Disclosure Risks in Releasing Output Based on Regression Residuals

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1. Introduction[1]

The U.S. Census Bureau's Center for Economic Studies (CES) and its network of Census Research Data Centers (RDCs) provide restricted access to non-publicly available Census Bureau data files for projects that benefit Census Bureau programs.[2] At RDCs, researchers estimate characteristics of the underlying populations, typically using various types of regression models. Regression model output, like any other, must be checked to make sure that it does not reveal confidential information about individual survey respondents.

Previous research by the authors (Reznek, 2003, Reznek and Riggs, 2004) has demonstrated that disclosure risks can exist in regression models, including Generalized Linear Models (GLMs). Coefficients in models that contain only fully-interacted (saturated) dummy variables on the right-hand side can be risky, since they allow recovery of cell means of the dependent variable in a table defined by the categories of the (fully interacted) dummy variables. Risks can also arise from correlation and covariance matrices of the variables, and variance-covariance matrices of model coefficients. If these matrices include dummy variables, then they can also allow recovery of cell means in tables defined by the dummy variable categories. Whether these tables present disclosure risks can be evaluted with the statistical agency's standard techniques for tabular data.

All of these results have assumed that the model residuals are independent and identically distributed (iid). This paper focuses on output from models in which the residuals are not necessarily iid. Section 2 reviews the layout of a model with possibly correlated residuals, and describes when it reduces to the standard linear regression model. We consider Generalized Least Squares (GLS) estimators, which are generalizations of the basic Ordinary Least Squares (OLS) estimators. In the next three sections, we consider examples of GLS models. Again, we focus on situations in which the model coefficients and variance-covariance matrices of the coefficients might allow recovery of tables of means of the left-hand-side variable, broken down by the categories in the right-hand-side dummy variables. Section 3 discusses models with heteroskedastic residuals, which are independent but do not have the same variance across observations. Section 4 considers models with residuals that are correlated across observations (autocorrelated). Section 5 analyzes a more complex 'error correction' model, in which the residuals are both heteroskedastic and autocorrelated. Section 6 gives conclusions.

2. Model Setup and Notation

The standard linear regression setup is summarized as follows:

$$
\begin{aligned}
y &= X\beta + \varepsilon && \text{Model}\\
E(\varepsilon\,|X) &= 0 && \text{iid error terms}\\
E(\varepsilon\varepsilon') &= \sigma^2 I &&\\
y &= Xb + e && \text{Estimated model} \quad (1)\\
b &= (X'X)^{-1}X'y && \text{Parameter vector}\\
s_b^2 &= s^2(X'X)^{-1} && \text{Var-covar matrix of } b\\
s^2 &= \frac{1}{n-k}\,\Sigma_{i=1}^{n} e^2 && \text{Residual variance}
\end{aligned}
$$

Here:

y is the dependent variable;

X is an $n \times k$ matrix of n observations each on k independent variables

β is a column vector of k parameters

ε is a column vector of n independent and identically distributed (iid) error terms:

b is the OLS estimate of β

e is the OLS residual, an estimate of ε.

[1] This report is to inform interested parties of research and to encourage discussion. The views expressed on statistical, methodological, or operational issues are those of the authors and not necessarily those of the Census Bureau. We thank Phil Steel and participants in a Center for Economic Studies seminar for helpful comments.

[2] For more detail on CES and the RDC program, see `http://www.ces.census.gov`.

For a wide variety of models[3], the assumption that the errors are iid is invalid, and is replaced by the assumption

$$E(\varepsilon\varepsilon') = \sigma^2\Omega \tag{2}$$

where Ω is symmetric and positive definite. By Aitken's theorem, there is a matrix P that can be used to transform the model into the form

$$Py = PX + \epsilon$$

or

$$y^* = X^*\beta^* + \epsilon^* \tag{3}$$

where

$$\begin{aligned} y^* &= Py \\ X^* &= PX \\ \epsilon^* &= P\epsilon^* \\ P'P &= \Omega^{-1} \end{aligned}$$

The OLS estimator of the transformed model (3) is the Generalized Least Squares (GLS) estimator:

$$\begin{aligned} b_{GLS} &= (X^{*\prime}X^*)^{-1}X^{*\prime}y \\ &= (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y \end{aligned} \tag{4}$$

To estimate (4), the error covariance matrix Ω must be known. If (as is typical), Ω is not known, we cannot estimate it in full generality because it contains n(n-1)/2 distinct parameters, which is more than the number of observations. Therefore, we must impose some structure to reduce the number of independent parameters, and then estimate this structure. This gives rise to the wide variety of Feasible Generalized Least Squares (FGLS) estimators that are applied in diverse situations: heteroskedasticity, autocorrelation, panel data, and more.

The problem here is whether the GLS or FGLS estimates allow us to estimate means tables of subsets of the data. This parallels what we did in earlier papers.

3. Simplest Example 1: Heteroskedastic Residuals

In models with heteroskedastic residuals, the matrix Ω is diagonal, but the variances are not constant:

$$\sigma^2\Omega = \begin{bmatrix} \sigma_1^2 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma_n^2 \end{bmatrix} \tag{5}$$

[3]Here, we follow closely discussions in (Greene, 2000), chapter 11.

In this situation, GLS or FGLS produce a Weighted Least Squares (WLS) estimate. In either case, a model that includes only fully interacted FGLS dummy variable coefficients produces regression coefficients that are weighted means. In this situation, it is possible to recover cells of tables of weighted means.

We illustrate with a model from Chapter 12 of Greene (2000). Here, we assume that Ω is known. (This is sometimes reasonable in models with heteroskedasticity.) The model includes data for individual persons on credit card expenditures and certain variables affecting them:[4]

exp is credit card expenditures.

age is the person's age.

income is the person's income.

incomesq is income squared.

hiinc is a dummy variable with 1 for high-income persons, 0 othewise.

old is a dummy variable with 1 for older persons, 0 otherwise.

hiold = *hiint* · *old*, the interaction between *hiint* and *old*.

ownrent is a dummy variable with 1 if the person owns a home, 0 otherwise.

From an initial OLS model, it is reasonable to assume that the variance of the residuals increases with income. To investigate possible disclosure risks, we estimate unweighted and income-weighted models including combinations of the right-hand side variables shown in the following model:

$$\begin{aligned} exp = b_0 + b_1 \cdot hiincome + b_2 \cdot old + b_3 \cdot hiold \\ + b_4 \cdot ownrent + b_5 \cdot incomesq + e \end{aligned} \tag{6}$$

We also compare this model with the model estimated by Greene:

$$\begin{aligned} exp = b_0 + b_6 \cdot age + b_7 \cdot income \\ + b_5 \cdot incomesq + b_4 \cdot ownrent + e \end{aligned} \tag{7}$$

We consider unweighted (OLS) as well as weighted models in part because researchers often begin the estimation of complex models by estimating OLS models, and they may often wish to report these estimates. Also, the more complex estimation procedures themselves often begin by calculating and

[4]The underlying data set is available at `http://pages.stern.nyu.edu/~wgreene/Text/tables/Tbl5_1.htm`.

reporting OLS estimates. Therefore, we need to consider the risks of OLS models even in the estimation of more complex models.

Table 1[5] gives unweighted and income-weighted means of income (low and high) by age (young and old). We compare these tables with parameter estimates from Tables 2 and 3. In tables 2 and 3, the first four models contain only dummy variables. Models (1), (2), and (4) are "fully" interacted. Their parameter estimates produce unweighted means that can be used to recover entries in Table 1. (Whether these are disclosure risks depends on whether the underlying weighted or unweighted means pose disclosure risks.) For example, the parameter estimates of model (1) in Table 2 (shown in bold) can be used to recover the entries in the unweighted "All" column of Table 1 (also shown in bold): we have $b_0 = 113.5869$, which is the mean of credit card expenditures for low income persons, and $b_0 + b_1 = 403.4262$, the mean expenditure for high-income persons.[6] Similarly, the parameters in models (2) and (4) in Table 2 can be used to recover other cells in the unweighted portion of Table 1. But the other models in table 2 cannot be used in this way. Model (3) is not fully interacted; Models (5) and (6) include another dummy and continuous variables.[7]

Models (1), (2), and (4) in Table 3 can be used in exactly the same way to recover the corresponding cells in the weighted portion of Table 1.

We end this section with two notes. First, models are estimated with weighted data for reasons other than heteroskedasticity. For example, regression models using complex survey data are often estimated using survey weights. Second, OLS parameter estimates in models with heteroskedastic data are consistent but not efficient, and the parameter variance estimates can be biased. Several methods have been developed to adjust the standard errors for heteroskedasticity. For these methods, the estimated parameters are the OLS parameters. [8]

4. Simplest Example 2: Autocorrelated Residuals

The simplest model with autocorrelated residuals called the first-order autoregressive, or AR(1), model with parameter ρ:

$$y = \beta X + \epsilon \tag{8}$$

where

$$\epsilon = \mu_t - \mu_{t-1} \tag{9}$$

and the μ_t are iid, $i = 0 \dots n$. The residual variance-covariance matrix is

$$\sigma^2\Omega = \frac{\sigma_\mu^2}{(1-\rho)} \begin{bmatrix} 1 & \rho & \cdots & \rho^3 \\ \rho^3 & 1 & \cdots & \rho^2 \\ \vdots & \vdots & \ddots & \rho \\ \rho & \rho^2 & \rho^3 & 1 \end{bmatrix} \tag{10}$$

If ρ is known, then we can carry out GLS estimation by performing OLS on the following transformed data:

$$y_* = \begin{bmatrix} \sqrt{1-\rho^2}y_1 \\ y_2 - \rho y_1 \\ \vdots \\ y_n - \rho y_{n-1} \end{bmatrix}, \mathbf{x}_* = \begin{bmatrix} \sqrt{1-\rho^2}\mathbf{x}_1 \\ \mathbf{x}_2 - \rho \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_n - \rho \mathbf{x}_{n-1} \end{bmatrix}, \tag{11}$$

where $\mathbf{x}_i$ is a row vector containing the i^{th} observation on each of the right-hand side variables, including the transformed constant term.

Usually, ρ is not known and must be estimated; several methods are available that estimate all the model parameters jointly ((Greene, 2000) sec. 13.7).

We now estimate a very simple model, adapted from example 13.1, p. 525 of Greene, in which real gross private investment depends on real gross national product (GNP) and the real rate of interest.[9] The investment data are autocorrelated, and Greene fits models in which the residuals have an AR(1) structure.[10] The original data are continuous, so for our purposes we constructed two dummy variables, representing years of high and low GNP and high and low real interest rates. Table 4 shows the means of real investment by these categories of GNP and interest rates. To investigate possible disclosure risks, we estimate OLS models that include four combinations of the right-hand side variables in the following specification:

$$rinvest = b_0 + b_1 \cdot hiint + b_2 \cdot hignp + b_3 \cdot hiintgnp + e \tag{12}$$

where

[5] All tables are at the end of the paper.

[6] The values do not agree exactly because of inconsistent rounding.

[7] Model (6) is the first model in Table 12.3 of Greene (2000) p. 506.

[8] Table 12.2 of Greene (2000) gives different estimates of standard errors for three estimators. Stata output all these models is available at `http://www.ats.ucla.edu/stat/stata/examples/greene/greene12.htm`.

[9] Such a simple model, with aggregated data, is not often used at CES. However, researchers at CES do occasionally estimate models based on aggregated data that the Census Bureau does not publish.

[10] The underlying variables, with annual observations covering years 1963 through 1982, are defined in (Greene, 2000) A13.1, p. 954. The data set is available at `http://pages.stern.nyu.edu/~wgreene/Text/tables/Tbl13_1.htm`.

rinvest is real investment

hiint is a dummy variable with 1 for real high interest rate, 0 otherwise.

hignp is a dummy variable with 1 for high real GNP, 0 otherwise.

$hiintgnp = hiint \cdot hignp$, the interaction between *hiint* and *hignp*.

Table 5 shows the OLS parameter estimates from the four models. Exactly as in the previous section, models (1), (2), and (4) are fully saturated, and can be used to recover entries from Table 4; the relevant entries for model (1) are shown in bold in both tables.

Table 6 shows the same four models estimated using FGLS.[11] Again, we concentrate on model (1). The parameter estimates in this model do not allow recovery of the means shown in bold in Table 4. (The same is true for the other three models in the table.) Table 8 gives both the transformed and untransformed data for this example and reveals the reason. The first column is a time index $t, t = 1 \ldots 20$, representing years 1963-1982. The transformed variables are calculated using data outside the original dummy variable categories. The transformed dummy variables (and the transformed constant) do not have the same categories as the original varsiables. Without access to the microdata, it would be impossible to recover the means of the original data.

We conclude that use of FGLS for models with autocorrelated residuals appears to pose few disclosure risks; at least, the risk of recovering tables of means based on categories of dummy variables is small. However, we should always remember that a researcher may report an initial OLS model. Also, it is possible (though unlikely) that FGLS might reduce to OLS; this happens when the estimated parameter ρ is zero.

5. Error Corrections in Models

This section focuses on the models used for estimation of time-series and panel data that use error correction terms to adjust for correlation in the error terms. Given the potential disclosure risks of releasing the covariance matrix of the residuals, further investigation on the impact of these error correction adjustments seems warranted. Typically, these models first estimate the structure of the error term using OLS. These OLS estimates are then put back into the more complicated model to obtain the adjustment. Then, does releasing the OLS estimates, the means of the variables, and the adjusted estimates pose disclosure risk?

The most basic of these models is corrected OLS where the absolute value of the minimum error (e_i) is added to the constant term of the original OLS model, $\hat{\beta}_0$. If both the original constant term and the error-corrected constant term are released, the minimum error would be revealed. Since this error term is from one observation, this would be considered an inappropriate disclosure.

This model is very simplistic compared to typical models estimated at RDCs. A more realistic model for panel uses data with N cross-sectional units and T time series observations:

$$\begin{aligned} y_{it} &= \Sigma_{k=1}^{K} X_{itk}\beta_k + u_{it} \\ & \quad i = 1, \ldots, N;\ t = 1, \ldots, T \end{aligned} \tag{13}$$

where K is the number of independent variables.

Specification of the error term defines the type of model to be estimated. For example, $u_{it} = v_i + \epsilon_{it}$ specifies a one-way (dependent only on the cross section) fixed or random effects model; whereas, $u_{it} = v_i + e_t + \epsilon_{it}$ specifies a two-way (dependent on the cross section and the time series) fixed or random effects model. Further, $u_{it} = \rho_i u_{i,t-1} + \epsilon_{it}$ specifies a first-order autoregressive model with contemporaneous correlation. The mixed variance-component moving average error process is specified with the following:

$$\begin{aligned} u_{it} &= v_i + b_t + e_{it} \\ e_{it} &= \alpha_0\epsilon_t + \alpha_1\epsilon_{t-1} + \cdots + \alpha_m\epsilon_{t-m} \end{aligned} \tag{14}$$

Both the first-order autoregressive model and the moving average model specified above are estimated using a two-stage GLS-type procedure.

For the autoregressive model, assume the following:

$$\begin{aligned} E(u_{it}^2) &= \sigma_{ii} \\ E(u_{it}u_{jt}) &= \sigma_{ij} \\ u_{it} &= \rho_i u_{i,t-1} + \epsilon_{it} \end{aligned} \tag{15}$$

where

$$\begin{aligned} E(\epsilon_{it}) &= 0 \\ E(u_{i,t-1}\epsilon_{it}) &= 0 \\ E(\epsilon_{it}\epsilon_{jt}) &= \phi_{ij} \qquad (s \neq t) \\ E(\epsilon_{it}\epsilon_{js}) &= 0 \\ E(u_{i0}) &= 0 \\ E(u_{i0}u_{j0}) &= \sigma_{ij} = \phi_{ij}/(1 - \rho_i\rho_j) \end{aligned} \tag{16}$$

[11] Several methods are available; we use the Prais-Winsten method implemented in Stata.

Then, the covariance matrix for the residuals can be written as follows:

$$
\begin{aligned}
E(uu') &= V \qquad (17)\\
&= \begin{bmatrix} \sigma_{11}P_{11} & \sigma_{12}P_{12} & \cdots & \sigma_{1N}P_{1N} \\ \sigma_{21}P_{21} & \sigma_{22}P_{22} & \cdots & \sigma_{2N}P_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{N1}P_{N1} & \sigma_{N2}P_{N2} & \cdots & \sigma_{NN}P_{NN} \end{bmatrix}
\end{aligned}
$$

where

$$
P_{ij} = \begin{bmatrix} 1 & \rho_j & \rho_j^2 & \cdots & \rho_j^{T-1} \\ \rho_i & 1 & \rho_j & \cdots & \rho_j^{T-2} \\ \rho_i^2 & \rho_i & 1 & \cdots & \rho_j^{T-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho_i^{T-1} & \rho_i^{T-2} & \rho_i^{T-3} & \cdots & 1 \end{bmatrix} \qquad (18)
$$

To estimate the covariance matrix, V, OLS is used to estimate β and to obtain the fitted residuals, $\hat{u} = y - X\hat{\beta}_{OLS}$. This also provides a consistent estimator of ρ_i:

$$\hat{\rho}_i = \frac{(\Sigma_{t=2}^{T}\hat{u}_{it}\hat{u}_{i,t-1})}{(\Sigma_{t=2}^{T}\hat{u}_{i,t-1}^2)} \qquad (19)$$

where $i = 1, 2, \ldots, N$.

The model is then transformed as follows:

$$
\begin{aligned}
y_{it} - \hat{\rho}_i y_{i,t-1} = &\Sigma_{k=1}^{p}(X_{i1k} - \hat{\rho}_i X_{i,t-1,k})\beta_k \qquad (20)\\
&+ u_{it} - \hat{\rho}_i u_{i,t-1}
\end{aligned}
$$

where $t = 2, 3, \ldots, T$. Then,

$$y_{it}^* = \Sigma_{k=1}^{p} X_{itk}^* \beta_k + u_{it}^*$$

where $t = 1, 2, \ldots, T$.

The second step is estimating the transformed model using FGLS:

$$
\begin{aligned}
\hat{u}^* &= y^* - X^*\beta_{OLS}^* \qquad (21)\\
\hat{\beta}_P &= (X'\hat{V}^{-1}X)^{-1}X'\hat{V}^{-1}y
\end{aligned}
$$

Further, the covariance matrix of the coefficients is as follows:

$$Var(\hat{\beta}_P) = (X'\hat{V}^{-1}X)^{-1}$$

Note that $\hat{\beta}_P$ can be derived directly from the transformed model:

$$\hat{\beta}_P = (X'(\hat{\Phi}^{-1} \otimes I_T)X)^{-1}X'(\hat{\Phi}^{-1} \otimes I_T)y \qquad (22)$$

where $\hat{\Phi} = [\hat{\phi}_{ij}]\ i, j = 1, \ldots, N$.

This is simply the GLS model outlined in Reznek and Riggs (2004). Hence, when the transformation is known, this model has the same disclosure risks as OLS with respect to the release of the variance-covariance matrix of the coefficients.

6. Conclusions

In this paper, we have discussed disclosure risks in three types of models in which the residuals are not independent and identically distributed, as in the standard OLS model. The first two types, in which the residuals are heteroskedastistic or autocorrelated, are perhaps the simplest possible ones. Models incorporating heteroskedasticity, described in section 3, can lead to weighted regressions and can pose risks similar to those in OLS models. Models incorporating autocorelation, described in section 4, appear to present few risks. More realistic models, illustrated by a simple error correction model described in section 5, incorproate both autocorrelation and heteroskedasticity in various ways. Their complexity seems to prevent most disclosure risks. However, we should always remember that a researcher may report an initial OLS model, which of course might pose risks. Moreover, it is possible (though unlikely) that a relatively complex model might reduce to OLS. Overall, we conclude that the disclosure risks of these models are usually small - at least, the risk of recovering tables of means based on categories of dummy variables is small.

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Arnold P. Reznek and T. Lynn Riggs. Disclosure Risks in Regression Models: Some Further Results. In *Proceedings of the American Statistical Association, Government Statistics Section [CD-ROM]*, Alexandria, VA, 2004. American Statistical Association.

Table 1: Means of Credit Card Expenditures: Unweighted and Weighted

Income	Unweighted			Weighted		
	Age			Age		
	Young	Old	All	Young	Old	All
Low	116.42151	66.81500	**113.58686**	114.49718	66.88952	**112.11997**
High	399.45482	414.14900	**403.42622**	371.56891	395.65982	**377.07915**
All	243.78650	356.26000	262.53208	195.48488	304.21312	207.94463

Table 2: Coefficents of Models of Credit Card Expenditures: Unweighted

	Dependent Variable: exp (Credit card expenditures)					
	Model					
Indep. Var	(1)	(2)	(3)	(4)	(5)	(6)
hiincome (b_1)	**289.83940**		289.52180	283.03330	227.84180	
old (b_2)		112.47350	1.49020	-49.60650	-68.71840	
hiold (b_3)				64.30070	23.01660	
ownrent (b_4)					54.68620	27.94090
incomesq (b_5)					3.01390	-14.99680
age (b_6)						-3.08180
income (b_7)						234.34700
Constant (b_0)	**113.58690**	243.78650	113.50170	116.42150	89.00770	-237.14650

Table 3: Coefficients of Models of Credit Card Expenditures – Weighted

	Dependent Variable: exp (Credit card expenditures)					
	Model					
Indep. Var.	(1)	(2)	(3)	(4)	(5)	(6)
hiincome (b_1)	**264.95920**		264.77800	257.07170	203.56200	
old (b_2)		108.72820	1.01310	-47.60770	-71.00780	
hiold (b_3)				71.69860	25.89480	
ownrent (b_4)					65.45680	50.49360
incomesq (b_5)					3.95530	-12.11360
age (b_6)						-2.93500
income (b_7)						202.16940
Constant (b_0)	**112.12000**	195.48490	112.06940	114.49720	81.66630	-181.87060

Table 4: Means of Investment by GNP and Interest Rate

		GNP		
		Low	High	Total
Interest	Low	142.1596	202.2608	**180.4058**
Rate	High	163.7402	228.6995	**199.8287**
	Total	152.9499	213.2769	189.1461

Table 5: Investment Models – OLS Coefficient Estimates

	Dependent variable: Real investment (rinvest)			
	Model			
Indep. Var.	(1)	(2)	(3)	(4)
hiint (b_1)	**19.4229**		24.4625	21.5806
hignp (b_2)		60.3270	62.3655	60.1011
hiinteract (b_3)				4.8581
Constant (b_0)	**180.4058**	152.9499	140.7187	142.1596
N=20				

Table 6: Investment Models – FGLS (Autocorrelation) Coeff. Estimates

	Dependent variable: Real investment (rinvest)			
	Model			
Indep. Var.	(1)	(2)	(3)	(4)
hiint (b_1)	14.4217		21.6560	14.3980
hignp (b_2)		46.8686	54.2330	47.6627
hiinteract (b_3)				12.6784
Constant (b_0)	176.8380	158.7417	145.6316	149.4874
rho	0.8146	0.5513	0.5732	0.5922
N=20				
Estimated with Stata "Prais" procedure				

Table 7: Means of Transformed Real Investment

Interest Rate	Transformed Investment
Low	43.1811
High	37.3731

Table 8: Data for Investment Model - Original and Transformed

	Original Data			Transformed Data		
(1) time (t)	(2) $rinvest$	(3) $constant$	(4) $hiint$	(5) $trinvest$	(6) $tconstant$	(7) $thiint$
1	126.8313	1	0	73.5675	0.5800	0.0000
2	133.8464	1	0	30.5313	0.1854	0.0000
3	152.6358	1	0	43.6064	0.1854	0.0000
4	163.7572	1	1	39.4221	0.1854	1.0000
5	155.3251	1	0	21.9307	0.1854	-0.8146
6	161.4975	1	1	34.9718	0.1854	1.0000
7	172.0244	1	1	40.4708	0.1854	0.1854
8	157.6818	1	1	17.5530	0.1854	0.1854
9	173.3153	1	0	44.8698	0.1854	-0.8146
10	195.0000	1	0	53.8197	0.1854	0.0000
11	217.3050	1	1	58.4606	0.1854	1.0000
12	198.7313	1	1	21.7176	0.1854	0.1854
13	163.8445	1	0	1.9607	0.1854	-0.8146
14	194.8768	1	0	61.4113	0.1854	0.0000
15	231.4173	1	0	72.6733	0.1854	0.0000
16	257.0137	1	0	68.5043	0.1854	0.0000
17	258.8423	1	1	49.4824	0.1854	1.0000
18	225.2550	1	1	14.4056	0.1854	0.1854
19	243.3637	1	1	59.8740	0.1854	0.1854
20	200.3577	1	0	2.1169	0.1854	-0.8146
Source: Greene (2000) chap. 13.						