A bridge between the GREG and the Linear Regression Estimators

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Abstract

We discovered there is a choice of weights that builds a bridge between the GREG proposed by Deville and Sarndal (1992) and the linear regression estimator due to Hansen, Hurwitz, and Madow (1953) while using one auxiliary variable. It gives the same result as given in Singh (2003, 2004, 2006a, 2006b) and Stearns and Singh (2005) for unequal probability sampling by using two calibration constraints in the presence of one auxiliary variable. Thus, these approaches can be considered as alternative to each other while considering use of one auxiliary variable. The bridge developed here reconfirms that the sum of the calibrated weights should be equal to the sum of the design weights in a given sample. The approach by Singh (2003, 2004, 2006a, 2006b), and Stearns and Singh (2005) seems simple while using multiauxiliary information.

Keywords: Calibration; Linear regression estimator; GREG; Estimation of total and variance.

1. Introduction

The problem of calibration of design weights is well known in the literature of survey sampling. Deville and Särndal (1992) used the method of calibration of estimators using auxiliary information. Their calibration method provides a class of estimators. Some of the well-known estimators such as classical ratio estimator belong to this class. Several authors including Singh (2003, 2004, 2006a, 2006b), Farrell and Singh (2002, 2005), Wu and Sitter (2001), Sitter and Wu (2002), Wu (2003), Estevao and Särndal (2003), Kott (2003, 2006) and Montanari and Ronalli (2005) among others considered the Deville and Särndal (1992) method and derived important calibrated estimators. But, so far derivation of the traditional linear regression estimator from the class of calibrated estimators derived by Deville and Särndal (1992) method has not been found in the literature. Here we have considered a subclass of the class of calibrated estimators provided by Deville and Särndal (1992). In this proposed subclass, the sum of calibrated weights remains equal to the sum of design weights as pointed out by Singh (2003, 2004, 2006a, 2006b). The traditional regression estimator is found to belong to the proposed sub class.

Consider a finite population $\Omega = \{1, 2, .., i, .., N\}$ of N units, from which a probability sample $s(s \subset \Omega)$ of fixed size n is drawn with probability $p(s)$ according to a given sampling design p. The inclusion probabilities $\pi_i = \Pr(i \in s)$ and $\pi_{ij} \in \Pr(i \neq j \in s)$ are assumed to be strictly positive and known. Let y_i be the value of the variable of interest, y, for the i^{th} unit of the population, with which is also associated an auxiliary variable x_i. For the element $i \in s$, we observe (y_i, x_i). The population total of the auxiliary variable x, $X = \sum_{i \in \Omega} x_i$, is assumed to be known. The objective is to estimate the population total $Y = \sum_{i \in \Omega} y_i$. Deville and Särndal (1992) proposed the calibrated estimator:

$$\hat{Y}_{\text{ds}} = \sum_{i \in s} w_i y_i \tag{1.1}$$

for the Horvitz and Thompson (1952) estimator:

$$\hat{Y}_{\text{HT}} = \sum_{i \in s} \frac{y_i}{\pi_i} = \sum_{i \in s} d_i y_i \tag{1.2}$$

where $d_i = 1/\pi_i$ and the calibrated weights w_i, $i \in s$ are obtained by minimizing chi-square type distance function:

$$\sum_{i \in s} \frac{(w_i - d_i)^2}{d_i q_i} \tag{1.3}$$

subject to the calibration constraint:

$$\sum_{i \in s} w_i x_i = X \tag{1.4}$$

Here q_i, $i \in s$ are suitably chosen weights. In many situations the value of $q_i = 1$. The form of the estimator (1.1) depends upon the choice of q_i. Minimization of (1.3) subject to calibration equation (1.4), leads to the calibrated weights:

$$w_i = d_i + \frac{d_i q_i x_i}{\sum_{i \in s} d_i q_i x_i^2} \left(X - \sum_{i \in s} d_i x_i \right) \tag{1.5}$$

Substitution of the value of w_i from (1.5) in (1.1) leads to the generalized regression (GREG) estimator of the population total Y as:

$$\hat{Y}_{\text{GREG}} = \sum_{i \in s} d_i y_i + \hat{\beta}_{ds} \left(X - \sum_{i \in s} d_i x_i \right) \tag{1.6}$$

where

$$\hat{\beta}_{\text{ds}} = \left(\sum_{i \in s} d_i q_i x_i y_i \right) \Big/ \left(\sum_{i \in s} d_i q_i x_i^2 \right) \tag{1.7}$$

An approximate variance of the calibrated estimator $\hat{Y}_{\text{GREG}}$ for a large sample size provided by Deville and Särndal (1992) as:

$$V_{DS}(\hat{Y}_{GREG}) = \frac{1}{2}\sum_{i \neq j \in \Omega}\sum D_{ij}\pi_{ij}(d_i E_i - d_j E_j)^2 \qquad (1.8)$$

where $D_{ij} = (\pi_i \pi_j - \pi_{ij})/\pi_{ij}$, $B = \sum_{i\in\Omega} q_i x_i y_i \big/ \sum_{i\in\Omega} q_i x_i^2$ and $E_i = y_i - Bx_i$.

A consistent and approximate unbiased estimator of variance proposed by them is:

$$\hat{V}_{DS}(\hat{Y}_{GREG}) = \frac{1}{2}\sum_{i\in s}\sum_{j\in s} D_{ij}(w_i e_i - w_j e_j)^2 \qquad (1.9)$$

with $e_i = y_i - \hat{\beta}_{ds} x_i$.

2. Linear regression estimator using calibration

Deville and Särndal (1992) imposed the constraint $\sum_{i\in s} w_i x_i = X$ under the assumption that the value of the calibrated estimator $\hat{Y}_{ds} = \sum_{i\in s} w_i y_i$ for the total Y should be equal to the known total X if y_i is replaced by x_i . In this section we find how one can derive the ordinary linear regression estimator using the calibrating weights derived by Deville and Särndal (1992).

Let us substitute:

$$q_i = q_i^* \left(\frac{\sum_{i\in s} d_i q_i^*}{\sum_{i\in s} d_i q_i^* x_i} - \frac{1}{x_i} \right) \qquad (2.1)$$

in the expression (1.5). The substitution yields:

$$w_i = w_{i0}$$
$$= d_i + \frac{d_i q_i^* \left(x_i \sum_{i\in s} d_i q_i^* - \sum_{i\in s} d_i q_i^* x_i \right)}{\left(\sum_{i\in s} d_i q_i^* x_i^2\right)\left(\sum_{i\in s} d_i q_i^*\right) - \left(\sum_{i\in s} d_i q_i^* x_i\right)^2} \left(X - \sum_{i\in s} d_i x_i \right) \qquad (2.2)$$

Finally putting $w_i = w_{i0}$ in the equation (1.1), we get:

$$\hat{Y}_{ds} = \hat{Y}_{LR} = \hat{Y}_{HT} + \hat{\beta}_{ols}\left(X - \hat{X}_{HT}\right) \qquad (2.3)$$

with

$$\hat{\beta}_{ols} = \frac{\left(\sum_{i\in s} d_i q_i^*\right)\left(\sum_{i\in s} d_i q_i^* x_i y_i\right) - \left(\sum_{i\in s} d_i q_i^* y_i\right)\left(\sum_{i\in s} d_i q_i^* x_i\right)}{\left(\sum_{i\in s} d_i q_i^*\right)\left(\sum_{i\in s} d_i q_i^* x_i^2\right) - \left(\sum_{i\in s} d_i q_i^* x_i\right)^2}$$

It should be worth noting that the calibrated weights w_{i0}, $i \in s$ satisfy the constraints:

$$\sum_{i\in s} w_i x_i = X \qquad (2.4)$$

and

$$\sum_{i\in s} w_i = \sum_{i\in s} d_i \qquad (2.5)$$

Note that the condition (2.5) is due to Singh (2003, 2004, 2006a, 2006b). It builds a bridge between the GREG due to Deville and Sarndal (1992) and the linear regression estimator due to Hansen, Hurwitz and Madow (1953). Asymptotic properties of the estimator (2.3) are studied by Sampath and Chandra (1990). It reconfirms that there is a strong need to set constraint (2.5) into all the statistical packages like GES, SUDDAN etc. while doing calibration of design weights. ***Caution:*** Choice of weights q_i in (2.1) may lead to a negative chi-square distance due to Deville and Sarndal (1992).

Note that for simple random and without replacement (SRSWOR) sampling, the Wu and Sitter (2001), and the Estevao and Sarndal (2003) calibration constraint is also a special case of (2.5) for $d_i = N/n$.

For SRSWOR, $\pi_i = n/N$ and the estimator (2.3) reduces to:

$$\hat{Y}_{LR} = N\left[\bar{y}_s + \hat{\beta}_{ols}\left(\bar{X} - \bar{x}_s\right)\right] \qquad (2.6)$$

where $\bar{y}_s = \sum_{i\in s} y_i / n$, $\bar{x}_s = \sum_{i\in s} x_i / n$, and $\bar{X} = \sum_{i\in s} x_i / N$.

In particular $q_i^* = 1$, $\hat{\beta}_{ols}$ reduces to $s_{xy} \big/ s_x^2$ and we get:

$$\hat{Y}_{LR} = N[\bar{y} + \frac{s_{xy}}{s_x^2}(\bar{X} - \bar{x})] \qquad (2.7)$$

where $s_{xy} = (n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$. The estimator (2.7) is the famous traditional linear regression estimator due to Hansen, Hurwitz and Madow (1953) in the presence of a single auxiliary variable.

3. Variance estimation

Following Singh, Horn and Yu (1998), a calibrated estimator of the variance of the linear regression estimator $\hat{Y}_{LR}$ in (2.3) is given by:

$$\hat{V}_s(\hat{Y}_{LR}) = \frac{1}{2}\sum_{i\neq j\in s}\sum D_{ij}\Phi_{ij}(s) \qquad (3.1)$$

where $\Phi_{ij}(s) = (w_{i0}\hat{e}_i^* - w_{j0}\hat{e}_j^*)^2$ and $\hat{e}_i^*$ can be obtained by: $\min . \sum_{i\in s} d_i q_i^* \hat{e}_i^{*2}$. Further, we consider a new calibrated estimator of variance of the linear regression as:

$$\hat{V}_{ss}(\hat{Y}_{LR}) = \frac{1}{2}\sum_{i\neq j\in s}\sum \Omega_{ij}(s)\Phi_{ij}(s) \qquad (3.2)$$

where $\Omega_{ij}(s)$ are weights such that the chi-square distance:

$$D = \frac{1}{2}\sum_{i\neq j\in s}\sum \frac{\left(\Omega_{ij}(s) - D_{ij}\right)^2}{D_{ij}Q_{ij}(s)} \qquad (3.3)$$

is minimum subject to a calibration constraint, given by:

$$\frac{1}{2}\sum_{i\neq j\in s}\sum \Omega_{ij}(s)\delta_{ij} = V_{syg}(\hat{X}_{HT}) \qquad (3.4)$$

where $V_{syg} = \frac{1}{2}\sum_{i\neq j\in\Omega}\sum D_{ij}\pi_{ij}\delta_{ij}$ and $\delta_{ij} = (d_i x_i - d_j x_j)^2$.

Obviously, for the minimization of (3.3) subject to (3.4), the Lagrange function is given by:

$$LM = \frac{1}{2}\sum_{i\neq j\in s}\sum \frac{\left(\Omega_{ij}(s) - D_{ij}\right)^2}{D_{ij}Q_{ij}(s)} - \mu\left\{\frac{1}{2}\sum_{i\neq j\in s}\sum \Omega_{ij}(s)\delta_{ij} - V_{syg}(\hat{X}_{HT})\right\} \qquad (3.5)$$

with μ as a Lagrange multiplier. On setting $\partial LM / \partial \Omega_{ij}(s) = 0$, we have:

$$\Omega_{ij}(s) = D_{ij} + \mu\left(\frac{D_{ij}Q_{ij}(s)}{2}\right)\delta_{ij} \quad (3.6)$$

On using (3.4) in (3.6) we have:

$$\mu = 4\left\{V_{syg}(\hat{X}_{HT}) - \hat{V}_{syg}(\hat{X}_{HT})\right\} \Big/ \sum_{i \neq j \in s}\sum D_{ij}Q_{ij}(s)\delta_{ij}^2 \quad (3.7)$$

and noting $\hat{V}_{syg}(\hat{X}_{\text{HT}}) = \frac{1}{2}\sum_{i \neq j \in s}\sum D_{ij}\delta_{ij}$ denotes the Sen (1953) and Yates and Grundy (1953) form of the estimator of variance. On substituting (3.7) into (3.2), we obtain a new calibrated estimator of variance of the linear regression estimator $\hat{Y}_{\text{LR}}$ in (2.3) as:

$$\hat{V}_{\text{ss}}(\hat{Y}_{\text{LR}}) = \hat{V}_{\text{s}}(\hat{Y}_{\text{LR}}) + \hat{B}_2\left[V_{\text{syg}}(\hat{X}_{\text{HT}}) - \hat{V}_{\text{syg}}(\hat{X}_{\text{HT}})\right] \quad (3.8)$$

where

$$\hat{B}_2 = \frac{\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}(s)\delta_{ij}\Phi_{ij}(s)}{\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}(s)\delta_{ij}^2} \quad (3.9)$$

Now choosing:

$$Q_{ij}(s) = Q_{ij}^*(s) = q_{ij}(s)\left[\frac{\sum_{i \neq j \in s}\sum D_{ij}q_{ij}(s)}{\sum_{i \neq j \in s}\sum D_{ij}q_{ij}(s)\delta_{ij}} - \frac{1}{\delta_{ij}}\right] \quad (3.10)$$

with $q_{ij}(s)$ as a suitable weight we get:

$$\hat{B}_2 = \frac{\left(\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}(s)\delta_{ij}\Phi_{ij}(s)\right)\left(\sum_{i \neq j \in s}\sum D_{ij}q_{ij}(s)\right) - \left(\sum_{i \neq j \in s}\sum D_{ij}q_{ij}(s)\delta_{ij}\right)\left(\sum_{i \neq j \in s}\sum D_{ij}q_{ij}(s)\Phi_{ij}(s)\delta_{ij}\right)}{\left(\sum_{i \neq j \in s}\sum D_{ij}q_{ij}(s)\delta_{ij}^2\right)\left(\sum_{i \neq j \in s}\sum D_{ij}q_{ij}(s)\right) - \left(\sum_{i \neq j \in s}\sum D_{ij}q_{ij}(s)\delta_{ij}\right)^2} \quad (3.11)$$

The choice of $Q_{ij}^*(s)$ in (3.10) satisfies constraints:

$$\frac{1}{2}\sum_{i \neq j \in s}\sum \Omega_{ij}^*(s)\delta_{ij} = V_{\text{syg}}\left(\hat{X}_{\text{HT}}\right) \quad (3.12)$$

and

$$\sum_{i \neq}\sum_{j \in s}\Omega_{ij}^*(s) = \sum_{i \neq}\sum_{j \in s} D_{ij} \quad (3.13)$$

Again note that the condition (3.13) is due to Singh (2003, 2004, 2006a, 2006b). Thus it builds a bridge between the estimator of variance due to Singh, Horn and Yu (1998) and Singh (2003, 2004, 2006a).

Remark: Note carefully if $w_i^* = d_i$ and $e_i^* = y_i$, then the estimator $\hat{Y}_{\text{LR}} = \hat{Y}_{\text{HT}}$, and by following Singh, Horn, Chowdhury and Yu (1999), the ratio $\hat{V}_{\text{ss}}(\hat{Y}_{\text{LR}}) \Big/ \{N^2(1-f)/n\}$ becomes a traditional linear regression estimator of finite population variance, $\sigma_y^2 = N^{-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$, under SRSWOR sampling given by:

$$\hat{\sigma}_{\text{dt}}^2 = s_y^2 + \hat{\beta}_2(S_x^2 - s_x^2) \quad (3.14)$$

where $(n-1)s_y^2 = \sum_{i=1}^{n}(y_i - \bar{y})^2$, $\hat{\beta}_2 = (\hat{\mu}_{22} - \hat{\mu}_{20}\hat{\mu}_{02}) / (\hat{\mu}_{04} - \hat{\mu}_{02}^2)$

$(N-1)S_x^2 = \sum_{i=1}^{N}(X_i - \bar{X})^2$, and $(n-1)\hat{\mu}_{rs} = \sum_{i \in s}(y_i - \bar{y})^r (x_i - \bar{x})^s$;

which was obtained by Das and Tripathi (1978). Note that the estimator (3.14) has also independently studied by Srivastava and Jhajj (1980) and Isaki (1983).

Now the question arises of how the calibration can be done if there are two or more auxiliary variables. To answer this question we have the following section:

4. Use of Multi-Auxiliary Information

4.1. GREG with two auxiliary variables

Suppose X_1 and X_2 are the known totals of two auxiliary characters X_{1i} and X_{2i}, for $i = 1, 2, ..., N$. The minimization of the chi-square distance function:

$$D = \sum_{i \in s}(w_i - d_i)^2 (d_i q_i)^{-1} \quad (4.1.1)$$

where q_i are suitably chosen constants such that the estimator depends upon its choice, subject to the two linear calibration constraints given by:

$$\sum_{i \in s} w_i x_{1i} = X_1 \quad (4.1.2)$$

and

$$\sum_{i \in s} w_i x_{2i} = X_2 \quad (4.1.3)$$

In this case the Lagrange function L is given by:

$$L = \sum_{i \in s}(w_i - d_i)^2 (d_i q_i)^{-1} - 2\lambda_1\left[\sum_{i \in s} w_i x_{1i} - X_1\right] - 2\lambda_2\left[\sum_{i \in s} w_i x_{2i} - X_2\right] \quad (4.1.4)$$

On differentiating (4.1.4) with respect to w_i and equating to zero we have:

$$w_i = d_i + \lambda_1 d_i q_i x_{1i} + \lambda_2 d_i q_i x_{2i} \quad (4.1.5)$$

On substituting (4.1.5) in (4.1.2) and (4.1.3), respectively we have:

$$\lambda_1\left(\sum_{i \in s} d_i q_i x_{1i}^2\right) + \lambda_2\left(\sum_{i \in s} d_i q_i x_{1i} x_{2i}\right) = \left(X_1 - \sum_{i \in s} d_i x_{1i}\right) \quad (4.1.6)$$

and

$$\lambda_1\left(\sum_{i\in s} d_i q_i x_{1i} x_{2i}\right) + \lambda_2\left(\sum_{i\in s} d_i q_i x_{2i}^2\right) = \left(X_2 - \sum_{i\in s} d_i x_{2i}\right) \qquad (4.1.7)$$

The system of equations given by (4.1.6) and (4.1.7) can be written as:

$$\begin{bmatrix} \sum_{i\in s} d_i q_i x_{1i}^2, & \sum_{i\in s} d_i q_i x_{1i} x_{2i} \\ \sum_{i\in s} d_i q_i x_{1i} x_{2i}, & \sum_{i\in s} d_i q_i x_{2i}^2 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} X_1 - \sum_{i\in s} d_i x_{1i} \\ X_2 - \sum_{i\in s} d_i x_{2i} \end{bmatrix} \qquad (4.1.8)$$

Solving the above system of equations for λ_1 and λ_2 and on substituting these values in (4.1.5) we obtain the optimal weights given by:

$$w_i = d_i + \frac{d_i q_i x_{1i}\left[\left(X_1 - \sum_{i\in s} d_i x_{1i}\right)\sum_{i\in s} d_i q_i x_{2i}^2 - \left(X_2 - \sum_{i\in s} d_i x_{2i}\right)\sum_{i\in s} d_i q_i x_{1i} x_{2i}\right]}{\left(\sum_{i\in s} d_i q_i x_{1i}^2\right)\left(\sum_{i\in s} d_i q_i x_{2i}^2\right) - \left(\sum_{i\in s} d_i q_i x_{1i} x_{2i}\right)^2}$$
$$+ \frac{d_i q_i x_{2i}\left[\left(X_2 - \sum_{i\in s} d_i x_{2i}\right)\sum_{i\in s} d_i q_i x_{1i}^2 - \left(X_1 - \sum_{i\in s} d_i x_{1i}\right)\sum_{i\in s} d_i q_i x_{1i} x_{2i}\right]}{\left(\sum_{i\in s} d_i q_i x_{1i}^2\right)\left(\sum_{i\in s} d_i q_i x_{2i}^2\right) - \left(\sum_{i\in s} d_i q_i x_{1i} x_{2i}\right)^2} \qquad (4.1.9)$$

On inserting this value of w_i in the GREG estimator:

$$\hat{y}_G = \sum_{i\in s} w_i y_i \qquad (4.1.10)$$

We obtain the GREG with two auxiliary variables as:

$$\hat{y}_G = \sum_{i\in s} d_i y_i + \hat{\beta}_{1(ds)}\left(X_1 - \hat{X}_1\right) + \hat{\beta}_{2(ds)}\left(X_2 - \hat{X}_2\right) \qquad (4.1.11)$$

where $\hat{X}_j = \sum_{i\in s} d_i x_{ji}$, $j = 1,2$,

$$\hat{\beta}_{1(ds)} = \frac{\sum_{i\in s} d_i q_i x_{1i} y_i \sum_{i\in s} d_i q_i x_{2i}^2 - \sum_{i\in s} d_i q_i x_{2i} y_i \sum_{i\in s} d_i q_i x_{1i} x_{2i}}{\left(\sum_{i\in s} d_i q_i x_{1i}^2\right)\left(\sum_{i\in s} d_i q_i x_{2i}^2\right) - \left(\sum_{i\in s} d_i q_i x_{1i} x_{2i}\right)^2} \qquad (4.1.12)$$

and

$$\hat{\beta}_{2(ds)} = \frac{\sum_{i\in s} d_i q_i x_{2i} y_i \sum_{i\in s} d_i q_i x_{1i}^2 - \sum_{i\in s} d_i q_i x_{1i} y_i \sum_{i\in s} d_i q_i x_{1i} x_{2i}}{\left(\sum_{i\in s} d_i q_i x_{1i}^2\right)\left(\sum_{i\in s} d_i q_i x_{2i}^2\right) - \left(\sum_{i\in s} d_i q_i x_{1i} x_{2i}\right)^2} \qquad (4.1.13)$$

4.2 Linear regression with two auxiliary variables

Now let us consider the additional constraint:

$$\sum_{i\in s} w_i = \sum_{i\in s} d_i \qquad (4.2.1)$$

then the Lagrange's function becomes:

$$L = \sum_{i\in s}(w_i - d_i)^2 (d_i q_i)^{-1} - 2\delta_0\left[\sum_{i\in s}(w_i - d_i)\right]$$
$$- 2\delta_1\left[\sum_{i\in s} w_i x_{1i} - X_1\right] - 2\delta_2\left[\sum_{i\in s} w_i x_{2i} - X_2\right] \qquad (4.2.2)$$

where δ_0, δ_1 and δ_2 are called Lagrange's multipliers. On setting $\partial L / \partial w_i = 0$, we get:

$$w_i = d_i + \delta_0 d_i q_i + \delta_1 d_i q_i x_{1i} + \delta_2 d_i q_i x_{2i} \qquad (4.2.3)$$

The values of δ_0, δ_1 and δ_2 are obtained by solving a system of equations given by:

$$A_{3\times3}\delta_{3\times1} = D_{3\times1}$$

where

$$A_{3\times3} = \begin{bmatrix} \sum_{i\in s} d_i q_i, & \sum_{i\in s} d_i q_i x_{1i}, & \sum_{i\in s} d_i q_i x_{2i} \\ \sum_{i\in s} d_i q_i x_{1i}, & \sum_{i\in s} d_i q_i x_{1i}^2, & \sum_{i\in s} d_i q_i x_{1i} x_{2i} \\ \sum_{i\in s} d_i q_i x_{2i}, & \sum_{i\in s} d_i q_i x_{1i} x_{2i}, & \sum_{i\in s} d_i q_i x_{2i}^2 \end{bmatrix},$$

$$\delta_{3\times1} = \begin{bmatrix} \delta_0 \\ \delta_1 \\ \delta_2 \end{bmatrix} \text{ and } D_{3\times1} = \begin{bmatrix} 0 \\ (X_1 - \sum_{i\in s} d_i q_i x_{1i}) \\ (X_2 - \sum_{i\in s} d_i q_i x_{2i}) \end{bmatrix}$$

The linear regression estimator with two auxiliary variables X_1 and X_2 is given by:

$$\hat{y}_{reg(2)} = \sum_{i\in s} d_i y_i + \hat{\beta}_{1(ols)}(X_1 - \hat{X}_1) + \hat{\beta}_{2(ols)}(X_2 - \hat{X}_2) \qquad (4.2.4)$$

where

$$\hat{\beta}_{1(ols)} = \frac{1}{\Delta}\Bigg[\left\{\left(\sum_{i\in s} d_i q_i x_{2i}\right)\left(\sum_{i\in s} d_i q_i x_{1i} x_{2i}\right) - \left(\sum_{i\in s} d_i q_i x_{1i}\right)\left(\sum_{i\in s} d_i q_i x_{2i}^2\right)\right\}\left(\sum_{i\in s} d_i q_i y_i\right)$$
$$+ \left\{\left(\sum_{i\in s} d_i q_i\right)\left(\sum_{i\in s} d_i q_i x_{2i}^2\right) - \left(\sum_{i\in s} d_i q_i x_{2i}\right)^2\right\}\left(\sum_{i\in s} d_i q_i x_{1i} y_i\right)$$
$$+ \left\{\left(\sum_{i\in s} d_i q_i x_{1i}\right)\left(\sum_{i\in s} d_i q_i x_{2i}\right) - \left(\sum_{i\in s} d_i q_i\right)\left(\sum_{i\in s} d_i q_i x_{1i} x_{2i}\right)\right\}\left(\sum_{i\in s} d_i q_i x_{2i} y_i\right)\Bigg] \qquad (4.2.5)$$

and

$$\hat{\beta}_{2(ols)} = \frac{1}{\Delta}\Bigg[\left\{\left(\sum_{i\in s} d_i q_i x_{1i}\right)\left(\sum_{i\in s} d_i q_i x_{1i} x_{2i}\right) - \left(\sum_{i\in s} d_i q_i x_{2i}\right)\left(\sum_{i\in s} d_i q_i x_{1i}^2\right)\right\}\left(\sum_{i\in s} d_i q_i y_i\right)$$
$$+ \left\{\left(\sum_{i\in s} d_i q_i x_{1i}\right)\left(\sum_{i\in s} d_i q_i x_{2i}\right) - \left(\sum_{i\in s} d_i q_i\right)\left(\sum_{i\in s} d_i q_i x_{1i} x_{2i}\right)\right\}\left(\sum_{i\in s} d_i q_i x_{1i} y_i\right)$$
$$+ \left\{\left(\sum_{i\in s} d_i q_i\right)\left(\sum_{i\in s} d_i q_i x_{1i}^2\right) - \left(\sum_{i\in s} d_i q_i x_{1i}\right)^2\right\}\left(\sum_{i\in s} d_i q_i x_{2i} y_i\right)\Bigg] \qquad (4.2.6)$$

with $\Delta = \det\left(A_{3\times3}\right)$.

Now it is **not** easy to find q_i such that $\hat{\beta}_{1(ds)}$ reduces to $\hat{\beta}_{1(ols)}$ and $\hat{\beta}_{2(ds)}$ reduces to $\hat{\beta}_{2(ols)}$.

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