Improved Evaluation of the Quantitative Survey Response Variance

Patrick E. Flanagan
U.S. Bureau of the Census

Abstract

When evaluating the survey response variance of quantitative survey data using a reinterview, the Index of Inconsistency has an advantage over the Pearson correlation coefficient and Lin's concordance correlation coefficient in that it estimates the more illustrative ratio of the simple response variance to the overall variance (though under restrictive conditions they all produce the same results). The typical formula for the Index of Inconsistency for quantitative data is biased if the equal means or equal variances assumptions are violated. I propose an improved Index of Inconsistency "testimator" to adjust the estimation for violations of those assumptions, particularly focusing on the equal variance assumption. The results of simulations illustrate the effect of violation of assumptions on four possible estimators, comparatively illustrating their properties. The approach is applied to actual data from the Current Population Survey.

Keywords: Surveys, index of inconsistency, response variance, response error, reinterview.

1. Introduction

Measure-remeasure experiments are often used for comparing measurement instruments in a validation process, as discussed in Lin (1989). Similarly, in surveys, they are used to estimate the response variance of questions responses to identify questions that induce an unreasonable variation, potentially producing unreliable estimates. The survey application is still a measure of instrument effect in that the questionnaire is the instrument. However, the survey case differs in that its primary purpose is not comparing two instruments, but using the same instrument twice measuring the same source to estimate variance of response error. This is most frequently done with an interview-reinterview experiment.

In an interview-reinterview experiment designed to measure response variance, a simple random sub-sample of the original survey cases is selected. The interview is conducted a second time in a manner so as to duplicate the original interview conditions, but far enough after the original interview to achieve a level of independence of the response errors. From a population of size N, let n_o be the size of the original interview sample and let n_r be the size of the reinterview sample, such that $n_r \leq n_o$. The original interview response to a given question is $x_{1i} = \mu_i + \beta_{1i} + \epsilon_{1i}$, i = 1, 2, ..., n_o and the reinterview responses are $x_{2i} = \mu_i + \beta_{2i} + \epsilon_{2i}$, i = 1, 2, ..., n_r, where μ_i is the "true" answer to the survey question, β_{1i} and β_{2i} are the systematic error for each respondent's original and reinterview, respectively, (i.e., they are the expected values for error in the two interviews) and ϵ_{1i} and ϵ_{2i} are the variable response error components for the interview and reinterview, respectively. The errors are regarded as random events at the point of observation.

For a response variance experiment, the population characteristic of interest is the population variance of the error terms across the population, called the simple response variance (SRV), or

$$SRV = \frac{1}{N}\sum_{i=1}^{N}(\beta_{1i} - \bar{\beta})^2 + \frac{1}{N}\sum_{i=1}^{N} Var(\epsilon_{1i}).$$

Given the interview-reinterview sample response pairs, a simple estimator for SRV is

$\widehat{SRV} = \frac{1}{2n_r}\sum_{i=1}^{n_r}(x_{1i} - x_{2i})^2$, making the assumptions of equal means, equal variances, and case-wise independence of response errors between interview and reinterview. If those assumptions are met, it is an unbiased estimator of SRV, given that the original survey were a simple random sample, and

$$E\left[\frac{1}{2n_r}\sum_{i=1}^{n_r}(x_{1i} - x_{2i})^2\right] = \frac{1}{N}\sum_{i=1}^{N} p_i \frac{N}{n_o}(\beta_{1i} - \bar{\beta})^2 + \frac{1}{N}\sum_{i=1}^{N} p_i \frac{N}{n_o} Var(\epsilon_{1i})$$

(where p_i = probability of selection for case i in the

This paper is released to inform interested parties of research and to encourage discussion. The views expressed on statistical, methodological, technical, and operational issues are those of the author and not necessarily those of the U.S. Census Bureau.

original interview) if the survey is complex,[1] which has proven to be negligibly biased in practice.

Further, since the interpretation of SRV is relative to the overall population variance of the data, the index of inconsistency (Census 1985) is used as a more easily interpretable ratio of SRV to overall variance,

$$I = \frac{\frac{1}{N}\sum_{i=1}^{N}(\beta_{1i}-\bar{\beta})^2 + \frac{1}{N}\sum_{i=1}^{N}\mathrm{Var}(\epsilon_{1i})}{\frac{1}{N}\sum_{i=1}^{N}(\mu_i-\bar{\mu})^2 + \frac{1}{N}\sum_{i=1}^{N}(\beta_{1i}-\bar{\beta})^2 + \frac{1}{N}\sum_{i=1}^{N}\mathrm{Var}(\epsilon_{1i})},$$

where $\bar{\mu} = \frac{1}{N}\sum_{i=1}^{N}\mu_i$. The Index of inconsistency for quantitative variables can be estimated by

$$\hat{I}_{Census} = \frac{\frac{1}{2n_r}\sum_{i=1}^{n_r}(x_{1i}-x_{2i})^2}{\frac{1}{2}\left(s_1^2 + s_2^2\right)}, \text{ where}$$

$$s_1^2 = \frac{1}{n_r-1}\sum_{i=1}^{n_r}(x_{1i}-\bar{x}_1)^2 \text{ and}$$

$$s_2^2 = \frac{1}{n_r-1}\sum_{i=1}^{n_r}(x_{2i}-\bar{x}_2)^2,$$

though, within those same assumptions, it is also frequently estimated by $\hat{I} = 1 - r$, where

$$r = \frac{\frac{1}{n_r-1}\sum_{i=1}^{n_r}(x_{1i}-\bar{x}_1)x_{2i}}{\sqrt{s_1^2 s_2^2}}.$$

2. Problem Statement

The estimate of the SRV used in the Index of Inconsistency makes assumptions of case-wise equal means and equal variances between interview and reinterview. In practice, there is always some difference and frequently statistically significant differences due to a variety of reasons (Forsman and Schreiner 1991):

- Time lag between interview and reinterview
- Interviewer type differences (e.g., field representative. vs. telephone center)
- Mode differences (e.g., personal visit vs. telephone)

[1] This does ignore case to case dependence caused by things like interviewer effects and cluster sampling.

So, if, in fact, the means and variances are different, but the independence assumption is still met, the numerator of the Index of Inconsistency will have a bias, in that the numerator will be biased from the inequality of means and both the numerator and denominator will be biased from the inequality of the variances. So letting $\delta_i = \mathrm{Var}(\epsilon_{2i}) - \mathrm{Var}(\epsilon_{1i})$, and the population variance, $PV = \frac{1}{N}\sum_{i=1}^{N}(\mu_i-\bar{\mu})^2$, the numerator of the index of inconsistency would be estimating:

$SRV + \frac{1}{2}(\beta_1-\beta_2)^2 + \frac{1}{2N}\sum_{i=1}^{N}\delta_i$ and the denominator would be estimating $PV + SRV + \frac{1}{2N}\sum_{i=1}^{N}\delta_i$, thus the estimator for the Index of Inconsistency,

$$\hat{I}_{Census} = \frac{\frac{1}{2n_r}\sum_{i=1}^{n_r}(x_{1i}-x_{2i})^2}{\frac{1}{2}\left(s_1^2 + s_2^2\right)},$$

would actually estimate

$$\frac{SRV + \frac{1}{2}(\bar{\beta}_1-\bar{\beta}_2)^2 + \frac{1}{2N}\sum_{i=1}^{N}\delta_i}{PV + SRV + \frac{1}{2N}\sum_{i=1}^{N}\delta_i}.$$

3. Discussion

Given an interview-reinterview paired sample (Ω), with the random errors,

$\epsilon_{1i} \sim (0, \mathrm{Var}(\epsilon_{1i})) \perp \epsilon_{2i} \sim (0, \mathrm{Var}(\epsilon_{2i})) \quad \forall$ i=1..n,

then $\begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\end{pmatrix} \sim \left[\begin{pmatrix}\tilde{\mu}+\tilde{\beta}_1\\ \tilde{\mu}+\tilde{\beta}_2\end{pmatrix}, \Sigma_\epsilon\right]$, where

$$\Sigma_\epsilon = \begin{pmatrix}\mathbf{V}_{\epsilon_1} & \mathbf{0}_{nxn}\\ \mathbf{0}_{nxn} & \mathbf{V}_{\epsilon_2}\end{pmatrix}, \text{ with}$$

$$\mathbf{V}_{\epsilon_1} = \begin{bmatrix}\mathrm{Var}(\epsilon_{11}) & 0 & \cdots & 0\\ 0 & \mathrm{Var}(\epsilon_{12}) & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \mathrm{Var}(\epsilon_{1n})\end{bmatrix} \text{ and}$$

$$\mathbf{V}_{\epsilon_2} = \begin{bmatrix} \mathrm{Var}(\epsilon_{21}) & 0 & \cdots & 0 \\ 0 & \mathrm{Var}(\epsilon_{22}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathrm{Var}(\epsilon_{2n}) \end{bmatrix}.$$

This makes a number of assumptions, including no correlation between survey cases, case-wise independence between interview and reinterview errors, but does not assume equal means or equal variances.

An unbiased estimator for SRV if the equal means assumption is not true, but the variance are equal would be $\hat{SRV}_{Means} = \frac{1}{2(n-1)}\sum_{i=1}^{n}(x_{1i} - x_{2i} - (\bar{x}_1 - \bar{x}_2))^2$. This is in fact the Best Quadratic Unbiased Estimator (BQUE) under those assumptions (See Flanagan 2001).

However, if both the means and variances are not equal, an unbiased estimator for SRV would be

$$\hat{SRV}_{Var} = \frac{1}{n-1}\sum_{i=1}^{n}(x_{1i} - \bar{x}_1)^2 - \frac{1}{n-1}\sum_{i=1}^{n}(x_{1i} - \bar{x}_1)x_{2i}$$

and an unbiased estimator for total variance in the denominator of the Index of Inconsistency would be

$$\frac{1}{n-1}\sum_{i=1}^{n}(x_{1i} - \bar{x}_1)^2.$$

4. An Improved Index of Inconsistency "Testimator"

The following is a procedure for testing the equal means and equal variances assumptions and choosing an estimator depending on the results, that is unbiased for the numerator and the denominator of Index of Inconsistency:

STEP 1 => Test for equal means:

Test for equal means: $H_0: \mu_1 - \mu_2 = 0$; $H_A: \mu_1 - \mu_2 \neq 0$; let $D_i = x_{1i} - x_{2i}$, $\forall$ i,

$$Z = \frac{\bar{D}}{\sqrt{\frac{\sum(D_i - \bar{D})^2}{n_r(n_r - 1)}}} \quad ; \quad \text{if Z is significant}$$

(using a standard Normal), reject the null hypothesis and conclude that the means are not equal.

STEP 2 => Test for equal variances:

Run the paired sample Pitman test (Snedecor and Cochran 1989) for equal variances:

- Let $F = s_1^2/s_2^2$, where $s_1^2 = \frac{1}{n_r - 1}\sum_{i=1}^{n_r}(x_{1i} - \bar{x}_1)^2$ and $s_2^2 = \frac{1}{n_r - 1}\sum_{i=1}^{n_r}(x_{2i} - \bar{x}_2)^2$
- Compute the sample correlation coefficient of x_1 and x_2, $$r = \frac{\frac{1}{n_r - 1}\sum_{i=1}^{n_r}(x_{1i} - \bar{x}_1)x_{2i}}{\sqrt{s_1^2 s_2^2}}.$$
- Let $r_{DS} = (F - 1)/\sqrt{(F+1)^2 - 4r^2F}$. Then
- Look up r_{DS} in a correlation coefficient table using n_r - 2 degrees of freedom.
- If $|r_{DS}|$ is greater than the table value, reject the null hypothesis that $\rho_{DS} = 0$, which is equivalent to an $Ho: \sigma_1^2 = \sigma_2^2$.

STEP 3 => Chose an estimator based upon the results:

If	Then
$\mu_1 = \mu_2$ and $\sigma_1^2 = \sigma_2^2$	Use $\hat{I}_1 = \dfrac{\frac{1}{2n_r}\sum_{i=1}^{n_r}(x_{1i} - x_{2i})^2}{\frac{1}{2}(s_1^2 + s_2^2)}$
$\mu_1 \neq \mu_2$ and $\sigma_1^2 = \sigma_2^2$	Use $\hat{I}_2 = 1 - \dfrac{s_{12}}{\frac{1}{2}(s_1^2 + s_2^2)}$
$\mu_1 = \mu_2$ and $\sigma_1^2 \neq \sigma_2^2$ or $\mu_1 \neq \mu_2$ and $\sigma_1^2 \neq \sigma_2^2$	Use $\hat{I}_3 = \dfrac{s_1^2 - s_{12}}{s_1^2} = 1 - \dfrac{s_{12}}{s_1^2}$

where $s_1^2 = \frac{1}{n_r - 1}\sum_{i=1}^{n_r}(x_{1i} - \bar{x}_1)^2$,

$s_2^2 = \frac{1}{n_r - 1}\sum_{i=1}^{n_r}(x_{2i} - \bar{x}_2)^2$ and ,

$s_{12} = \frac{1}{n_r - 1}\sum_{i=1}^{n_r}(x_{1i} - \bar{x}_1)x_{2i}$.

5. Simulations

For each simulation, 1000 sets consisting of a "true" value, an original survey error term, and a reinterview survey error term were generated and estimates of SRV and the Index of Inconsistency by four methods were calculated. This process was, in turn, repeated 1000 times to allow calculation of variance, bias, and mean square error. For an income-like scenario, simulations were run varying the mean and variance of the reinterview to illustrate violation of those assumptions to varying degrees.

For all simulations, a mean true value of 40,000, a variance for the true value of $(13{,}000)^2$, an original interview bias of 3,000, and an original interview response variance of $(6000)^2$ were used. Nine simulations were then run with the simulation input values varied as follows:

→ Equal variances and equal means
→ Equal variances and reinterview bias 70% of interview bias
→ Equal variances and reinterview bias 50% of interview bias
→ Reinterview error variance 70% of interview error variance and equal means
→ Reinterview error variance 70% of interview error variance and reinterview bias 70% of interview bias
→ Reinterview error variance 70% of interview error variance and reinterview bias 50% of interview bias
→ Reinterview error variance 50% of interview error variance and equal means
→ Reinterview error variance 50% of interview error variance and reinterview bias 70% of interview bias
→ Reinterview error variance 50% of interview error variance and reinterview bias 50% of interview bias

The mean square errors were compared for the following statistics:

$$\hat{SRV}_{Census} = \frac{1}{2n_r}\sum_{i=1}^{n_r}(x_{1i} - x_{2i})^2,$$

$$\hat{I}_{Census} = \frac{\hat{SRV}_{Census}}{\frac{1}{2}(s_1^2 + s_2^2)}, \quad \hat{I}_{Corr} = 1 - \frac{s_{12}}{\sqrt{s_1^2 s_2^2}},$$

$$\hat{SRV}_{Means} = \frac{1}{2(n_r - 1)}\sum_{i=1}^{n_r}(x_{1i} - x_{2i} - (\bar{x}_1 - \bar{x}_2))^2,$$

$$\hat{I}_{Means} = \frac{\frac{1}{2(n_r - 1)}\sum_{i=1}^{n_r}(x_{1i} - x_{2i} - (\bar{x}_1 - \bar{x}_2))^2}{\frac{1}{2}(s_1^2 + s_2^2)},$$

$\hat{SRV}_{Var} = s_1^2 - s_{12}$, and

$\hat{I}_{var} = 1 - \frac{s_{12}}{s_1^2}$, where $s_1^2 = \frac{1}{n_r - 1}\sum_{i=1}^{n_r}(x_{1i} - \bar{x}_1)^2$,

$s_2^2 = \frac{1}{n_r - 1}\sum_{i=1}^{n_r}(x_{2i} - \bar{x}_2)^2$, and

$s_{12} = \frac{1}{n_r - 1}\sum_{i=1}^{n_r}(x_{1i} - \bar{x}_1)x_{2i}$.

Results of the simulations are provided in Table 1 with the low bias and low mean square error in bold for each.

6. Variances Estimators

From Flanagan (2001), the following are formulas for estimating variance for the estimators discussed.

Given equal means and variances, an estimate of the variance of $\hat{I}_{Census} = \frac{\frac{1}{2n_r}\sum_{i=1}^{n_r}(x_{1i} - x_{2i})^2}{\frac{1}{2}(s_1^2 + s_2^2)}$, is

$$\hat{Var}(\hat{I}_{Census}) = \frac{2}{n_r}\hat{I}_{Census}^2 - \frac{2}{n_r}\left(1 - \left(1 + \frac{1}{n_r - 1}\right)\frac{s_{12}}{\frac{1}{2}(s_1^2 + s_2^2)}\right)\hat{I}_{Census}^3 + \frac{1}{n_r - 1}\hat{I}_{Census}^4,$$

where $s_{12} = \frac{1}{n_r - 1}\sum_{i=1}^{n_r}(x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2)$.

Given differing means and equal variances between interview and reinterview, an approximation of the variance of the estimator of

$$\hat{I}_{Means} = \frac{\frac{1}{2(n_r-1)}\sum_{i=1}^{n_r}(x_{1i} - x_{2i} - \bar{x}_1 + \bar{x}_2)^2}{\frac{1}{2}(s_1^2 + s_2^2)}, \text{ is}$$

$$\hat{Var}\left(\hat{I}_{Means}\right) \approx \frac{1}{n_r - 1}\left[2\hat{I}^2_{Means} - 2\left(1 - \frac{s_{12}}{\frac{1}{2}(s_1^2 + s_2^2)}\right)\hat{I}^3_{Means} + \hat{I}^4_{Means}\right],$$

where

$$s_{12} = \frac{1}{n_r - 1}\sum_{i=1}^{n_r}(x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2).$$

Given differing variances between interview and reinterview, an approximation of the variance of the estimator $\hat{I}_{Var} = \frac{s_1^2 - s_{12}}{s_1^2} = 1 - \frac{s_{12}}{s_1^2}$ is

$$\hat{Var}\left(\hat{I}_{Var}\right) \approx \frac{1}{n_r - 1}\left[\frac{s_2^2 - s_{12}}{s_1^2} + \hat{I}_{Var}(1 - \hat{I}_{Var}) - 4\hat{I}_{Var}(1 - \hat{I}_{Var})^2 + 2\hat{I}^2_{Var}(1 - \hat{I}_{Var})^2 + 4\hat{I}_{Var}(1 - \hat{I}_{Var})^3\right],$$

where $s_{12} = \frac{1}{n_r - 1}\sum_{i=1}^{n_r}(x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2)$.

7. Application to the Current Population Survey

The following is an application of the methods discussed herein to the Reinterview of the March 1998 Current Population Survey (CPS) Income Supplement. The data was collected by the U. S. Bureau of the Census in March 1998 and results were published in U.S. Bureau of the Census (1999).

The CPS is a monthly survey of approximately 60,000 households from across the United States. From those households, all individuals 15 years old and older, not in the Armed Forces, are interviewed. Thus, the survey collects information on over 125,000 people every month. The survey's primary purpose is to collect employment information. However, a wide range of other information is also collected.

Each March, the CPS asks additional questions related to income to all of its sample households plus an additional oversample of 2,500 households with Hispanic members.

In March 1998, a reinterview was conducted on the Income Supplement questions for a sample of the households that responded to the March Income Supplement to the CPS. An initial sample of 1,666 households in two strata. One stratum consisted of "poverty" households, as defined by OMB's Statistical Policy Directive 14. The other consisted of non-poverty households. Each stratum had 833 households in the initial sample. That had the effect of oversampling poverty. Of the 1,666 households, 1,346 responded to the March Income Supplement, making them eligible for reinterview. Of those eligible households, 1,008 households responded to reinterview, with some or all questions answered by or for 2,731 individuals.

"How much did you earn from your longest job in 1997?"

This question is an illustration of the continuous data "testimator."

For this question, 830 sample persons answered both interview and reinterview. Of those, 5 were removed from calculations as outliers probably due to keying errors.

Conventional Calculation

The estimate of the Index of Inconsistency using the standard estimator, $\hat{I} = \frac{\frac{1}{2n_r}\sum_{i=1}^{n_r}\left(x_{1i} - x_{2i}\right)^2}{\frac{1}{2}\left[s_1^2 + s_2^2\right]}$, is 17.1% with a 90% confidence interval [15.4, 18.7]. The test for equal means does not reject the null hypothesis. However:

Problem: The standard deviation of the original interview is 24,020.31, while the standard deviation of the reinterview is 22,957.80.

Testing for equal variances:

F = 1.09470, so r_{DS} = 0.081512, which is significant($p \approx 0.02$).

Testimator Adjustment

The significant difference in variances implies that the use of $\hat{I} = \dfrac{\frac{1}{2n_r}\sum_{i=1}^{n_r}\left(x_{1i} - x_{2i}\right)^2}{\frac{1}{2}\left[s_1^2 + s_2^2\right]}$ will tend to underestimate the true Index of Inconsistency, so we will use

$\hat{I}_\sigma = \dfrac{s_1^2 - s_{12}}{s_1^2} = 1 - \dfrac{s_{12}}{s_1^2}$. Thus the estimate for the Index of Inconsistency using $\hat{I}_\sigma$ is 20.4%. The confidence interval is [17.1%, 23.6%].

8. Conclusions

The simulation did show that when the equal means assumption was violated, use of

$S\hat{R}V_{Means} = \dfrac{1}{2(n_r - 1)}\sum_{i=1}^{n_r}(x_{1i} - x_{2i} - (\bar{x}_1 - \bar{x}_2))^2$ in the numerator reduced the mean square error of the Index of Inconsistency. However, it also showed that using 1 - r (correlation coefficient between X_1 and X_2) produced an estimate slightly lower in MSE when means are not equal. The strongest result of the simulation was the dramatic reduction in MSE when using $\hat{I}_{var} = 1 - \dfrac{s_{12}}{s_1^2}$, in the case of unequal variances. As a result, I would recommend using 1 - r to estimate the Index of Inconsistency, except where the test for equal variances indicate that the assumption has been violated.

References

Flanagan, P. (2001) "Measurement Errors in Survey Response," Ph.D. dissertation, University of Maryland Baltimore County, 85 - 92.

Forsman, G. And Schreiner, I. (1991), "The Design and Analysis of Reinterview: An Overview," *Measurement Errors in Surveys*, ed. Biemer, P., Groves, R., Lyberg, L., Mathiowetz, N., and Sudman, S., Wiley, NY, 279 - 301.

Lin, L.I. (1989), "A Concordance Correlation Coefficient to Evaluate Reproducibility," *Biometrics*, 45, 255-268.

Snedecor, G. And Cochran, W. (1989), Statistical Methods, 8th ed., Iowa State University Press, Ames, Iowa, 192 - 193.

U.S. Bureau of the Census (1985), Evaluating Censuses of Population and Housing, Statistical Training Document ISP-TR-5.

U.S. Bureau of the Census (1999), Response Variance in the March 1998 Current Population Survey Income Supplement.

Table 1

Simulation Results (Means Square Error over Bias for each)

#	Description	SRV_{Census}	SRV_{Means}	SRV_{Var}	I_{Census}	I_{Corr}	I_{Means}	I_{Var}
1	= means = variances	2.47266E12 **-51260.14**	**2.47261E12** -51366.22	1.61138E13 -56182.99	**1.10254E-4** 1.37678E-4	1.10274E-4 **2.35757E-6**	1.10301E-4 1.37296E-4	3.41875E-4 -1.28600E-4
2	$mean_O$ =43K, $mean_R$=42.1K, = variances	2.99795E12 4.74392E5	**2.72799E12** **7.75390E4**	1.63685E13 9.51640E4	1.33118E-4 2.79200E-3	**1.22710E-4** 6.86417E-4	1.22991E-4 8.20944E-4	3.33634E-4 **6.35234E-4**
3	$mean_O$ =43K, $mean_R$=41.5K, = variances	3.73815E12 1.08253E6	**2.38175E12** **-3.48686E4**	1.69093E13 -2.75680E5	1.56842E-4 6.07290E-3	**1.11833E-4** **4.59140E-4**	1.12037E-4 5.97674E-4	3.46551E-4 -6.55600E-4
4	= means, var_o=205M, var_o=186.64M	8.59107E13 -9.18921E6	8.59352E13 -9.19051E6	**1.23105E13** **3.07287E4**	1.55290E-3 -3.85109E-2	1.63600E-3 -3.95853E-2	1.55340E-3 -0.385176E-2	**2.43915E-4** **8.30360E-5**
5	$mean_O$ =43K, $mean_R$=42.1K var_o=205M, var_o=186.64M	7.87067E13 -8.78461E6	8.59054E13 -9.18699E6	**1.20711E13** **2.1338.91E4**	1.41990E-3 -3.65993E-2	1.65550E-3 -3.97212E-2	1.57220E-3 -3.86561E-2	**2.43909E-4** **-1.54963E-4**
6	$mean_O$ =43K, $mean_R$=41.5K var_o=205M, var_o=186.64M	6.72326E13 -8.10582E6	8.67643E13 -9.23865E6	**1.23959E13** **-1.49567E5**	1.14820E-3 -3.26650E-2	1.63460E-3 -3.95144E-2	1.55450E-3 3.84723E-2	**2.50205E-4** **-3.45763E-4**
7	= means var_o=205M, var_o=178M	1.83517E14 -1.35109E7	1.83500E14 -1.35101E7	**1.07919E13** **-7.14448E4**	3.39110E-3 -5.77557E-2	3.65910E-3 -6.00412E-2	3.39070E-3 -5.77520E-2	**2.09564E-4** **4.43446E-6**
8	$mean_O$ =43K, $mean_R$=42.1K var_o=205M, var_o=178M	1.73356E14 -1.31269E7	1.83912E14 1.35242E7	**9.53630E12** **-1.13370E5**	3.18930E-3 -5.59645E-2	3.69090E-3 -6.03047E-2	3.42420E-3 -5.80445E-2	**1.87244E-4** **-4.55297E-4**
9	$mean_O$ =43K, $mean_R$=41.5K var_o=205M, var_o=178M	1.55261E14 -1.24165E7	1.84019E14 -1.35274E7	**1.00803E13** **2.12065E4**	2.78910E-3 -5.22285E-2	3.69740E-3 -6.03597E-2	3.42470E-3 -5.80462E-2	**2.01190E-4** **1.07821E-4**