varying unit response rates, it is difficult to predict at what point the cumulative effect of the shortcut procedure overestimation will become severe enough to affect confidence interval coverage. In fact, it is likely that corresponding fully reweighted and shortcut variance estimates will often be comparable; the two MSE components from the auxiliary variable will nearly
cancel, and the majority of the variance will derive from the respondent sample ratio estimate.

The justification for the use of a shortcut procedure in a replicate variance estimation method is to save time and computing resources. If these are truly issues and the program has consistently high unit response-rates in all weighting cells, then while there are clearly theoretical advantages to fully replicating the weight adjustment procedure, there may be little or no practical advantage. Having said that, our applications demonstrated excellent approximations to stratified jackknife variance estimates with our linearized jackknife variance estimators, both of which are computationally quick and computer overhead “free” (in terms of replicate storage). Given these viable alternatives, it is difficult to justify the use of a shortcut procedure variance estimator -- replicated or linearized -- over a fully replicated procedure variance, at least in the case of weighting adjustment for unit non-response.

In conclusion, note that our findings rely on several assumptions. In Section 2, we assume that 2nd order and higher derivatives are negligible for the linearization. Moreover, these derivations rely on a Bernoulli sample of respondents. We make statements about the expected “overestimation” with a shortcut procedure stratified jackknife ratio estimator and a ratio adjustment procedure for unit non-response, assuming a positive correlation between characteristic(s) and auxiliary variables. The empirical results in Section 3 provide some support for the first two assumptions. More general statements about the variance estimation effects with a ratio adjustment procedure or the degree of variance overestimation using a shortcut procedure and a count adjustment are not possible without a controlled study; and in fact, the next steps of our research will explore these issues via a simulation study. Finally, the results presented in this paper are applicable to data sets with missing-completely-at-random (uniform) or missing-at-random (MAR) response mechanisms. However, the view of unit non-response as a second random stage of sampling is not necessarily realistic for a voluntary survey. It is equally likely that several non-respondents are fixed in the population, so that the unit non-response is an estimation-bias problem. A limitation of our results is they do not apply to survey data that has a nonignorable response mechanism. And of course, without conducting a survey of nonresponding units, it is impossible to assess the validity of the model assumptions used.

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References