GOLDEN AND SILVER JUBILEE YEAR-2003 OF THE LINEAR REGRESSION ESTIMATORS

Sarjinder Singh, Department of Statistics, St. Cloud State University, St. Cloud, MN 56301-4498, USA.
E-mail: sarjinder@yahoo.com

Abstract

In the present investigation, a new model assisted chi-square distance function has been introduced. The estimators due to Hansen, Hurwitz and Madow (1953) and Das and Tripathi (1978) are shown to have celebrated the Golden and Silver Jubilee Year-2003 for their outstanding performances and uniqueness. The proposed methodology is based on the recent work of Singh (2003a, 2004a, 2004b). An important modification in the statistical packages such as GES, SUDDAN etc. has been strongly recommended.

Key words: Model assisted calibration; Linear regression estimator; GREG; Estimation of total and variance.

1. Introduction

The present investigation is an answer to a question raised by Deville and Sarndal's (1992) calibration approach to all eminent survey statisticians working at Govt. Organizations such as the U.S. Bureau of Census, Statistics Canada, Australian Bureau of Statistics, private organizations such as RAND, WESTSTAT etc., and their consultants from different universities across the world. Note that during the last decade the survey statisticians working for these institutes have tried a lot to construct a traditional linear regression estimator through calibration approach, but with little success. For example, as reported by Farrell and Singh (2002) that Wu and Sitter (2001) and Sitter and Wu (2002) rediscovered the Deville and Särndal (1992) estimator by setting one of the auxiliary variable at constant level while in the search of a traditional linear regression estimator using calibration approach, and same thing has been observed in recent papers by Estevao and Särndal (2003), Wu (2003), Wu and Luan (2003), Patel and Chaudhari (2003), Singh (2003b), Farrell and Singh (2004), Arnab and Singh (2003, 2004) and Harms and Duchesne (2004a, 2004b).

Here we discovered that the traditional linear regression estimator can also be shown as a special case of calibration approach under any unequal sampling design. It seems that all the papers related to minimizing chi-square distance function in survey sampling need modifications. There is a series of such papers by many followers of Deville and Särndal (1992) and it seems that everyone, for example see Rao (1994), Sarndal (1996) and Brewer (1999), skipped a very important point while using chi-square distance function. The technique developed in the present investigation is logically more accurate than whatever is done by survey statisticians during the last decade. The traditional linear regression estimators owed to Hansen, Hurwitz, and Madow (1953) and Das and Tripathi (1978) are shown to be unique in their classes of estimators.

It is fact that the survey statisticians are often interested in the precision of survey estimates. The most commonly used estimator of population total/ mean is the generalized linear regression (GREG) estimator. Let us consider the simplest case of the GREG where information on a single auxiliary variable is available (Note that problems with the use of multi-auxiliary information in calibration approach have been discussed in the recent decent book by Singh (2003a)).

Consider a population $\Omega = \{1,2,..,i,..,N\}$, from which a probability sample $s(s \subset \Omega)$ of fixed size (for simplicity) is drawn with a given sampling design, $p(.)$. The condition of the fixed sample size design can be relaxed by following Singh and Horn (1999). The inclusion probabilities $\pi_i = \Pr(i \in s)$ and $\pi_{ij} \in \Pr(i \,\&\, j \in s)$ are assumed to be strictly positive and known. Let y_i be the value of the variable of interest, y, for the i^{th} population element, with which is also associated an auxiliary variable x_i. For the element $i \in s$, we observe (y_i, x_i). The population total of the auxiliary variable x, $X = \sum_{i \in \Omega} x_i$, is assumed to be accurately known. Please note that the first order inclusion probabilities cannot be made directly proportional to the same auxiliary variable to be being used at estimation stage as shown by Mahajan and Singh (1997). The objective is to estimate the population total $Y = \sum_{i \in \Omega} y_i$. Deville and Särndal (1992) used calibration on known population x total to modify the basic sampling design weights, $d_i = 1/\pi_i$, that appear in the Horvitz and Thompson (1952) estimator

$$\hat{Y}_{\mathrm{HT}} = \sum_{i \in s} \frac{y_i}{\pi_i} = \sum_{i \in s} d_i y_i \tag{1.1}$$

Deville and Särndal (1992) considered an estimator of population total Y as

$$\hat{Y}_{\mathrm{ds}} = \sum_{i \in s} w_i y_i \tag{1.2}$$

with weights w_i are as close as possible in an average sense to the d_i, while respecting the calibration equation

$$\sum_{i\in s} w_i x_i = X . \tag{1.3}$$

A simple case due to Deville and Särndal (1992) is the minimization of chi square type distance function given by

$$\sum_{i\in s} \frac{(w_i - d_i)^2}{d_i q_i} \tag{1.4}$$

where q_i are suitably chosen weights. In many situations the value of $q_i = 1$. The form of the estimator depends upon the choice of q_i. By minimizing (1.4) subject to calibration equation (1.3), the calibration weights are given by

$$w_i = d_i + \frac{d_i q_i x_i}{\sum_{i\in s} d_i q_i x_i^2}\left(X - \sum_{i\in s} d_i x_i \right) \tag{1.5}$$

Substitution of the value of w_i from (1.5) in (1.2) leads to the GREG estimator of total given by

$$\hat{Y}_{\text{GREG}} = \sum_{i\in s} d_i y_i + \hat{\beta}_{\text{ds}}\left(X - \sum_{i\in s} d_i x_i \right) \tag{1.6}$$

where

$$\hat{\beta}_{\text{ds}} = \left(\sum_{i\in s} d_i q_i x_i y_i \right) \Big/ \left(\sum_{i\in s} d_i q_i x_i^2 \right) \tag{1.7}$$

Note that several researchers tried to improve the estimator $\hat{\beta}_{ds}$ to get an estimator close to the traditional linear regression estimator of the Hansen, Hurwitz and Madow (1953), but unfortunately with little success because every one skipped a very important point when in considering minimization of chi-square distance function.

An estimator for estimating the variance of GREG in (1.6) is given by

$$\hat{V}_{\text{DS}}\left(\hat{Y}_{\text{GREG}}\right) = \frac{1}{2}\sum_{i\in s}\sum_{j\in s} D_{ij}\left(w_i e_i - w_j e_j\right)^2 \tag{1.8}$$

where $e_i = y_i - \hat{\beta}_{\text{ds}} x_i$ and $D_{ij} = (\pi_i \pi_j - \pi_{ij}) / \pi_{ij}$. Note that (1.8) is an estimator of variance of GREG, but may not be an estimator of the variance of the traditional linear regression estimator because the error tem is free from intercept.

Singh, Horn and Yu (1998) have shown that the usual estimator of variance of the linear regression estimator and the class of estimators due to Deng and Wu (1987) are a special cases of (1.8), but their claim is incorrect because the error term $e_i = y_i - \hat{\beta}_{\text{ds}} x_i$ is different from the Ordinary Least Square (OLS) error term with intercept. Singh, Horn and Yu (1998) also claimed an improved estimator of variance of GREG as

$$\hat{V}_{\text{shy}}\left(\hat{Y}_{\text{GREG}}\right) = \frac{1}{2}\sum_{i\neq j}\sum_{\in s} \Omega_{ij}\left(w_i e_i - w_j e_j\right)^2 \tag{1.9}$$

where Ω_{ij} are calibrated weights such that the chi-square distance function

$$D^* = \frac{1}{2}\sum_{i\neq j}\sum_{\in s} \frac{(\Omega_{ij} - D_{ij})^2}{D_{ij} Q_{ij}} \tag{1.10}$$

is minimum subject to a calibration constraint

$$\frac{1}{2}\sum_{i\neq}\sum_{j\in s} \Omega_{ij}\left(d_i x_i - d_j x_j\right)^2 = V_{\text{syg}}\left(\hat{X}_{\text{HT}}\right) \tag{1.11}$$

where $V_{\text{syg}}\left(\hat{X}_{\text{HT}}\right) = \frac{1}{2}\sum_{i\neq}\sum_{j\in\Omega} \Theta_{ij}\left(d_i x_i - d_j x_j\right)^2$ with $\Theta_{ij} = (\pi_i \pi_j - \pi_{ij})$.

Minimization of (1.10) subject to (1.11) leads to calibrated weights given by

$$\Omega_{ij} = D_{ij} + \frac{D_{ij} Q_{ij}\left(d_i x_i - d_j x_j\right)^2}{\frac{1}{2}\sum_{i\neq}\sum_{j\in s} D_{ij} Q_{ij}\left(d_i x_i - d_j x_j\right)^4}\left[V_{\text{syg}}\left(\hat{X}_{\text{HT}}\right) - \hat{V}_{\text{syg}}\left(\hat{X}_{\text{HT}}\right)\right]$$

where $\hat{V}_{\text{syg}}\left(\hat{X}_{\text{HT}}\right) = \frac{1}{2}\sum_{i\neq}\sum_{j\in s} D_{ij}\left(d_i x_i - d_j x_j\right)^2$. Singh, Horn and Yu (1998) estimator of the variance of GREG is given by

$$\hat{V}_{\text{shy}}\left(\hat{Y}_{\text{GREG}}\right) = \hat{V}_{\text{DS}}\left(\hat{Y}_{\text{GREG}}\right) + \hat{B}_1\left[V_{\text{syg}}\left(\hat{X}_{\text{HT}}\right) - \hat{V}_{\text{syg}}\left(\hat{X}_{\text{HT}}\right)\right] \tag{1.12}$$

where

$$\hat{B}_1 = \frac{\sum_{i\neq}\sum_{j\in s} D_{ij} Q_{ij}\left(d_i x_i - d_j x_j\right)^2\left(w_i e_i - w_j e_j\right)^2}{\sum_{i\neq}\sum_{j\in s} D_{ij} Q_{ij}\left(d_i x_i - d_j x_j\right)^4} \tag{1.13}$$

Two obvious problems are noted in the estimator (1.12): first is the use of $e_i = y_i - \hat{\beta}_{\text{ds}} x_i$ and the second is $\hat{B}_1$ is not exactly in the form as considered by Das and Tripathi (1978), and their followers Srivastava and Jhajj (1980) and Isaki (1983). A review of this work can be had from a decent and great book of Singh (2003a).

In this paper, a new improved method of minimizing chi-square distance function has been introduced, and which in turn can result a series of research articles.

2. Hansen, Hurwitz and Madow (1953) estimator

We consider an estimator of the population total Y as

$$\hat{Y}_s = \sum_{i\in s} w_i^* y_i \tag{2.1}$$

Under the model

$$\text{M} : y_i = \alpha + \beta\, x_i + e_i^* \tag{2.2}$$

where α is an intercept, β is a slope, $E_M\left(e_i^*\right) = 0$, $E_M\left(e_i^{*2}\right) = \sigma^2\sqrt{v(x_i)}$, and $v(x_i)$ is known, (see Royall 1970a, 1970b, 1970c, 1971, Hajek 1981, Bellhouse 1984, Pokropp 2002), we consider in (2.1) the w_i^* as the calibrated weights such that the **model assisted chi-square** distance function defined as

$$D^* = \frac{1}{2}\sum_{i\in s}\left[\frac{v(x_i)\left(w_i^* - d_i\right)^2}{d_i q_i^*}\right] \tag{2.3}$$

is minimum subject to the two constraints, given by

$$\sum_{i\in s} w_i^* = \sum_{i\in s} d_i \tag{2.4}$$

and

$$\sum_{i\in s} w_i^* x_i = X . \tag{2.5}$$

Note that the condition (2.4) is a requirement of the chi square test given by Prof. Karl Pearson (1900, 1922)/Sir R.A. Fisher (1915,1922), which is **ignored** by all the followers of Deville and Särndal (1992). Further note that the condition (2.4) shows that from a given sample the sum of observed weights should be equal to sum of the calibrated weights, and hence controls any kind of extra variation in the resultant estimators. Obviously the Lagrange function is

$$L=\frac{1}{2}\sum_{i\in s}\left[\frac{v(x_i)\left(w_i^*-d_i\right)^2}{d_iq_i^*}\right]-\lambda_1\left\{\sum_{i\in s}w_i^*-\sum_{i\in s}d_i\right\}-\lambda_2\left\{\sum_{i\in s}w_i^*x_i-X\right\}.$$

On setting $\partial L/\partial w_i^*=0$ we obtain

$$w_i^*=d_i+\left(\lambda_1 d_i q_i^*+\lambda_2 d_i q_i^* x_i\right)/v(x_i). \qquad (2.6)$$

On using (2.6) in (2.4) we obtain

$$\lambda_1\left\{\sum_{i\in s}\left(d_iq_i^*/v(x_i)\right)\right\}+\lambda_2\left\{\sum_{i\in s}\left(d_iq_i^*x_i/v(x_i)\right)\right\}=0\,. \qquad (2.7)$$

On using (2.6) in (2.5) we obtain

$$\lambda_1\left\{\sum_{i\in s}\left(d_iq_i^*x_i/v(x_i)\right)\right\}+\lambda_2\left\{\sum_{i\in s}\left(d_iq_i^*x_i^2/v(x_i)\right)\right\}=\left(X-\hat{X}_{\text{HT}}\right). \qquad (2.8)$$

where $\hat{X}_{\text{HT}}=\sum_{i\in s}d_ix_i$. On solving (2.7) and (2.8) for λ_1 and λ_2, and on substituting these values in (2.6) we obtain the calibrated weights as

$$w_i^*=d_i+\left\{\frac{\dfrac{d_iq_i^*x_i}{v(x_i)}\left(\sum_{i\in s}\dfrac{d_iq_i^*}{v(x_i)}\right)-\dfrac{d_iq_i^*}{v(x_i)}\left(\sum_{i\in s}\dfrac{d_iq_i^*x_i}{v(x_i)}\right)}{\left(\sum_{i\in s}\dfrac{d_iq_i^*}{v(x_i)}\right)\left(\sum_{i\in s}\dfrac{d_iq_i^*x_i^2}{v(x_i)}\right)-\left(\sum_{i\in s}\dfrac{d_iq_i^*x_i}{v(x_i)}\right)^2}\right\}\left(X-\hat{X}_{\text{HT}}\right)$$

On substituting it in (2.1), we have a great success as

$$\hat{Y}_{\text{s}}=\sum_{i\in s}d_iy_i+\hat{\beta}_{ols}\left(X-\sum_{i\in s}d_ix_i\right)=\hat{Y}_{\text{reg}}\ \text{(say)} \qquad (2.9)$$

where

$$\hat{\beta}_{\text{ols}}=\frac{\left(\sum_{i\in s}\dfrac{d_iq_i^*x_iy_i}{v(x_i)}\right)\left(\sum_{i\in s}\dfrac{d_iq_i^*}{v(x_i)}\right)-\left(\sum_{i\in s}\dfrac{d_iq_i^*y_i}{v(x_i)}\right)\left(\sum_{i\in s}\dfrac{d_iq_i^*x_i}{v(x_i)}\right)}{\left(\sum_{i\in s}\dfrac{d_iq_i^*}{v(x_i)}\right)\left(\sum_{i\in s}\dfrac{d_iq_i^*x_i^2}{v(x_i)}\right)-\left(\sum_{i\in s}\dfrac{d_iq_i^*x_i}{v(x_i)}\right)^2} \qquad (2.10)$$

which is clearly usual traditional linear regression estimator for any kind of probability proportional to size and without replacement (PPSWOR) sampling. No doubt $\hat{\beta}_{\text{ols}}$ in (2.10) is a competitor of Royall (1970a, 1970b, 1970c, 1971), Hajek (1981) and other related work in decent papers by Bellhouse (1984) and Pokropp (2002).

For example, note that if $q_i^*=1$, $v(x_i)=1$ and for SRSWOR sampling where $d_i=N/n$, the estimator (2.9) reduces to

$$\hat{Y}_{\text{reg}}=N\left[\bar{y}+\hat{\beta}_{\text{ols}}\left(\bar{X}-\bar{x}\right)\right] \qquad (2.11)$$

where $\hat{\beta}_{\text{ols}}=s_{xy}/s_x^2$, $s_{xy}=(n-1)^{-1}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})$, $\bar{y}=n^{-1}\sum_{i=1}^{n}y_i$, $\bar{x}=n^{-1}\sum_{i=1}^{n}x_i$ and $s_x^2=(n-1)^{-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$, and it is the famous traditional linear regression estimator owed to Hansen, Hurwitz and Madow (1953). Note the calibration constraint due to Wu and Sitter (2001), Estevao and Sarndal (2003) etc., which is valid only for a couple of designs, is also a special case of (2.4) for $d_i=N/n$. Further note that Wu and Sitter (2001) and Sitter and Wu (2002) made the linear regression estimator by neglecting an important term, and for details refer to Farrell and Singh (2002). Note that there is no choice of weights q_i^* that reduces the estimator (2.9) into ratio or product estimator. Only choice of $q_i^*=1$ reduces to the estimator in (2.9) to the traditional linear regression estimator under any unequal probability sampling, unlike Wu and Sitter (2001) and Estevao and Sarndal (2003), and celebrates Golden Jubilee Year-2003 for its outstanding performance, and leads to the theorem:

***Theorem 1**. The traditional linear regression estimator due to Hansen, Hurwitz and Madow (1953) is unique in its class of estimators.*

It is more interesting to note that Deville and Särndal (1992) assumed the model $M_1: y_i=\beta\ x_i+e_i$ such that $e_i\overset{iid}{\sim}N(0,\ \sigma^2)$. Under model M_1, an estimator of β can be obtained by $\min.\sum_{i\in s}d_iq_i\hat{e}^2_i$ where $\hat{e}_i=y_i-\hat{\beta}_{\text{ds}}x_i$. In contrast the proposed method relaxes the condition of zero intercept, which is a requirement of traditional linear regression estimator to provide efficient results, that is we can consider any linear model of the form $\text{M}: y_i=\alpha+\beta\ x_i+e_i^*$, where α is intercept and β is a slope, $v(x_i)$ being any function of x_i, such that $e_i^*\overset{iid}{\sim}N\left(0,\ \sigma^2\sqrt{v(x_i)}\right)$.Under the model M , estimates of intercept and slope are given by $\min.\sum_{i\in s}d_iq_i^*\hat{e}_i^{*2}/v(x_i)$, where $\hat{e}_i^*=y_i-\hat{\alpha}_{\text{ols}}-\hat{\beta}_{\text{ols}}x_i$. In simple words, the new method relaxes the assumption of Deville and Särndal (1992) that the regression line should pass through the origin, and the investigator or researcher is not supposed to be worried about the status of the regression line while applying the proposed methodology.

3. Das and Tripathi (1978) estimator of variance

Following Singh, Horn and Yu (1998), a low level calibrated estimator of the variance of the linear regression estimator $\hat{Y}_{\text{reg}}$ is given by

$$\hat{V}_{\text{s}}\left(\hat{Y}_{\text{reg}}\right)=\frac{1}{2}\sum_{i\neq}\sum_{j\in s}D_{ij}\left(w_i^*\hat{e}_i^*-w_j^*\hat{e}_j^*\right)^2 \qquad (3.1)$$

where $\hat{e}_i^*=y_i-\hat{\alpha}_{\text{ols}}-\hat{\beta}_{\text{ols}}x_i$. Further, we consider a new higher order calibrated estimator of variance as

$$\hat{V}_{\text{ss}}\left(\hat{Y}_{\text{reg}}\right)=\frac{1}{2}\sum\sum_{i\neq j\in s}\Omega_{ij}^*\left(w_i^*\hat{e}_i^*-w_j^*\hat{e}_j^*\right)^2 \qquad (3.2)$$

where Ω_{ij}^* are weights such that chi-square distance function

$$D^*=\frac{1}{2}\sum\sum_{i\neq j\in s}\frac{\left(\Omega_{ij}^*-D_{ij}\right)^2}{D_{ij}Q_{ij}} \qquad (3.3)$$

is minimum subject to two calibration constraints, given by

$$\sum\sum_{i\neq j\in s}\Omega_{ij}^*=\sum\sum_{i\neq j\in s}D_{ij} \qquad (3.4)$$

and

$$\frac{1}{2}\sum_{i\neq}\sum_{j\in s}\Omega_{ij}^*\left(d_ix_i-d_jx_j\right)^2=V_{\text{syg}}\left(\hat{X}_{\text{HT}}\right). \qquad (3.5)$$

Note that the condition (3.4) is same as given by Prof. Karl Pearson/Sir R.A. Fisher for a $r \times c$ contingency table, and is ignored by all the followers of Singh, Horn and Yu (1998), like Singh, Horn, Chowdhury, and Yu (1999), Singh (2001), Wu and Sitter (2001), Sitter and Wu (2002), Wu (2003), and Farrell and Singh (2004). Further note that for SRSWOR sampling the condition (3.4) is same as reported by Sitter and Wu (2002) indicating that their technique is a special case of it. Again note carefully that from a given sample the sum of the two dimensional design weights should be equal to that of calibrated weights and it will control extra variation in the resultant estimators. Obviously, for the minimization of (3.3) subject to (3.4) and (3.5), the Lagrange function is given by

$$\mathrm{LM} = \frac{1}{2}\sum_{i \neq j \in s}\sum \frac{\left(\Omega_{ij}^* - D_{ij}\right)^2}{D_{ij}Q_{ij}^*} - \lambda_{11}\left\{\sum_{i \neq j \in s}\sum \Omega_{ij}^* - \sum_{i \neq j \in s}\sum D_{ij}\right\} - \lambda_{22}\left\{\frac{1}{2}\sum_{i \neq j \in s}\sum \Omega_{ij}^*\left(d_i x_i - d_j x_j\right)^2 - V_{\text{syg}}\left(\hat{X}_{\text{HT}}\right)\right\} \quad (3.6)$$

where λ_{11} and λ_{22} are Lagrange multipliers. On setting $\partial LM / \partial \Omega_{ij}^* = 0$, we have

$$\Omega_{ij}^* = D_{ij} + \lambda_{11} D_{ij} Q_{ij}^* + \lambda_{22}\frac{D_{ij}Q_{ij}^*}{2}\left(d_i x_i - d_j x_j\right)^2 \quad (3.7)$$

On using (3.7) in (3.4) we have

$$\lambda_{11}\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^* + 0.5\lambda_{22}\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\left(d_i x_i - d_j x_j\right)^2 = 0 \quad (3.8)$$

and on using (3.7) in (3.5) we have

$$0.5\lambda_{11}\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\left(d_i x_i - d_j x_j\right)^2 + 0.25\lambda_{22}\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\left(d_i x_i - d_j x_j\right)^4 = \left\{V_{\text{syg}}\left(\hat{X}_{\text{HT}}\right) - \hat{V}_{\text{syg}}\left(\hat{X}_{\text{HT}}\right)\right\} \quad (3.9)$$

On solving (3.8) and (3.9) for λ_{11} and λ_{22}, and putting back in (3.7), we have

$$\Omega_{ij}^* = D_{ij} + \frac{\left(D_{ij}Q_{ij}^*\left(d_i x_i - d_j x_j\right)^2\right)\left(\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\right) - \left(D_{ij}Q_{ij}^*\right)\left(\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\left(d_i x_i - d_j x_j\right)^2\right)}{0.5\left[\left(\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\right)\left(\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\left(d_i x_i - d_j x_j\right)^4\right) - \left\{\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\left(d_i x_i - d_j x_j\right)^2\right\}^2\right]}\left\{V_{\text{syg}}\left(\hat{X}_{\text{HT}}\right) - \hat{V}_{\text{syg}}\left(\hat{X}_{\text{HT}}\right)\right\} \quad (3.10)$$

On substituting (3.10) into (3.2), we obtain a new higher order calibrated estimator of variance of the traditional linear regression estimator as:

$$\hat{V}_{\text{ss}}\left(\hat{Y}_{\text{reg}}\right) = \hat{V}_{\text{s}}\left(\hat{Y}_{\text{reg}}\right) + \hat{B}_2\left\{V_{\text{syg}}\left(\hat{X}_{\text{HT}}\right) - \hat{V}_{\text{syg}}\left(\hat{X}_{\text{HT}}\right)\right\} \quad (3.11)$$

where

$$\hat{B}_2 = \frac{\left(\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\left(d_i x_i - d_j x_j\right)^2\left(w_i^*\hat{e}_i^* - w_j^*\hat{e}_j^*\right)^2\right)\left(\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\right) - \left(\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\left(d_i x_i - d_j x_j\right)^2\right)\left(\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\left(w_i^*\hat{e}_i^* - w_j^*\hat{e}_j^*\right)^2\right)}{\left(\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\left(d_i x_i - d_j x_j\right)^4\right)\left(\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\right) - \left(\sum_{i \neq j \in s}\sum D_{ij}Q_{ij}^*\left(d_i x_i - d_j x_j\right)^2\right)^2} \quad (3.12)$$

Remark: Note carefully if $w_i^* = d_i$ and $\hat{e}_i^* = y_i$, then the estimator $\hat{Y}_{\text{reg}} = \hat{Y}_{\text{HT}}$ (No low level calibration is performed), then following Singh, Horn, Yu and Chowdhury (1999), the ratio $\hat{V}_{\text{ss}}\left(\hat{Y}_{\text{reg}}\right) \Big/ \left\{\frac{N^2(1-f)}{n}\right\}$ becomes traditional linear regression estimator of finite population variance, $\sigma_y^2 = N^{-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2$, under SRSWOR sampling given by

$$\hat{\sigma}_{\text{dt}}^2 = s_y^2 + \hat{\beta}_2\left(S_x^2 - s_x^2\right) \quad (3.13)$$

where $s_y^2 = (n-1)^{-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2$, $S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2$, $\hat{\beta}_2 = \frac{\hat{\mu}_{22} - \hat{\mu}_{20}\hat{\mu}_{02}}{\hat{\mu}_{04} - \hat{\mu}_{02}^2}$ with $\hat{\mu}_{rs} = (n-1)^{-1}\sum_{i \in s}\left(y_i - \bar{y}\right)^r\left(x_i - \bar{x}\right)^s$, which is owed to Das and Tripathi (1978) and celebrates Silver Jubilee Year-2003. Note that the estimator (3.13) was also independently studied by Srivastava and Jhajj (1980) and Isaki (1983). Thus we have the following theorem:

Theorem 2. The traditional linear regression type estimator of the finite population variance σ_y^2 is also unique in its class of estimators.

In the next section, we consider stratified random sampling design that has more practical utility in survey sampling applications.

4. Stratified sampling design

Suppose the population consists of L strata with N_h units in the h^{th} stratum from which a simple random sample of size n_h is taken without replacement, then the total population size $N = \sum_{h=1}^{L} N_h$ and sample size $n = \sum_{h=1}^{L} n_h$. Let the i^{th} unit of the h^{th} stratum be associated with two values y_{hi} and x_{hi} with $x_{h_i} > 0$ being the covariate. For h^{th} stratum, let $W_h = N_h / N$ be the stratum weights, $f_h = n_h / N_h$ the sample fraction, $\bar{y}_h$, $\bar{x}_h$, $\bar{Y}_h$, $\bar{X}_h$ the y and x sample and population means respectively. Assume $\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h$ is known. The purpose is to estimate $\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$, possibly by incorporating the covariance information x. The usual estimator of population mean $\bar{Y}$ is given by

$$\bar{y}_{\text{st}} = \sum_{h=1}^{L} W_h \bar{y}_h \tag{4.1}$$

Singh, Horn and Yu (1998) and Tracy, Singh and Arnab (2003) considered an estimator, given by

$$\bar{y}_{\text{St}}^{*} = \sum_{h=1}^{L} W_h^{*} \bar{y}_h \tag{4.2}$$

with new weights W_h^*. The new weights W_h^* are chosen such that chi square type distance given by

$$\sum_{h=1}^{L} \frac{\left(W_h^* - W_h\right)^2}{W_h q_h} \tag{4.3}$$

is minimum subject to the condition

$$\sum_{h=1}^{L} W_h^{*} \bar{x}_h = \bar{X} \tag{4.4}$$

Minimization of (4.3) subject to calibration equation (4.4) leads to the combined regression type estimator given by

$$\bar{y}_{\text{St}}^{*} = \sum_{h=1}^{L} W_h \bar{y}_h + \frac{\sum_{h=1}^{L} W_h q_h \bar{x}_h \bar{y}_h}{\sum_{h=1}^{L} W_h q_h \bar{x}_h^2} \left[\bar{X} - \sum_{h=1}^{L} W_h \bar{x}_h \right] \tag{4.5}$$

Note that the estimator (4.5) suggested by Singh, Horn and Yu (1998) is not a traditional linear combined regression estimator for any choice of weights q_h.

5. Combined linear regression using calibration

We consider here a new estimator of the population mean $\bar{Y}$ in stratified sampling as

$$\bar{y}_{\text{St}}^{\otimes} = \sum_{h=1}^{L} W_h^{\otimes} \bar{y}_h \tag{5.1}$$

where $W_h^{\otimes}$ are the calibrated weights such that the chi square distance function

$$D^{\otimes} = \frac{1}{2} \sum_{h=1}^{L} \frac{\left(W_h^{\otimes} - W_h\right)^2}{W_h Q_h^{\otimes}} \tag{5.2}$$

is minimum subject to two constraints, defined as

$$\sum_{h=1}^{L} W_h^{\otimes} = \sum_{h=1}^{L} W_h \tag{5.3}$$

and

$$\sum_{h=1}^{L} W_h^{\otimes} \bar{x}_h = \bar{X} \tag{5.4}$$

where $Q_h^{\otimes}$ are some suitably chosen weights. The condition (5.3) implies that sum of observed weights should be equal to sum of expected weights across all strata and such condition will control any kind of extra variation in the resultant estimator. By proceeding as in section 2, the new calibrated estimator of the population mean $\bar{Y}$ becomes

$$\bar{y}_{\text{s}}^{\text{st}} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{\text{st}} \left[\bar{X} - \sum_{h=1}^{L} W_h \bar{x}_h \right] \tag{5.5}$$

where

$$\hat{\beta}_{\text{st}} = \frac{\left(\sum_{h=1}^{L} W_h Q_h^{\otimes} \bar{x}_h \bar{y}_h\right)\left(\sum_{h=1}^{L} W_h Q_h^{\otimes}\right) - \left(\sum_{h=1}^{L} W_h Q_h^{\otimes} \bar{y}_h\right)\left(\sum_{h=1}^{L} W_h Q_h^{\otimes} \bar{x}_h\right)}{\left(\sum_{h=1}^{L} W_h Q_h^{\otimes}\right)\left(\sum_{h=1}^{L} W_h Q_h^{\otimes} \bar{x}_h^2\right) - \left(\sum_{h=1}^{L} W_h Q_h^{\otimes} \bar{x}_h\right)^2}$$

If $Q_h^{\otimes} = 1$ then the estimator (5.5) reduces to the traditional combined stratified linear regression estimator, and hence better than the estimators developed by Singh, Horn and Yu (1998), and Tracy, Singh and Arnab (2003), and leads to the theorem:

Theorem 3. The combined linear regression estimator in stratified random sampling is unique in its class of estimators.

Again note that the claim of Singh, Horn and Yu (1998) to estimate the variance of combined linear regression estimator is incorrect, and their claim of Wu (1985) class of estimators is also incorrect because their low level calibration approach is not reducing to the traditional combined linear regression estimator in stratified random sampling.

6. Estimation of variance of the combined linear regression estimator

Here we consider a new estimator of the variance of the combined stratified linear regression estimator and correct the results of Singh, Horn and Yu (1998) as follows. The well-known estimator of variance of combined regression estimator is given by

$$\hat{v}\left(\bar{y}_{\text{St}}^{*}\right) = \sum_{h=1}^{L} \frac{W_h^2 (1 - f_h)}{n_h} s_{e^*h}^2 \tag{6.1}$$

where

$s_{e^*h}^2 = (n_h - 1)^{-1} \sum_{i=1}^{n_h} e_{hi}^{*2}$ is the h^{th} stratum sample variance and $e_{hi}^{*} = (y_{hi} - \bar{y}_h) - b_{st}(x_{hi} - \bar{x}_h)$ and b_{st} denote the traditional linear regression coefficient in stratified sampling. The lower level calibration approach yields an estimator of variance of the combined regression estimator as

$$\hat{v}_{\text{c}}\left(\bar{y}_{\text{St}}^{*}\right) = \sum_{h=1}^{L} \frac{D_h W_h^{\otimes 2}}{W_h^2} s_{e^*h}^2 \tag{6.2}$$

where $D_h = W_h^2 (1 - f_h) / n_h$ and $W_h^{\otimes}$ is same as used in (5.5). Note that there is no choice of $Q_h^{\otimes}$ such that the estimator (5.5) reduces to combined ratio or combined product estimator in stratified sampling. Again following Singh, Horn and Yu (1998), a higher-level calibrated estimator of the variance of the combined linear regression estimator is

$$\hat{v}_{St}\left(\hat{\bar{Y}}_s^{St}\right) = \sum_{h=1}^{L} \frac{\Omega_h^{\oplus} W_h^{\otimes 2}}{W_h^2} s_{e^*h}^2 \tag{6.3}$$

where $\Omega_h^{\oplus}$ are suitably chosen weights such that chi square distance function given by

$$\sum_{h=1}^{L} \frac{\left(\Omega_h^{\oplus} - D_h\right)^2}{D_h Q_h^{\oplus}} \qquad (6.4)$$

is minimum subject to two calibration equations defined as

$$\sum_{h=1}^{L} \Omega_h^{\oplus} = \sum_{h=1}^{L} D_h \text{ and } \sum_{h=1}^{L} \Omega_h^{\oplus} s_{hx}^2 = V(\bar{x}_{\text{St}}) \qquad (6.5)$$

Where $V(\bar{x}_{\text{St}}) = \sum_{h=1}^{L} W_h^2 n_h^{-1}(1 - f_h) S_{hx}^2$ is assumed to be known and $s_{hx}^2 = (n_h - 1)^{-1} \sum_{i=1}^{n_h} (x_{hi} - \bar{x}_h)^2$ is an unbiased estimator of $S_{hx}^2 = (N_h - 1)^{-1} \sum_{i=1}^{N_h} (X_{hi} - \bar{X}_h)^2$,and $\hat{v}(\bar{x}_{\text{St}}) = \sum_{h=1}^{L} W_h^2 n_h^{-1}(1 - f_h) s_{hx}^2$ is an unbiased estimator of $V(\bar{x}_{\text{St}})$. Again note that first condition in (6.5) is a requirement of chi-square test introduced by Prof. Karl Pearson/Sir R.A. Fisher and is being ignored by the followers of Singh, Horn and Yu (1998). This procedure leads to a new higher order calibrated estimator of the variance of the combined regression estimator given by

$$\hat{v}\left(\hat{y}_{\text{s}}^{\text{St}}\right)_{\text{ho}} = \hat{v}\left(\hat{Y}_{\text{s}}^{\text{St}}\right) + \hat{B}_{\text{St}}^{\oplus}\left[V(\bar{x}_{\text{St}}) - \hat{v}(\bar{x}_{\text{St}})\right] \qquad (6.6)$$

where

$$\hat{B}_{\text{St}}^{\oplus} = \frac{\left(\sum_{h=1}^{L} \frac{D_h Q_h^{\oplus} W_h^{\otimes 2} s_{hx}^2 s_{e*h}^2}{W_h^2}\right)\left(\sum_{h=1}^{L} D_h Q_h^{\oplus}\right) - \left(\sum_{h=1}^{L} \frac{D_h Q_h^{\oplus} s_{hx}^2 s_{e*h}^2}{W_h^2}\right)\left(\sum_{h=1}^{L} D_h Q_h^{\oplus} s_{hx}^2\right)}{\left(\sum_{h=1}^{L} D_h Q_h^{\oplus}\right)\left(\sum_{h=1}^{L} D_h Q_h^{\oplus} s_{hx}^4\right) - \left(\sum_{h=1}^{L} D_h Q_h^{\oplus} s_{hx}^2\right)^2}$$

which seems to be completely new development although it is a corrected version of the estimator of Singh, Horn and Yu (1998), and hence that of Tracy, Singh, and Arnab (2003).

7. Conclusion

In this paper, we modify the methodology of Deville and Särndal (1992), and hence that of Rao (1994), Särndal (1996), Singh, Horn and Yu (1998), Brewer (1999), Singh (2001), Wu and Sitter (2001), Sitter and Wu (2002), and Estevao and Sarndal (2003) in addition to several more papers, while considering the problem of estimation of population total and variance of the linear regression estimator in survey sampling. A theoretical justification about the stability of the proposed estimators has also been given based on the weighted least square methods on estimation. In addition to that the proposed method relaxes the assumption of zero intercept in Deville and Särndal (1992) model assisted calibration technique. The statistical package GES developed by Statistics Canada can be modified to obtain the traditional linear regression type estimates of population total, and to estimate its variance using the proposed modified calibration approach. Similar changes in other statistical packages such as SUDAAN, CALMAR, SAS, and STATA etc. are also suggested. Note that under raking and poststratified sampling we consider the design weights $d_i = N/n$, and the constraint (2.4) works well. It shows claims of Wu (2003) and Wu and Luan (2003) to form optimal calibration approaches are also wrong in case of unequal probability proportional to size and without replacement designs, and their optimality is shown to be pseudo optimality by Singh (2004c).

8. Further study

Thus looking at my new results the concern is that we have to check all the estimates we are producing at the US Bureau of Census, Statistics Canada, Australian Bureau of Statistics, and Statistics Sweden etc. from the last 12 years based on Deville and Särndal (1992) techniques (DS) implemented especially in the statistical package GES developed by Statistics Canada and sold to different organizations across the world. Who knows that some of the results obtained using GES may be inaccurate? Those estimates based on the wrong methodology may harm a lot of innocents in the world, and change the Govt. policies or thinking towards certain sensitive estimates such as drug used estimates, abortion estimates, and alcohol consumption estimates, and all other such estimates which have major social implications.

Note that the extension of the present work to the dual frame survey sampling due to Singh and Wu (2003) is in progress by following Singh (2003a, 2004a, 2004b). Tracy and Singh (1999) calibration approach for scrambled responses can also be modified. Just as a reminder, the work of Rao (1994) and hence that of Singh (2001) related to the estimation of distribution function using calibration; and two-phase calibration methodology of Hidiroglou and Sandal (1995, 1998) and hence of Singh (2000) have been repaired by Singh (2004a, 2004b) respectively.

In the same way, the other distance functions also need to be investigated with the new set of constraints considered in the present investigation, which the author is leaving for the readers because of limited resources.

9. Appendix: Simulation study

Under the simulation study, we have considered two types of populations, *viz.* finite populations as well as infinite populations to cover almost all practical types of situations.

(a) Finite populations: We have taken a population consisting of $N = 20$ units from Horvitz and Thompson (1952) where the study variable, y^*, is the number of house holds on i^{th} block and known auxiliary character, x^*, is the eye estimated number of house holds on the i^{th} block. All possible samples of size n were selected by SRSWOR sampling. Then the estimator $\hat{\bar{y}}_{\text{lr}}|_k = \bar{y} + \hat{\beta}_{\text{ols}}(\bar{X} - \bar{x})$ and $\hat{\bar{y}}_{\text{ds}}|_k = \bar{y} + \hat{\beta}_{\text{ds}}(\bar{X} - \bar{x})$ were computed from the k^{th} sample. The percent relative efficiency (RE) of the proposed estimator with respect to Deville and Särndal (1992) estimator was computed as

$$\text{RE} = \sum_{k=1}^{\binom{N}{n}} \left[\hat{\bar{y}}_{\text{ds}}|_k - \bar{Y}\right]^2 \Big/ \sum_{k=1}^{\binom{N}{n}} \left[\hat{\bar{y}}_{\text{lr}}|_k - \bar{Y}\right]^2 \times 100\,. \qquad (\text{A.1})$$

Table 1: Relative Efficiency of the proposed estimator.

Transformations		ρ_{xy}	n	RE
$y=\sqrt{y^*}$	$x=x^*$	0.890	5	403.32
			6	405.55
			7	401.38
$y=y^*$	$x=\sqrt{x^*}$	0.867	5	122.23
			6	129.23
			7	131.77
$y=\log(y^*)$	$x=x^*$	0.897	5	1404.98
			6	1423.30
			7	1412.93
$y=y^*$	$x=\log(x^*)$	0.856	5	138.12
			6	153.12
			7	159.67

Note that the results given in Table 1 are exact and can be reproduced at any time, and show that the relative efficiency of the proposed techniques is really appreciable.

(b) Infinite populations: The size N of these populations is unknown. We generated a pair of n independent random numbers y_i^* and x_i^* (say), $i=1,2,...,n$, from a subroutine VNORM with PHI = 0.7, seed(y) = 13031963, and seed(x) = 19630313 following Bratley, Fox, and Scharge (1983). For fixed $S_y^2=50$ and $S_x^2=50$, we generated variables:

$$y_i=100.0+\sqrt{S_y^2\left(1-\rho_{xy}^2\right)}y_i^*+\rho_{xy}S_y x_i^* \qquad (A.2)$$

and

$$x_i=90.0+S_x x_i^* \qquad (A.3)$$

for different values of the correlation coefficient ρ_{xy}. For the k^{th} sample, the estimators $\hat{\bar{y}}_{lr}|_k=\bar{y}+\hat{\beta}_{ols}\left(\bar{X}-\bar{x}\right)$ and $\hat{\bar{y}}_{ds}|_k=\bar{y}+\hat{\beta}_{ds}\left(\bar{X}-\bar{x}\right)$ were computed. Empirical mean squared error of these estimators were approximated as

$$\text{MSE}\left(\hat{\bar{y}}_{lr}\right)=(15000)^{-1}\sum_{k=1}^{15000}\left[\hat{\bar{y}}_{lr}|_k-\bar{Y}\right]^2 \qquad (A.4)$$

and

$$\text{MSE}\left(\hat{\bar{y}}_{ds}\right)=(15000)^{-1}\sum_{k=1}^{15000}\left[\hat{\bar{y}}_{ds}|_k-\bar{Y}\right]^2 \qquad (A.5)$$

The percent relative efficiency of the proposed estimator with respect to Deville and Särndal (1992) estimator was:

$$\text{RE}=\frac{\text{MSE}\left(\hat{\bar{y}}_{ds}\right)}{\text{MSE}\left(\hat{\bar{y}}_{lr}\right)}\times 100 \qquad (A.6)$$

The results so obtained are presented in Table 2.

Table 2. Relative Efficiency of $\hat{\bar{y}}_{lr}$ with respect to $\hat{\bar{y}}_{ds}$

Sample Size	Correlation coefficient				
	0.1	0.3	0.5	0.7	0.9
25	160.24	141.32	130.69	116.75	108.82
50	161.62	146.60	131.34	119.37	114.45
75	162.58	145.47	128.70	118.63	113.43
100	165.88	145.49	129.80	117.46	113.13

Table 2 illustrates that for moderate sample sizes the relative efficiency of the proposed techniques also remains better than its competitors. From Table 2, we observed that if the value of correlation is high, say 0.9 then as the sample size becomes 50 the relative efficiency is maximum, and as the sample size becomes 100 the relative efficiency decreases, which makes sense because when the sample size approaches infinity then both estimators may be conversing to each other. More simulation study results with large and practicable sample sizes can be had from the author on request via: sarjinder@yahoo.com.

Caution! *Note that the logic of construction of different forms of estimators in the General Estimation System (GES) developed by Statistics Canada with different choice of weights looks misleading, because the linear regression estimator automatically reduces to the ratio estimator if the regression line passes through the origin, and thus the second requirement of choice of weights to form ratio type estimator using the GES becomes ridiculous.*

Acknowledgement: *The author would like to thank God who sent this genius idea to him at the right time of completion of the book, "Advanced Sampling Theory with Applications: How Michael 'selected' Amy". All opinions/results are author's own, and feels it is the best paper in the life.*

References

Arnab, R. and Singh, S. (2003). On estimation of population total using generalized regression predictor. *JSM Proceedings, California, 1-6* (Available of the CD).

Arnab, R. and Singh, S. (2004). A note on the estimation of variance of the generalized linear regression predictor. *Australian and New Zealand J. Statist.*, (Accepted)

Bellhouse, D.R. (1984). A review of optimal designs in survey sampling. *Canadian J. Statist.*, 12, 53--65.

Bratley, P., Fox, B. L. and Schrage, L.E. (1983). *A Guide to Simulation*. Springer, NY.

Brewer, K.R.W. (1999). Cosmetic calibration with unequal probability sampling. *Survey Methodology*, 25(2), 205-212.

Das, A.K. and Tripathi, T.P. (1978). Use of auxiliary information in estimating finite population variance. *Sankhya*, C, 40, 139-148.

Deng, L.Y. and Wu, C.F.J. (1987). Estimation of variance of the regression estimator. *J. Amer. Statist. Assoc.*,82,568-575.

Deville, J.C. and Sarndal, C.E. (1992). Calibration estimators in survey sampling. *J. Amer. Statist. Assoc.*, 87, 376-382.

Estevao, V.M. and Sarndal, C.E. (2003). A new perspective on calibration estimators. *JSM—Survey Research Meth., 1346—1356.*

Farrell, P.J. and Singh, S. (2002). Penalized chi square distance function in survey sampling. *JSM Proceedings NY.* (On the CD).

Farrell, P.J. and Singh, S. (2004). Model-assisted higher order calibration of estimators of variance. *Australian and New Zealand J. Statist.*, (Accepted)

Fisher, R.A. (1915). Frequency distribution of the values of the correlation coefficient in sample from an indefinitely large populations. *Biometrika*, 10, 507—521.

Fisher, R.A. (1922). On the interpretation of chi square from contingency tables. *J. Roy. Staist. Soc.*, 85, 87—94.

Hajek, J. (1981). *Sampling from a Finite Population*. Marcel Dekker, New York.

Hansen, M.H., Hurwitz, W.N. and Madow, W.G. (1953). *Sample survey method and theory.* John Wiley and Sons, NY, 456-464.

Harms, T. and Duchesne, P. (2004a). Calibration estimators for quantiles. *Paper presented at SSC, Montreal.*

Harms, T.N.J. and Duchesne, P. (2004b). Calibration estimation for quantiles. *Paper presented at JSM, Toronto*.

Hidiroglou, M.A. and Särndal, C.E. (1995). Use of auxiliary information for two-phase sampling. *JSM proceedings*, 873-878.

Hidiroglou, M.A. and Särndal, C.E. (1998). Use of auxiliary information for two-phase sampling. *Survey Meth*, 24, 11-20.

Horvitz, D.G. and Thompson, D.J. (1952). A generalisation of sampling without replacement from a finite universe. *J. Amer. Statist. Assoc*., 47, 663-685.

Isaki, C.T. (1983).Variance estimation using auxiliary information. *J. Amer. Statist. Assoc.* ,78, 117-123.

Mahajan, P.K. and Singh, S. (1997). Almost unbiased ratio and product type estimators. *Biom. J.* 39, 509-516.

Patel, P.A. and Chaudhari, R.D. (2003). Estimation of a finite population variance in the presence of auxiliary information. *Statistics in Transition*, 6(4), 533-542.

Pearson, K. (1900). On the criterion that a given system of deviations from the probable, in the case of a correlated system of variables, is such that it can be reasonably supposed to have arisen from random sampling. *Phil. Mag.*, 50, 157-175

Pearson, K. (1922). On the chi square test of goodness of fit. *Biometrika,* 14, 186—418.

Pokropp, F. (2002). Imposed linear structures in conventional sampling theory. *Alligemeines Statist. Archiv,* 86, 333—352.

Rao, J.N.K. (1994). Estimating totals and distribution functions using auxiliary information at the estimation stage. *J. Off. Statist*., 10(2), 153-165.

Royall, R.M. (1970a). On finite population sampling theory under certain linear regression models. *Biometrika*,57, 377--387.

Royall, R.M. (1970b). Finite population sampling: on labels in estimation. *Ann. Math. Statist.,* 41, 1774--1779.

Royall, R M. (1970c). On finite population sampling theory under certain linear regression models. *Biometrika*, 57, 377--387.

Royall, R.M. (1971). Linear regression models in finite population sampling theory. *Foundations of Statist. Infer.* (V.P. Godambe and D.A. Sprott, eds). Holt, Rinehart & Winston, Toronto, 259--274.

Särndal, C.E. (1996). Efficient estimators with simple variance in unequal probability sampling. *J. Amer. Statist. Assoc*., 91, 1289--1300.

Singh, A.C. and Wu, S. (2003). An extension of generalized regression estimator to dual frame surveys. *JSM—Section on Survey Research Methods*, 3911--3918

Singh, S. (2000). Estimation of variance of regression estimator in two-phase sampling. *Calcutta Statist. Assoc. Bull*., 50, 49-63.

Singh, S. (2001). Generalized calibration approach for estimating the variance in survey sampling. *Ann. Ins. Stat. Math.* 53, 404-417.

Singh, S. (2003a). *Advanced sampling theory with applications: How Michael 'Selected' Amy*. Kluwer Academic Publishers, The Netherlands.

Singh, S. (2003b). On Farrell and Singh's penalized chi-square distance function in survey sampling. *SCC Proceedings*, *Halifax* (Available on the CD).

Singh, S. (2004a). Survey statisticians celebrate Golden Jubilee Year–2003 of the linear regression estimator. *Metrika* (Accepted).

Singh, S. (2004b). Repair of two-phase calibration methodology in survey sampling. *Presented at SSC-2004, Montreal (To appear on the CD)*

Singh, S. (2004c). Optimality versus Pseudo-optimality. *Working paper at St. Cloud State University, St. Cloud, MN.*

Singh, S. and Horn, S. (1999). An improved estimator of the variance of the regression estimator. *Biom. J.*, 359—369.

Singh, S. Horn, S. and Yu, F. (1998). Estimation of variance of the general regression estimator: Higher level calibration approach. *Survey Meth.* 24(1), 41-50.

Singh, S., Horn, S., Chowdhury, S. and Yu, F. (1999). Calibration of the estimators of variance. *Austral. & New Zealand J. Statist.*, 41(2) , 199-212

Sitter, R.R. and Wu, C. (2002). Efficient estimation of quadratic finite population functions. *J. Amer. Statist. Assoc*., 97, 535-543

Srivastava, S.K. and Jhajj, S.K (1980). A class of estimators using auxiliary information for estimating finite population variance. *Sankhya*, C, 42,87-96.

Tracy, D.S. and Singh, S. (1999). Calibration estimators for randomized response sampling. *Metron*, 57,47-68.

Tracy, D.S., Singh, S. and Arnab, R. (2003). Note on calibration estimator in stratified and double sampling. *Survey Meth.* 99-106.

Wu, C. (2003). Optimal calibration estimation in survey sampling. *Biometrika*, 90(4), 937—951.

Wu, C. and Luan, Y. (2003). Optimal calibration estimators under two-phase sampling. *J. Off. Statist.* 19(2), 119-131**.**

Wu, C. and Sitter, R.R. (2001). A model-calibration approach to using complete auxiliary information from survey data. *J. Amer. Statist. Assoc.*, 96, 185-193.

Wu, C.F.J. (1985). Variance estimation for combined ratio and combined regression estimators. *J. R. Statist. Soc*., 47, B, 147-154.

-☺- (320)-308--5324