Small Area Estimation for Business Surveys

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Abstract

In business surveys, data typically are skewed and the standard approach for small area estimation (SAE) based on linear mixed models lead to inefficient estimates. In this paper, we discuss SAE techniques for skewed data that are linear following a suitable transformation. In this context, implementation of the empirical best linear unbiased prediction (EBLUP) approach under transformation to a linear mixed model is complicated. However, this is not the case with the model-based direct (MBD) approach (Chambers and Chandra, 2006), which is based on weighted linear estimators. We extend the MBD approach to skewed data using sample weights derived via model calibration based on a log transform model with random area effects. Our results show this estimator is both efficient and robust with respect to the distribution of these random effects. An application to real data demonstrates the satisfactory performance of the method.

Keywords: Small areas, Skewed data, MBD, Model Calibration, Expected value model.

1. Introduction

The standard methods for SAE assume a linear mixed model can be used to characterize the small areas of interest. However, it happens (typically for skewed data) that the variable of interest Y is linear on some transformed scale (e.g. in business surveys, often variables are linear on log scale). In this context, estimation based on linear model for Y leads to inefficient estimates. In such situation, an appropriate technique for SAE should essentially be based on a linear mixed model for a transformed variable. In this paper we explore transform variable based estimation in context of SAE for skewed data, focussing on the widely used log transformation. In this paper we extend the MBD approach of Chambers and Chandra (2006) to SAE for skewed data. In particular, we consider the use of sample weights derived via model calibration (Wu and Sitter, 2001) based on a log transform model with random area effects.

In the following section we summarize the model calibration approach for estimation of population quantities. In section 3 we then discuss the expected value model derived from a transform linear mixed model for SAE of skewed data. Section 4 introduces the survey weights based on expected value model derived from a transform linear mixed model and describes the MBD estimator for SAE in this case. In section 5 we provide illustrative empirical results. Finally, in section 6 some concluding remarks are made.

2. Model Calibration for Population Estimation

In this section we briefly review model calibration for estimation of population level quantities. To start, we fix our notation. Let Y denote an N-vector of population values of a characteristic of interest, and suppose that our primary aim is estimation of the total T_y of the values in Y (or their mean). In order to assist us in this objective, we shall assume that we have 'access' to X, an $N \times p$ matrix of values of p auxiliary variables that are related, in some sense, to the values in Y. In particular, we assume that the individual sample values in X are known. The non-sample values in X may not be individually known, but are assumed known at some aggregate level. At a minimum, we know the population totals T_x of the columns of X. Given this set up, Deville and Särndal (1992) introduced the notation of a calibration estimator of population total of Y as $\hat{T}_y = \sum_{j\in s} w_j y_j$, where the calibration weights w_j's are chosen to minimise their average distance, from the basic design weights, subject to the calibration constraint $\sum_{j\in s} w_j x_j = \sum_{j=1}^{N} x_j = T_x$. There is an implicit underlying assumption that Y and X are linearly related that makes this a valid argument. If the underlying model is non-linear then the calibrated estimator derived under a linearity assumption cannot be very efficient. Let us assume the relationship between Y and X can be described by a super population model

$$E_\xi(Y \mid X) = h(X;\eta) \text{ and } V_\xi(Y \mid X) = \Omega \qquad (1)$$

where η typically vector-valued model parameter, and the mean function $h(X;\eta)$ is a known function of X and η, the variance Ω is a function of X and $h(X;\eta)$. Here E_ξ and V_ξ denotes the expectation and variance with respect to model. The model (1) is quite general and includes linear, non-linear, and generalized linear models as special cases. In this context, Wu and Sitter, (2001) proposed the use of sample weights derived via model calibration. They defined the calibration estimator for population mean of Y as $\hat{\hat{Y}}_c = N^{-1}\sum_{j\in s} w_j y_j$ with weights sought to minimize the distance measure under the constraints: $\sum_{j\in s} w_j = N$ and $\sum_{j\in s} w_j \hat{h}_j = \sum_{j=1}^{N} \hat{h}_j$, where $\hat{\eta}$ is a design consistent estimator for η. Provided the model (1) is a reasonable one, y_j is then (at least approximately) a linear function of its 'fitted values' $h(x_j;\hat{\eta})$ under this model. The basic idea of this approach is then we can carry out linear estimation using these 'fitted or expected values' as auxiliary variables. That is calibration is performed with respect to the population mean of the 'fitted values' $\hat{h}_j = h(x_j;\hat{\eta})$ of $h(x_j;\eta)$.

The above discussion represents what might be referred to the design-based interpretation of model calibration. A model-based perspective on model calibration can be described as follows. We assume that Y and $h(X;\eta)$ are related by the linear model of the form

$$Y = \alpha J + \varepsilon \qquad (2)$$

where J denotes the 'design matrix' for the linear model (3) linking Y and $h(X;\eta)$, $\alpha = (\alpha_0, \alpha_1)'$ is a vector of unknown parameters, ε denotes a N-vector of random variables with $E_\xi(\varepsilon) = 0$ and $V_\xi(\varepsilon) = \Omega = [\omega_{jk}]$. We called model (2) the 'expected value' or 'fitted value' model defined by (1). For $\alpha_0 = 0$ in model (2) we refer as ratio specification of this model, otherwise regression specification. The model (2) can have either ratio or regression specification. Without loss of generality, we arrange the vector Y so that the first n elements correspond to the sample units, and partition Y, J and Ω according to sample and non-sample units. Where J_s denoted the $n \times 1$ vector of 'fitted values' of the auxiliary variables and Ω_{ss} is the $n\times n$ covariance matrix associated with the n sample units that make up the $n\times 1$ sample vector Y_s. A subscript of r is used to denote corresponding quantities defined by the $N-n$ non-sample units, with Ω_{rs} denoting the $(N-n)\times n$ matrix defined by $Cov(Y_r, Y_s)$. In what follows we denote 1_N, 1_n and 1_r as vectors of 1's and I_N, I_n and I_r as identity matrices of order N, n and $N-n$ respectively. In practice the variance components that define covariance matrix Ω are unknown and so need to be estimated from the sample data. We use a "hat" to denote such an estimate. Further, throughout this paper we assume that sampling is uninformative, so the sample data also follow the population model.

Given this notation, the sample weights that define the BLUP for population total of Y under a general linear 'fitted value' model (2) are

$$w^h_{BLUP} = 1_n + H_h'(J'1_N - J_s'1_n) + (I_n - H_h'J_s')\Omega_{ss}^{-1}\Omega_{sr}1_r \qquad (3)$$

where $H_h = (J_s'\Omega_{ss}^{-1}J_s)^{-1}J_s'\Omega_{ss}^{-1}$. See Royall (1976). The sample weights (3) derived via model calibration are calibrated on J. The weights (3) are based on a model appropriate for estimation of population as a whole and using these weights for SAE will be inefficient. The most commonly used class of models for small area estimation model is essentially a mixed model. The next section describes the models suitable for SAE.

3. Small Area Models under Transformation

Let Y_i be the $N_i \times 1$ vector of values of variable of interest in small area i $(i = 1,....,m)$ and let X_i be the $N_i \times p$ matrix of values of the auxiliary variables associated with Y_i. We assume that Y_i and X_i are not related by a linear model on themselves, but they are linearly related on logarithm (natural) transform model. We consider the following linear mixed model specification for the distribution of $l_i = \log(Y_i)$ given Z_i:

$$l_i = Z_i\beta + G_i u_i + e_i \qquad (4)$$

where $Z_i = (1_{N_i}, \log(X_i))$ is the $N_i \times (p+1)$ matrix of values of the auxiliary variables in area i, β is a $(p+1)\times 1$ vector of fixed effects, G_i is a $N_i \times q$ matrix of known covariates characterising differences between small areas, N_i is the number of population units in the small area i, 1_{N_i} is a vector of 1's of order N_i, u_i is a random area effect associated with the i^{th} small area and e_i is a $N_i \times 1$ vector of individual level random errors. The two random variables u_i and e_i are assumed to be independently normally distributed, with zero means and with variances $V(u_i) = \Sigma$ and $V(e_i) = \sigma_e^2 I_{N_i}$ respectively. The variance-covariance matrix of l_i is $V_i = G_i\Sigma(\theta)G_i' + \sigma_e^2 I_{N_i}$, with $v_{ijj} = G_{ij}\Sigma(\theta)G_{ij}' + Var(e_{ij})$ and $v_{ijk} = G_{ij}\Sigma(\theta)G_{ik}', j,k = 1,...,N_i$.

By grouping the area-specific models (4) over the population, we are led to the population level model:

$$l = Z\beta + Gu + e \qquad (5)$$

where $l = (l_1',....,l_m')'$, $Z = (Z_1',....,Z_m')'$, $G = diag(G_i; 1\le i\le m)$, $u = (u_1',...,u_m')'$ and $e = (e_1',....,e_m')'$. The variance matrix of l is $V = diag(V_i; 1\le i\le m)$. We assume that Z has full column rank. In practice the variance components of the model that define the covariance matrix V are unknown and we estimate them from the sample data under the model (5). The estimated variance matrix of l is $\hat{V} = diag(\hat{V}_i; 1\le i\le m)$ with $\hat{V}_i = \hat{\sigma}_e^2 I_{N_i} + G_i\hat{\Sigma}G_i'$. We consider the partition of l, Z, G and V into sample and non-sample components as mentioned before (3). We use similar notation at the small area level by introducing an extra subscript i to denote small area.

With this notation, and assuming (5) holds, the EBLUE of β is $\hat{\beta}=\left(\sum_{i=1}^{m} Z_{is}'\hat{V}_{iss}^{-1}Z_{is}\right)^{-1}\left(\sum_{i=1}^{m} Z_{is}'\hat{V}_{iss}^{-1}l_{is}\right)$ with $E_{\xi}(\hat{\beta})=\beta$ and $V_{\xi}(\hat{\beta})=\left(\sum_{i=1}^{m} Z_{is}'\hat{V}_{iss}^{-1}Z_{is}\right)^{-1}$. We denote $\hat{\phi}_i = Z_i\hat{\beta}$ with $E_{\xi}(\hat{\phi}_i)=Z_i\beta$ and $V_{\xi}(\hat{\phi}_i)=Z_iV_{\xi}(\hat{\beta})Z_i'$, where $a_{ijk}=Z_{ij}V_{\xi}(\hat{\beta})Z_{ik}' \to 0$ as $n\to\infty$. We denote by $a_i=(a_{i11},....,a_{iN_iN_i})'$ and $v_i=(v_{i11},...,v_{iN_iN_i})'$, the vectors of diagonal elements of the covariance matrices $V_{\xi}(\hat{\phi}_i)$ and $V_{\xi}(l_i)$ respectively.

In order to use the Chambers and Chandra (2006) MBD method to get estimates for small areas we require sample weights. For skewed data that follows a linear mixed model on the log scale (5), the sample weights can be derived via model calibration, so first we need to evaluate 'expected value' model (Section 2). In other words, we need to evaluate the first and second moments under the model (4) to derive the sample weights (3). We can use parameter estimates derived under model (4) to obtain the predicted values of the transform variable and then back-transform to get predicted values of Y. These lead to the naïve-lognormal predictor. However, this predictor is biased. Bias corrected first and second order moments that define the expected value model are expressed below.
Let us consider

$$E_{\xi}(Y_{ij})=e^{Z_{ij}\beta+v_{ijj}/2}\neq E_{\xi}(\hat{Y}_{ij}) \qquad (6)$$

Thus, we need to adjust this bias. Using two-step Taylor series approximation, a second order bias corrected estimate of $E_{\xi}(Y_{ij})$ is defined as

$$\hat{Y}_{ij}=h(Z_{ij};\hat{\eta})=\hat{k}_{ij}^{-1}\exp(Z_{ij}\hat{\beta}+\frac{\hat{v}_{ijj}}{2}),\ j=1,....,N_i \qquad (7)$$

so that $E_{\xi}(\hat{Y}_{ij})\approx e^{Z_{ij}\beta+v_{ijj}/2}=E_{\xi}(Y_{ij})$. Here $\hat{k}_{ij}=\left[1+\left(a_{ijj}+\hat{V}ar(\hat{v}_{ijj})/4\right)\big/2\right]$ is the bias correction and $Var(\hat{v}_{ijj})$ is the asymptotic covariance matrix of $\hat{v}_{ijj}$ given by inverse of the relevant information matrix (Rao, 2003). Under normality of the random errors u_i and e_i, covariance between Y_{ij} and Y_{ik} in small area i is

$$\omega_{ijk}=\begin{cases} e^{(Z_{ij}+Z_{ik})\beta}[e^{\frac{1}{2}(v_{ijj}+v_{ikk})}(e^{v_{ijk}}-1)] & if\ j\neq k \\ e^{2Z_{ij}\beta}[e^{v_{ijj}}(e^{v_{ijj}}-1)] & if\ j=k \end{cases} \qquad (8)$$

We group the bias corrected predictor (7) and the covariance (8) at the small area level as

$$\hat{Y}_i=h(Z_i;\hat{\eta})=\hat{k}_i^{-1}\exp(Z_i\hat{\beta}+\frac{\hat{v}_i}{2}), \qquad (9)$$

$$Var_{\xi}(Y_i)=\Omega_i=[\omega_{ijk}]=A_i\Delta_i A_i' \qquad (10)$$

where $A_i=\left\{diag(e^{Z_{ij}\beta});1\leq j\leq N_i\right\}$ and Δ_i is $N_i\times N_i$ positive definite matrix with $(j,k)^{th}$ elements as $\delta_{ijk}=\{e^{(v_{ijj}+v_{ikk})/2}(e^{v_{ijk}}-1)\}$.

The area-specific approximately bias corrected estimator (9) and covariance matrix (10), grouped at population level define the population level version of 'expected value' model

$$E_{\xi}(Y\mid h)=\alpha_0 1_N+\alpha_1 h=\alpha J \text{ and } V_{\xi}(Y\mid h)=\Omega \qquad (11)$$

where $h=(h_1',.......,h_m')'$ and $\Omega=diag(\Omega_i;1\leq i\leq m)$.

4. SAE under the Expected Value Model (11)

With appropriate sample and non-sample partition of Y, J and Ω, as in section 2, the EBLUP version of sample weights (3) under the model (11) are

$$w_{EBLUP}^{h}=1_n+\hat{H}_h'(J'1_N-J_s'1_n)+(I_n-\hat{H}_h'J_s')\hat{\Omega}_{ss}^{-1}\hat{\Omega}_{sr}1_r \qquad (12)$$

where $\hat{H}_h=(J_s'\hat{\Omega}_{ss}^{-1}J_s)^{-1}J_s'\hat{\Omega}_{ss}^{-1}$. We note that the weights (12) depend on random area effects of the mixed model (4) via the covariance structure of model (11) and are thus suitable for small area estimation. We now use the MBD approach of Chambers and Chandra (2006) to define estimator for small areas. They only consider the Hájek form of the MBD estimator for small areas using sample weights derived under a linear mixed model. However, the weights (12) are derived via model calibration under the expected value model (11) where estimator is defined as the HT form (Section 2). Thus, we consider both forms of MBD estimators. The sample weights (12) associated with the sample units in the small area i can be used to define the following MBD estimators for the i^{th} small area mean $\bar{Y}_i$:

- The Hájek form of the weighted sample for area i

$$\hat{\bar{Y}}_i^{Hájek}=\sum_{s_i} w_j y_j\Big/\sum_{s_i} w_j \qquad (13)$$

- The Horvitz-Thompson form of the weighted sample for area i

$$\hat{\bar{Y}}_i^{HT}=\sum_{s_i} w_j y_j\Big/N_i \qquad (14)$$

Both estimators (13) and (14) also depend on how the model calibration weights (12) are specified. In particular, we consider two different specifications for the expected value model (11), the ratio and the regression specification (see below equation (2)). This leads to four different MBD estimators that are set out below.

Estimator	Estimator type	Model specification
TrMBD1	Hájek type	Ratio
TrMBD2	H-T type	Ratio
TrMBD3	Hájek type	Regression
TrMBD4	H-T type	Regression

Estimation of MSE of (13) and (14) follows the approach of Chambers and Chandra (2006), and treats these expressions as simple weighted domain mean estimates under the population level model (3). Under this approach, the sample weights derived from (12) are treated as fixed and the prediction variance of (13) and (14) is estimated using a standard robust variance estimator. See Royall and Cumberland (1978). A "plug-in" estimate of the squared bias of (13) and (14) under this model is added to this estimated prediction variance to finally define a simple estimate of the MSE. Note that under this approach the EBLUP weights underlying (13) and (14) "borrow strength" via the assumed small area model (11), but this model is not used in inference. In particular, we treat the expected value model (11) as a vehicle for generating estimation weights, but base inference on the model (2), thus ensuring consistency with the way mean squared errors are estimated at population level.

5. Simulation Study

In this section we illustrate the performance of seven different small area estimators. These are the proposed MBD estimators (TrMBD1-TrMBD4) for skewed data (Section 4), the Hájek type (MBD1), and HT type (MBD2) MBD estimators based on sample weights derived under a linear mixed model and the EBLUP under a linear mixed model.

We consider two types of simulation studies. The first type of study uses model-based simulation to generate artificial population and sample data. These data are then used to contrast the performance of different estimators. The second type of simulation study was carried out using real data and design-based simulations to test these estimators in the context of a real population and realistic sampling methods. Three measures of estimation performance were computed using the estimates generated in the simulation study. These were the relative mean error and the relative root mean squared error (RMSE), both expressed as percentages, of regional mean estimates and the coverage rate (CR) of nominal 95 per cent confidence intervals for regional means.

5.1 The Model Based Simulation Study

In model-based simulations, we consider a population size N=1500 and generated randomly the small area population sizes N_i (i=1,...,m=30) so that $\sum_i N_i = N$. Further, we consider n=600 and generated $n_i = N_i(n/N)$ with $\sum_i n_i = n$. These were fixed for all simulations. We generated the population values y_{ij} from a multiplicative model $y_{ij} = 5.0 x_{ij}^{\beta} u_i e_{ij}$. The generated population is skewed on the raw scale and linear on the log transform scale. The random errors e_{ij} were independently generated from a lognormal (LN) distribution with LN $(0, \sigma_e)$. The random area effects u_i and auxiliary variables x_{ij} were generated from LN $(0, \sigma_u)$ and LN $(6, \sigma_x)$ respectively. The values of parameter σ_e and σ_u were chosen so that intra-area correlation varies between 0.20-0.25. We used six different sets of parameter to bring different level of variation in generated data as shown below:

Parameter	β	σ_u	σ_e	σ_x
Par1	0.5	0.30	0.50	3.00
Par2	0.8	0.35	0.60	2.50
Par3	1.0	0.40	0.70	2.25
Par4	1.3	0.45	0.80	1.75
Par5	1.5	0.50	0.90	1.50
Par6	2.0	0.60	1.00	1.20

Using this generated data we estimated the parameters using the *lme* function in R, and then calculated the estimates for small areas (Section 4). We replicated 1000 simulation runs. The results from this simulation study are reported in Table 1.

5.2 The Design Based Simulation Study

In design-based simulations, our basic data come from the same sample of 1652 Australian broadacre farms (AAGIS) that were used in the simulation study reported in Chandra and Chambers (2005). In particular, we use the same target population of 81982 farms (obtained by sampling with replacement from the original sample of 1652 farms with probabilities proportional to their sample weights). The same 1000 independent stratified random samples as used in Chandra and Chambers (2005) were then drawn from this (fixed) population, with total sample size in each draw equal to the original sample size (1652) and with the small areas of interest defined by the 29 Australian agricultural regions represented in this population. Sample sizes within these regions were fixed to be the same as in the original sample. Note that these varied from a low of 6 to a high of 117, allowing an evaluation of the performance of the different methods considered across a range of realistic small area sample sizes. Here, our aim is to estimate average annual farm costs (A$) in these regions with farm size (hectares) as auxiliary variable. We used random intercept model specification of the mixed model. Details of this simulated population are described in Chandra and Chambers (2005). Table 2 set out the results from this simulation study.

5.3 Results of the Simulation Studies

Results from Table 1 show that the average relative mean errors (RMEs) and the average relative RMSEs for Hajek type of estimators (TrMBD1 and

TrMBD3) under expected value model (11) are significantly large. Further, high coverage rates under these methods (TrMBD1 and TrMBD3) are the consequence of large biases. The HT type estimators (TrMBD2 and TrMBD4) derived under ratio and regression specifications for the expected value model are almost identical. Among conventional calibration weighting based MBD estimators, both Hajek type (MBD1) and HT type (MBD2) estimators are identical. Therefore, in further discussion we drop the Hajek type of estimator under model calibration and HT type estimator under classical calibration. Further, Table 1 shows that the average RMEs and the average relative RMSEs for TrMBD2 are consistently lower than both MBD1 and EBLUP. However, with same order of RMEs, the relative RMSEs of EBLUP is lower order than MBD1. The average coverage rates for TrMBD2 are relatively higher with smaller width as compare to MBD1 and EBLUP. With almost same coverage rates, EBLUP has smaller average widths than MBD1. We noticed that both the RME and the relative RMSEs of TrMBD2 are smaller than MBD1 and EBLUP in all regions. Further, the RMEs and the relative RMSE of MBD1 and EBLUP increase proportionate to non-linearity (Par1 to Par6) in the data. The coverage rate increases and the width decreases, hence accuracy increases in transformation-based methods. Further, the relative interval width under TrMBD2 reduced more rapidly as non-linearity in data increases. The results indicate a significant gain due to transformation based method of small area estimation for skewed data. This gain is proportionate to non-linearity in the data. Between MBD1 and EBLUP methods, the EBLUP appears to perform better.

The results from the design-based simulation using the real data (AAGIS) show that the average RME of TrMBD2 is smaller than EBLUP and but larger than MBD1. The relative RMSE of TrMBD2 is marginally larger and the average coverage rate higher (Table 2). However, Figure 1 indicates that the high RME and relative RMSE of TrMBD2 is due to an outlier in region 21. The TrMBD2 is more affected by this outlying point. If we discard the outlier contaminated estimates and examine the average based on 28 regions then TrMBD2 seems to be performing better. Overall transform variable based SAE methods for AAGIS data appears to provide efficient set of estimates. The TrMBD2 method provides significant gain under linearity on transform model. The gain may not be significant if linearity does not hold. However, it is safer to use TrMBD2 method even though transform model is approximately linear. For AAGIS data, fitted model on log scale is not exactly linear. Consequently, TrMBD2 method of SAE performs marginally better overall.

6. Conclusions and Further Research

Our results show that transformed variable based method for SAE of skewed data performs well. We note that the gain in efficiency by accounting non-linearity in data via log-transform linear model is quite significant. Further, even though assumed model deviates slightly from linearity on transform scale, the proposed method still works well with marginal gain. These results are based on normality assumption of random errors. However, we also investigated under gamma distribution for the random errors and noticed that method is robust with respect to distribution of random errors. The application of proposed SAE techniques to real data from AAGIS provides a satisfactory performance. In application of this method, identification of an appropriate transform model is crucial, otherwise results can be misleading.

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Table 1 Average relative mean error (ARME), average relative RMSE (ARRMSE), average coverage rate (ACR) and average 2-sigma confidence interval width (AW) for model based simulations. All averages are expressed as percentages.

Criterion	Estimator	Par1	Par2	Par3	Par4	Par5	Par6
ARME	TrMBD1	-86.02	-96.54	-98.43	-98.58	-98.45	-99.06
	TrMBD2	-0.01	-0.05	0.27	0.09	-0.43	0.76
	TrMBD3	-75.2	-95.97	-97.97	-98.55	-98.12	-98.66
	TrMBD4	0.02	-0.07	0.28	0.11	-0.39	0.75
	MBD1	10.98	4.11	-0.29	-6.28	-7.81	-9.59
	MBD2	12.63	5.47	0.48	-5.91	-7.58	-9.5
	EBLUP	12.65	5.44	0.49	-5.85	-7.68	-9.32
ARRMSE	TrMBD1	0.92	1.13	1.2	1.29	1.43	1.56
	TrMBD2	0.15	0.29	0.39	0.52	0.70	0.88
	TrMBD3	7.98	1.25	1.22	1.3	1.44	1.59
	TrMBD4	0.15	0.29	0.39	0.52	0.7	0.88
	MBD1	1.03	1.47	1.79	1.89	1.98	2.78
	MBD2	1.16	1.6	1.83	1.91	1.99	2.79
	EBLUP	0.76	0.69	0.61	0.75	0.98	1.29
ACR	TrMBD1	99	98	96	95	94	92
	TrMBD2	94	90	89	89	89	89
	TrMBD3	99	98	96	95	94	92
	TrMBD4	94	91	89	89	89	89
	MBD1	87	85	85	87	88	87
	MBD2	87	85	85	87	88	87
	EBLUP	85	85	85	87	87	87
AW	TrMBD1	1265	22389	140563	$27x10^4$	$35\ x10^5$	$44x10^6$
	TrMBD2	208	4326	33228	$7x10^4$	$11x10^5$	$15x10^6$
	TrMBD3	1753	22487	141001	$27x10^4$	$35\ x10^5$	$43x10^6$
	TrMBD4	220	4426	33722	$8x10^4$	$11x10^5$	$16x10^6$
	MBD1	1007	19318	13934	$28x10^4$	$38x10^5$	$56x10^6$
	MBD2	1033	19677	14062	$28x10^4$	$38x10^5$	$56\ x10^6$
	EBLUP	380	7253	55498	$13x10^4$	$20x10^5$	$31\ x10^6$

Table 2 Average relative mean error (ARME), average relative RMSE (ARRMSE) and average coverage rate (ACR) for AAGIS data. All averages are expressed as percentages.

Criterion	Estimator	Average of 29 areas	Average of 28 areas
ARME	TrMBD2	3.00	2.54
	MBD1	-2.49	-2.58
	EBLUP	4.24	4.74
ARRMSE	TrMBD2	22.00	17.15
	MBD1	20.55	17.33
	EBLUP	19.92	19.40
ACR	TrMBD2	99	99
	MBD1	92	93
	EBLUP	90	90

Figure 1 Regional performance of TrMBD2 (solid line), MBD1 (dashed line) and EBLUP (thin line) for AAGIS data.