On Non-Response Adjustment via Calibration

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Introduction

We consider estimation of finite population totals in the presence of non-response assuming that non-responses arise randomly within response classes. We compare in Section 1 two regression estimators: one of them is based on the adjusted for non-response probability weights and another is based on unadjusted weights. We show that when the auxiliary variables used for non-response adjustment are included in the estimators then they differ only very slightly. In this case the non-response adjustment step can be omitted from the estimation process without loss of generality (from Result 5 of Deville and Särndal 1992 it follows that the same remains correct for a wide class of calibration estimators). At the end of Section 1 we suggest a general idea of testing if regression estimators based on adjusted and unadjusted weights are significantly different. In Section 2 we consider a multivariate analog of a "regression through the origin" estimator, and show that the "adjusted" and "unadjusted" estimators coincide in this case. Then in Section 3 we consider the important practical case in which auxiliary variables are stratum indicators. We show that in this case all previous regression estimators coincide. In Section 4 we consider calibration estimators under restrictions on weights. We show that if there exists even one set of weights satisfying the calibration equations and restrictions then the regression through the origin estimator does not depend on the restrictions.

1. Linear Regression Estimator

Let $S^* = \cup_{k=1}^{K} S_k^*$, $S_k^* \cap S_{k'}^* = \varnothing$ when $k \neq k'$, and let s^* be a subset of S^* and put $s_k^* = s^* \cap S_k^*$. Let $(y_i, \tilde{x}_i = (x_{1i}, ..., x_{Ki})^T, d_i)_{i \in s^*}$ be such that $x_{ki} = 0$ if $i \notin s_k^*$ for any k. Let $c_1, ..., c_K$ be some constants. Denote $\mathbf{x}_i = (1, \tilde{x}_i^T)^T = (1, x_{1i}, ..., x_{Ki})^T$ and

$$d_i^* = d_i [\sum_{k=1}^{K} c_k 1_{i \in s_k^*}], \qquad (1)$$

where $1_A = 1$ if A is true and 0 otherwise.

Let $\mathbf{t}_x$ and $\tilde{t}_x$ be respectively $K+1$-dimensional and K-dimensional vectors of constants, corresponding to $\mathbf{x}_i$, $\tilde{x}_i$ respectively.

Consider $\hat{t}_{y,reg}(\mathbf{d}) = \sum_{i \in s^*} v_i y_i$, where $\sum_{i \in s^*} v_i \mathbf{x}_i = \mathbf{t}_\mathbf{x}$, $L < \frac{v_i}{d_i} < U$, and $\mathbf{v}$ minimizes $\sum_{i \in s^*} (v_i - d_i)^2 / d_i$, which in case $L = -\infty$ and $U = \infty$ is a Linear Regression Estimator (see Deville and Särndal 1992).
Until Section 4, we assume that $L = -\infty$ and $U = \infty$.

It can be shown (see Deville and Särndal 1992 and Valliant, et. al 2000) that $\hat{t}_{y,reg}(\mathbf{d}) = \hat{t}_y(\mathbf{d}) + [\mathbf{t}_x - \hat{\mathbf{t}}_x(\mathbf{d})]^T \mathbf{B}(\mathbf{d})$

where $\mathbf{t}_x$ is a vector of constants, $\hat{t}_{\mathbf{z}}(\mathbf{d}) = \sum_{i \in s^*} d_i \mathbf{z}_i$, and $\mathbf{B}(\mathbf{d})$ is a solution "$\mathbf{A}$" of the Weighted Least Squares (WLS) Normal Equations:

$$\sum_{i \in s^*} d_i (y_i - \mathbf{x}_i^T \mathbf{A}) = 0,$$

$$\sum_{i \in s^*} d_i (y_i - \mathbf{x}_i^T \mathbf{A}) x_{ki} = 0, \quad \text{k=1,...,K,}$$

In particular if $\sum_{i \in s^*} d_i \mathbf{x}_i \mathbf{x}_i^T$ is nonsingular then $\mathbf{B}(\mathbf{d}) = [\sum_{i \in s^*} d_i \mathbf{x}_i \mathbf{x}_i^T]^{-1} \sum_{i \in s^*} d_i \mathbf{x}_i y_i$.

COMMENT 1. S^* and s^* can be considered as a set indicating a population and a set of finally selected units (sample) respectively, $\{S_k^*\}$ and $\{s_k^*\}$ represent non-response adjustment groups in the population and the sample, y_i and $\mathbf{x}_i$'s represent values of target and auxiliary variables, d_i's are weights adjusted for non-response, and $\mathbf{t}_x$ is a vector of population totals of the auxiliary variables with the first coordinate equal to N, the number of population units (in particular if X_i is a variable used for non-response adjustment with known "totals by group", $T_X(k) = \sum_{i \in S_k^*} X_i$, then $x_{ki} = X_i$ if $i \in S_k^*$ and 0 otherwise); then $\hat{t}_{y,reg}$ is a linear regression estimator of the population total t_y. Assume that d_i^* are the original inverse inclusion probabilities. Then in the case of non-response, $1/d_i^*$ can differ from the ultimate probabilities of inclusion in the sample and thus to get consistent estimates based on these probabilities, non-response adjustment is done. Usually when adjustment is made based on auxiliary variables the adjusted weights and primary probability inverse weights are connected by (1) with some c_i's. In this section we try to estimate the difference of two regression estimators one of which is based on the adjusted weights and another on the original unadjusted weights, i.e. we would like to estimate $\hat{t}_{y,reg}(\mathbf{d}) - \hat{t}_{y,reg}(\mathbf{d}^*)$.

First we note the following very simple lemma.

Lemma 1. Let $\mathbf{A}$ satisfy

$$\sum_{i \in s^*} d_i (y_i - \mathbf{x}_i^T \mathbf{A}) = 0 \qquad (2)$$

and $\mathbf{A}^*$ satisfy

$$\sum_{i \in s^*} d_i^* (y_i - \mathbf{x}_i^T \mathbf{A}^*) = 0. \qquad (3)$$

Then for any vector $\mathbf{t}$

$$[\sum_{i \in s^*} d_i y_i + (\mathbf{t} - \sum_{i \in s^*} d_i \mathbf{x}_i)^T \mathbf{A}] - [\sum_{i \in s^*} d_i^* y_i + (\mathbf{t} - \sum_{i \in s^*} d_i^* \mathbf{x}_i)^T \mathbf{A}^*] = \mathbf{t}^T (\mathbf{A} - \mathbf{A}^*). \qquad (4)$$

This follows immediately from substitution of (2) and (3) into (4).

Corollary 1.

$$\hat{t}_{y,reg}(\mathbf{d}) - \hat{t}_{y,reg}(\mathbf{d}^*) = \mathbf{t}_x^T [\mathbf{B}(\mathbf{d}) - \mathbf{B}(\mathbf{d}^*)].$$

This follows from the fact that (2) and (3) are the first of the WLS Normal Equations respectively for $\mathbf{B}(\mathbf{d})$ and $\mathbf{B}(\mathbf{d}^*)$.

Lemma 2. Let N denote the size of a finite population, and let the following conditions be satisfied: as $N \to \infty$,

i) $\lim N^{-1} t_y$ and $\lim N^{-1} \mathbf{t}_{\mathbf{x}}$ exist.

ii) $N^{-1}(t_y - \hat{t}(\mathbf{d})) \to 0$ and $N^{-1}(\mathbf{t}_{\mathbf{x}} - \hat{\mathbf{t}}_{\mathbf{x}}(\mathbf{d})) \to 0$ in design probability.

Then $\hat{t}_{y,reg}(\mathbf{d}^*) - \hat{t}_{y,reg}(\mathbf{d}) \to 0$ in design probability if and only if

$$N^{-1} \sum_{i=1}^{K} \sum_{i \in s_k^*} d_i (c_k - 1)(y_i - \mathbf{x}_i^T \mathbf{B}(\mathbf{d})) \to 0 \text{ in design probability} \qquad (5)$$

In particular under i) and ii), (5) is equivalent to

$$N^{-1}\sum_{k=1}^{K}\sum_{i\in S_k^*}(c_k-1)(y_i-\mathbf{x}_i^T\mathbf{B}(\mathbf{d}))\to 0. \quad (5')$$

where summation is over the population. (Proof below.)

Corollary 2. Under i) and ii), $\hat{t}_{y,reg}(\mathbf{d}^*)-\hat{t}_{y,reg}(\mathbf{d})\to 0$ in design probability if

$$N^{-1}\sum_{i\in s_k^*} d_i(y_i-\mathbf{x}_i^T\mathbf{B}(\mathbf{d}))\to 0 \qquad (6)$$

in design probability for all $k=1,...,K$, or equivalently,

$$N^{-1}\sum_{i\in S_k^*}(y_i-\mathbf{x}_i^T\mathbf{B}(\mathbf{d}))\to 0 \qquad (6')$$

for all $k=1,...,K$.

Note that (6) and $(6')$ do not use the c_k's and (6) refers to a sample and therefore can be tested.

COMMENT 2. If we consider y_i and $1_{i\in S_k^*}$ as random variables generated by some super population (model) distribution then for some vector $\mathbf{b}$ $(5')$ converges in design probability to

$$E_\xi[(y_i-\mathbf{x}_i^T\mathbf{b})\sum_{k=1}^{K}(c_k-1)1_{i\in S_k^*}]=0 \quad (5M)$$

and (6) converges to,

$$E_\xi[(y_i-\mathbf{x}_i^T\mathbf{b})\mid i\in S_k^*]=0, \qquad (6M)$$

where E_ξ means expectation with respect to the model distribution. A usual basic model assumption in linear regression theory is

$$E_\xi[(y_i-\mathbf{x}_i^T\mathbf{b})\mid \mathbf{x}_i]=0$$

which is in general stronger than (6M) which is in turn stronger than (5M).

Proof of Lemma 2. It follows from Corollary 1 that we have to estimate the difference between $\mathbf{B}(\mathbf{d})$ and $\mathbf{B}(\mathbf{d}^*)$. Consider the WLS Normal equations for $\mathbf{B}(\mathbf{d})$ and $\mathbf{B}(\mathbf{d}^*)$:

$$\sum_{i\in s^*} d_i(y_i-\mathbf{x}_i^T\mathbf{A})=0,$$
$$\sum_{i\in s^*} d_i(y_i-\mathbf{x}_i^T\mathbf{A})x_{ki}=0,\quad k=1,...,K, \quad (7)$$

$$\sum_{i\in s^*} d_i^*(y_i-\mathbf{x}_i^T\mathbf{A})=0,$$
$$\sum_{i\in s^*} d_i^*(y_i-\mathbf{x}_i^T\mathbf{A})x_{ki}=0,\quad k=1,...,K. \quad (7')$$

Recall from Section 1 that $x_{ki}=0$ if $i\notin s_k^*$ for any k, *ex hypothesis.* Therefore

$$\sum_{i\in s^*} d_i(y_i-\mathbf{x}_i^T\mathbf{A})x_{ki}=$$
$$\sum_{i\in s_k^*} d_i(y_i-\mathbf{x}_i^T\mathbf{A})x_{ki}=0 \quad \Rightarrow$$
$$0=c_k\sum_{i\in s_k^*} d_i(y_i-\mathbf{x}_i^T\mathbf{A})x_{ki}=$$
$$\sum_{i\in s_k^*} d_i^*(y_i-\mathbf{x}_i^T\mathbf{A})x_{ki},\ \text{if}\ k=1,...K.$$

Therefore $(7')$ is equivalent to

$$\sum_{i\in s^*} d_i^*(y_i-\mathbf{x}_i^T\mathbf{A})=0,$$
$$\sum_{i\in s^*} d_i(y_i-\mathbf{x}_i^T\mathbf{A})x_{ki}=0,\quad \text{k=1,...,K.}$$

Now, under i) and ii), (7) is asymptotically equivalent to

$$N^{-1}\sum_{i\in S^*}(y_i-\mathbf{x}_i^T\mathbf{A})=0,$$
$$N^{-1}\sum_{i\in S^*}(y_i-\mathbf{x}_i^T\mathbf{A})x_{ki}=0,\quad \text{k=1,...,K.} \quad (8)$$

and (7') to

$$N^{-1}\sum_{i\in S^*}(y_i-\mathbf{x}_i^T\mathbf{A})[\sum_{k=1}^{K}c_k 1_{i\in S_k^*}]=0,$$
$$N^{-1}\sum_{i\in S^*}(y_i-\mathbf{x}_i^T\mathbf{A})x_{ki}=0,\quad \text{k=1,...,K.} \quad (8')$$

Then (8) and $(8')$ coincide asymptotically iff $N^{-1}\sum_{k=1}^{K}\sum_{i\in S_k^*}(y_i-\mathbf{x}_i^T\mathbf{A})(c_k-1)\to 0,$ proving the lemma.

Remark 1. Suppose (1), i) and ii) do not hold. We have in any case

$$\sum_{i\in s^*} d_i^*(y_i - \mathbf{x_i^T B(d^*)})x_{ki} =$$

$$\sum_{i\in s^*} d_i(\tilde{y}_i - \mathbf{x_i^T B(d^*)})x_{ki}\text{, where}$$

$$\tilde{y}_i = \frac{d_i^*}{d_i}(y_i - \mathbf{x_i^T B(d^*)}) + \mathbf{x_i^T B(d^*)}),$$

which implies

$$\mathbf{B(d^*)} = \left[\sum_{i\in s^*} d_i \mathbf{x_i x_i^T}\right]^{-1} \sum_{i\in s^*} d_i \mathbf{x_i} \tilde{y}_i,$$

$$\mathbf{B(d)} = \left[\sum_{i\in s^*} d_i \mathbf{x_i x_i^T}\right]^{-1} \sum_{i\in s^*} d_i \mathbf{x_i} y_i \text{ and}$$

thus,

$$[\hat{t}_{y,\mathrm{Re}\,g}(\mathbf{d}) - \hat{t}_{y,\mathrm{Re}\,g}(\mathbf{d}^*)] =$$

$$\mathbf{t_x}^T \left[\sum_{i\in s^*} d_i \mathbf{x_i x_i^T}\right]^{-1} \times$$

$$\sum_{i\in s^*}(d_i - d_i^*)\mathbf{x_i}(y_i - \mathbf{x_i^T B(d^*)}). \quad (9)$$

Remark 2. Based on (9) one can test whether calibrated estimators based on d-weights and on d^*-weights are significantly different or not. Under the assumption that $\frac{\mathbf{t_x}}{N}$ and $n\left[\sum_{i\in S} d_i \mathbf{x_i x_i^T}\right]^{-1}$ are bounded (9) implies that $N^{-1}[\hat{t}_{y,\mathrm{Re}\,g}(\mathbf{d}) - \hat{t}_{y,\mathrm{Re}\,g}(\mathbf{w})]$ is statistically insignificant if

$$H_0 : E_S(d_i - w_i)\mathbf{x_i}(y_i - \mathbf{x_i^T B(w)}) = 0,$$

where E_S denotes expectation over the sample distribution, $\Pr\{\cdot \mid i \in S\}$. H_0 can be tested for example by the following simple procedure: put $e_i^k = (y_i - \mathbf{x_i^T \hat{B}(w)})x_{ki}$, $k = 0,\ldots,K$ and $u_i = d_i - w_i$ and estimate coefficients of ordinary linear regression without intercept for $K+1$ linear models, $e_i^k = C^k u_i + \varepsilon_i$ (for example use PROC REG SAS). If for all k slope coefficients, C^k, are insignificant then H_0 is accepted.

2. Regression Through The Origin

In this section we consider a particular case of the regression estimator (regression without intercept) for which $\hat{t}_{y,reg}(\mathbf{d}) - \hat{t}_{y,reg}(\mathbf{d}^*) = 0.$

Lemma 3. Let $\tilde{\beta}$ be a solution of the following system of equations:

$$\sum_{i\in s_k^*} d_i(y_i - \tilde{x}_i^T\tilde{\beta}) = 0, \quad k = 1,\ldots,K \qquad (10)$$

and $\tilde{\beta}^*$ be a solution of the following system of equations:

$$\sum_{i\in s_k^*} d_i^*(y_i - \tilde{x}_i^T\tilde{\beta}^*) = 0, \quad k = 1,\ldots,K \quad (10')$$

Then $\tilde{\beta}^*$ is a solution for (10) and $\tilde{\beta}$ is a solution for $(10')$.

Proof. For $k = 1,\ldots,K$

$$0 = \sum_{i\in s_k^*} d_i(y_i - \tilde{x}_i^T\tilde{\beta}) \Rightarrow$$

$$c_k \sum_{i\in s_k^*} d_i(y_i - \tilde{x}_i^T\tilde{\beta}) =$$

$$\sum_{i\in s_k^*} d_i^*(y_i - \tilde{x}_i^T\tilde{\beta}) = 0$$ proving the lemma.

Now consider

$$\hat{t}^0_{y,reg}(\mathbf{d}) = \hat{t}_y(\mathbf{d}) + [\tilde{t}_x - \hat{\tilde{t}}_x(\mathbf{d})]^T\tilde{\beta}(\mathbf{d}) \quad (11)$$

where $\hat{\tilde{t}}_x(\mathbf{d}) = \sum_{i\in S^*} d_i\tilde{x}_i$,

$$\tilde{\beta}(\mathbf{d}) = [\sum_{i\in s^*} d_i \mathbf{z}_i \tilde{x}_i^T]^{-1} \sum_{i\in s^*} d_i \mathbf{z}_i y_i \quad \text{and}$$

$$\mathbf{z}_i = (1_{i\in s_1^*},\ldots,1_{i\in s_K^*})^T.$$

Note that the matrix $\sum_{i\in s^*} d_i \mathbf{z}_i \tilde{x}_i^T$ is diagonal and thus $[\sum_{i\in s^*} d_i \mathbf{z}_i \tilde{x}_i^T]^{-1}$ is also a diagonal matrix with k-th diagonal element equal to $[\sum_{i\in s_k^*} d_i x_{ki}]^{-1}$, and so

$$\tilde{\beta} = \begin{pmatrix} \dfrac{\sum_{i\in s_1^*} d_i y_i}{\sum_{i\in s_1^*} d_i x_{1i}} \\ . \\ . \\ . \\ \dfrac{\sum_{i\in s_K^*} d_i y_i}{\sum_{i\in s_K^*} d_i x_{Ki}} \end{pmatrix}.$$

Therefore $\tilde{\beta}(\mathbf{d})$ is a solution of (10). Thus terms cancel out in (11) so that
$\hat{t}_{y,reg}^{0}(\mathbf{d}) = \tilde{t}_x^T \tilde{\beta}(\mathbf{d}) =$

$$\sum_{k=1}^{K} t_{x_k} \frac{\sum_{i\in s_k^*} d_i y_i}{\sum_{i\in s_k^*} d_i x_{ki}} =$$

$$\sum_{k=1}^{K} \sum_{i\in s_k^*} y_i [d_i \frac{t_{x_k}}{\sum_{i\in s_k^*} d_i x_{ki}}] \qquad (12)$$

Additionally, from Lemma 3, $\hat{t}_{y,reg}^{0}(\mathbf{d}) = \hat{t}_{y,reg}^{0}(\mathbf{d}^*)$.

Remark 3. Note that if x_{ki} are nonnegative numbers and $x_{ki} > 0$ for any s_k^* then

$$\tilde{\beta}(\mathbf{d}) = [\sum_{i\in s^*} \frac{d_i}{\sigma_i^2} \tilde{x}_i \tilde{x}_i^T]^{-1} \sum_{i\in s^*} \frac{d_i}{\sigma_i^2} \tilde{x}_i y_i,$$

where $\sigma_i^2 = \sigma^2 \sum_{k=1}^{K} x_{ki}$. In particular $\hat{t}_{y,reg}^{0}$ is a ratio estimator if $K = 1$.

3. Calibration On Known Totals

Under the practically important case in which auxiliary variables are strata indicators the Linear Regression estimators with and without intercept coincide and thus $\hat{t}_{y,reg}(\mathbf{d}) = \hat{t}_{y,reg}(\mathbf{d}^*)$.

To see this, let $x_{ki} = 1_{i\in S_k^*}$. Then $\sum_{k=1}^{K} x_{ki} = 1$ for all i and $\mathbf{t_x} = (t_{\mathbf{x}_0}, ..., t_{\mathbf{x}_K})^T$ where $t_{\mathbf{x}_k} = \#\{$ units in a set $S_k^*\}$, the k-th total for $k = 1, ..., K$ and $t_{\mathbf{x}_0} = t_{\mathbf{x}_1} + ... + t_{\mathbf{x}_K}$ (equals the number of units in the population.). Consider $\hat{t}_{y,reg}^{0}(\mathbf{d}) = \hat{t}_y(\mathbf{d}) + [\tilde{t}_x - \hat{\tilde{t}}_x(\mathbf{d})]^T \tilde{\beta}(\mathbf{d})$,
where $\mathbf{B}(\mathbf{d})$ is any solution to

$$\mathbf{B}(\mathbf{d}) = arg \inf_{\mathbf{A}} \sum_{i\in s^*} d_i (y_i - \mathbf{x}_i^T \mathbf{A})^2. \qquad (13)$$

It can be shown that $[\mathbf{t_x} - \hat{\mathbf{t}}_\mathbf{x}(\mathbf{d})]^\mathbf{T} \mathbf{B}(\mathbf{d})$ is unique (compare Valliant, et. al 2000, Chapter 7.) Using $\sum_{k=1}^{K} x_{ki} = 1$ for all i and denoting $\tilde{A} = (a_1 + a_0, ..., a_K + a_0)^T$ we can write $\mathbf{x}_i^T \mathbf{A} = \tilde{x}_i^T \tilde{A}$ and thus (13) is equivalent to

$\hat{t}_{y,reg}^{0}(\mathbf{d}) = \hat{t}_y(\mathbf{d}) + [\tilde{t}_x - \hat{\tilde{t}}_x(\mathbf{d})]^T \tilde{\beta}(\mathbf{d})$,
where $\tilde{\beta}(\mathbf{d}) = arg \inf_{\mathbf{b}} \sum_{i\in s^*} d_i (y_i - \tilde{x}_i^T \mathbf{b})^2$

which implies
$\hat{t}_{y,reg}(\mathbf{d}) = \hat{t}_{y,reg}^{0}(\mathbf{d}) = \hat{t}_{y,reg}^{0}(\mathbf{d}^*) = \hat{t}_{y,reg}(\mathbf{d}^*)$.

4. Bounds For Adjusted Weights

Let us return to the general expression for the calibration estimator (see the beginning of section 1), $\hat{t}_{y,reg}(\mathbf{d}) = \sum_{i\in s^*} v_i y_i$ and $\hat{t}_{y,reg}^{0}(\mathbf{d}) = \sum_{i\in s^*} v_i^0 y_i$. One of the requirements which we have to follow often in practice is bounds for the ratio between the final weight, $\mathbf{v}$ or $\mathbf{v}^0$, and the initial (frame sample) weight, $\mathbf{d}^*$ in our case, that is

$$L < v_i / d_i^* < U. \qquad (14)$$

Multiplying (14) by $d_i^* \tilde{x}_i$ one can get $Ld_i^* \tilde{x}_i < v_i \tilde{x}_i < Ud_i^* \tilde{x}_i$ which implies (by summing over s^*) $L\hat{\hat{t}}_x < \tilde{t}_x < U\hat{\hat{t}}_x$ (component wise). Thus

$$L < \frac{\sum_{i \in S_k^*} x_{ki}}{\sum_{i \in s_k^*} d_i^* x_{ki}} < U, \; k = 1,...,K. \quad (15)$$

On the other hand suppose (15) holds. Comparing this to (12), one can note that the central part of (15) is the benchmark factor, that is, is the multiplier of d_i used to get the calibration weights v_i. Thus a set of weights satisfying (14) exists if and only if (15) holds. Therefore the following statement is correct.

Lemma 4. If there exists a set of weights satisfying

$$\sum_{i \in s^*} v_i \tilde{x}_i = \tilde{t}_x, \quad L < v_i / d_i^* < U$$

then $\hat{t}^0_{y,reg}(\mathbf{d})$ does not depend on L and U. In particular if auxiliary variables are strata indicators, i.e. $x_{ki} = 1_{i \in S_k^*}$ then the same remains true for $\hat{t}_{y,reg}(\mathbf{d}^*)$.

Remark 4. From Lemma 4 it follows that in the case of calibration on known totals the only way to get the restrictions (14) is to collapse cells s_k^*'s such that (15) is satisfied.

References

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