Adaptive Sampling with a Multiple Response Condition

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1. Introduction

Adaptive sampling designs can allow for the efficient sampling of populations whose observations have a rare and clustered structure. They do this by dividing the sampling effort into two stages: an initial stage conducted according to a standard design, and an adaptive stage where additional sampling effort is made near units revealed in the initial stage to be informative. A unit is considered informative if its response satisfies some predetermined condition. Adaptive sampling near informative units can decrease the variance of estimators at relatively low cost for samples drawn from suitable populations. For an overview of adaptive designs see Thompson and Seber (1996).

To date, adaptive designs have been developed where adaptive sampling is triggered by the responses of single units, with units satisfying the condition having their neighboring units sampled. In contrast, this paper will consider designs where the condition is triggered by a sum over the responses of multiple units. Such designs may allow more information to contribute to estimators by more closely sampling the shape of clusters. A variety of adaptive designs have been developed, including the adaptive cluster (Thompson, 1990), adaptive systematic (Thompson, 1991a) and stratified adaptive designs (Thompson, 1991b). This paper will focus on the adaptive cluster and related multiple response condition designs. Related work focusing on point patterns is in Mosquin and Thompson (Preprint).

Considered here is a population of N units, with associated unit responses $y_1, ..., y_N$. Of interest will be estimating the population total $\tau = \sum_{k=1}^{N} y_k$. To instead estimate the population mean $\mu = \frac{\tau}{N}$, the estimators given here for τ can be divided through by N, and estimates of its variance by N^2.

2. The initial stage sample

In the initial stage, n units are randomly sampled with replacement from the population of N units. The selection of these begins with the drawing of unit labels $s_1, ..., s_n$, and then including units $y_{s_1}, ..., y_{s_n}$ in the initial sample.

In addition, units close to the n randomly sampled units will be included in the sample. For each unit $k = 1, ..., N$, there is an associated set of unit labels S_k, where if unit of label s_j is selected on draw j, then all units with labels in S_{s_j} are included in the sample.

The final initial sample consists of the $s_1, ..., s_n$, the $S_{s_1}, ..., S_{s_n}$ and all associated responses $y_j : j \in S_{s_j}, j = 1, ..., n$.

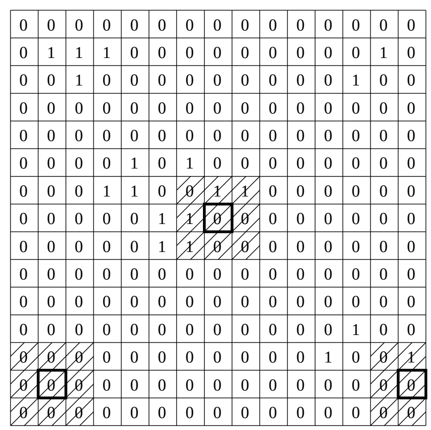


Figure 1: An initial stage sample of size $n = 3$.

To illustrate, one possible initial sample for a population of 225 units is given in figure 1. In this example, a random sample of size $n = 3$ units is first selected, as indicated by units with darkened edges. Label sets of units in this population consist of the unit itself as well as the eight adjacent and diagonally adjacent units. Label sets can thus be of size nine, six, or four, depending on whether the sampled unit lies on the interior, border, or corner of the study region. For the illustrated sample, two of the units are interior units and so have label sets of size nine, while one unit is a border unit with a label set of size six, as indicated by shaded regions.

3. The adaptive stage sample

The designs considered in this paper differ from other adaptive designs by having adaptive sampling triggered by more than one unit. Unit k (or label set S_k), $k = 1, ..., N$ satisfies adaptive condition c if $\sum_{j \in S_k} y_j \geq c$.

The adaptive stage proceeds by iterating:

1. Identify all units in the current sample which satisfy the condition. These are the currently sampled adaptive network units.

2. Sample all units in label sets of non-empty intersection with units adjacent or diagonally adjacent to the adaptive network units.

Iterations continue until no new units are sampled.

The first step then identifies the adaptive network units, while the second step expands the edge region around these units. At the completion of the iterations, the edge region surrounding networks will ensure that all possible randomly selected initial sample units that would lead to inclusion of the adaptive network are in the final sample.

An illustration of this adaptive design is provided in figure 2, where an adaptive stage is added to the initial stage sample of figure 1. For this adaptive stage, a total of two iterations of the above algorithm are required ((a),(b), and (c),(d)) to obtain the final sample.

In the first iteration, two units as indicated by the dark edged region (figure 2 (a)) are identified as satisfying condition $c = 3$. Sampling of all units within label sets intersecting units adjacent or diagonally adjacent to the identified adaptive network units produces the sample of figure 2 (b). Units subsequently identified as satisfying the condition are given as darkened regions of figure 2 (c). Additional sampling of edge region leads to the sample of figures 2 (d), after which no further sampling is possible.

The final sample consists of the initial stage final sample as well as responses and labels of units revealed in the adaptive stage.

Unlike the single response condition adaptive designs, some of the units in network edge regions contribute to the adaptive estimators for designs given here. This occurs since network membership is a property of units which index label sets, whereas responses of all units within a label set contribute to the condition being satisfied. Within figure 2 (d), units within the darkened border contribute to final estimators for the adaptively sampled region.

The above adaptive design is one of two designs given in this paper. Of the two designs, it is most similar to an adaptive cluster design, although does not allow all possible variance estimators because there is insufficient sample information for all required joint inclusion probabilities to be obtained.

Results and proofs are now given concerning Horvitz-Thompson-like estimator $\hat{\tau}$ and the related Hansen-Hurwitz-like estimator $\tilde{\tau}$ for this design.

4. Horvitz-Thompson-like estimator

An unbiased estimator for population total τ is

$$\hat{\tau} = \sum_{k=1}^{N} \frac{y_k I_k}{\pi_k} + \sum_{k \in B} \frac{c_k y_k J_k}{\pi_k^*}$$

where for $k = 1, ..., N$

- I_k indicates unit k present in the initial stage sample in a S_j, $j = \in \{s_1, ..., s_n\}$ where the condition is not satisfied, i.e. $\sum_{i \in S_j} y_i < c$

- π_k is the probability of inclusion if there were no adaptive stage:

$$\pi_k = 1 - \left(1 - \frac{m_k}{N}\right)^n$$

- m_k is the number of label sets containing unit k

- J_k indicates unit k is in a label set that satisfies the condition in the final sample.

- π_k^* is the adaptive inclusion probability of unit k:

$$\pi_k^* = 1 - \left(1 - \frac{m_k^*}{N}\right)^n$$

- m_k^* is the number of label sets containing unit k which satisfy the condition.

- c_k compensates for samples where unit k is sampled non-adaptively and given "inclusion probability" π_k

$$c_k = 1 - \frac{\pi_k^+}{\pi_k}$$

- π_k^+ is the probability that unit k is in the initial sample but did not satisfy the condition:

$$\pi_k^+ = 1 - \left(1 - \frac{m_k^+}{N}\right)^n$$

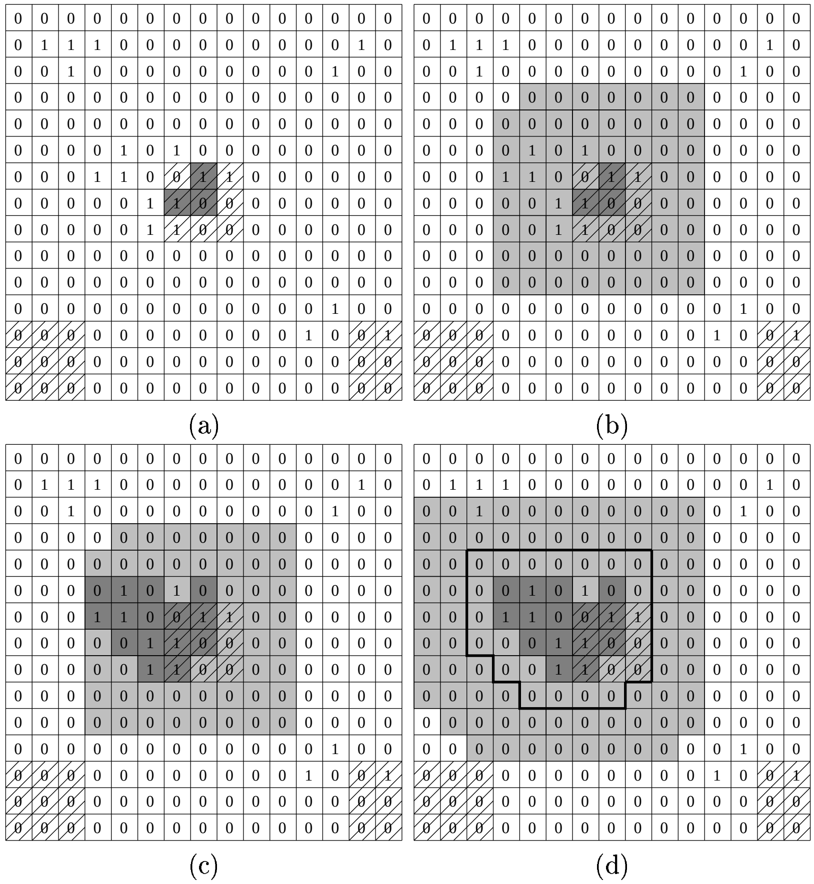


Figure 2: Two iterations ((a),(b) and (c),(d)) of an adaptive stage based on the initial sample of figure 1. Units satisfying the condition $c = 3$ are darkened regions of (a), (c). Units in edge region are lightly shaded regions of (b),(d). Units of the adaptive stage sample contributing to estimators lie within the bold line region.

- m_k^+ is the number of label sets containing k which do not satisfy the condition.

and set B is the union of all label sets satisfying the condition.

Result: Estimator $\hat{\tau}$ is unbiased for τ.

Proof: Partition the unit labels into sets B_1, B_2, and B_3 where for $k = 1, ..., N$

- $k \in B_1$ if all label sets containing k do not satisfy the condition.
- $k \in B_3$ if all label sets containing k satisfy the condition.
- $k \in B_2$ otherwise.

Estimator $\hat{\tau}$ is then unbiased since

$$
\begin{aligned}
E[\hat{\tau}] &= E[\sum_{i=1}^{N} \frac{1}{\pi_i} I_i + \sum_{i \in B_2 \cup B_3} \frac{c_i}{\pi_i^*} J_i y_i] \\
&= \sum_{i \in B_1} \frac{y_i}{\pi_i} E[I_i] \\
&+ \sum_{i \in B_2} \left(\frac{1}{\pi_i} E[I_i] + \frac{1 - \frac{\pi_i^+}{\pi_i}}{\pi_i^*} E[J_i] \right) y_i \\
&+ \sum_{i \in B_3} \left(\frac{1}{\pi_i} E[I_i] + \frac{1 - \frac{\pi_i^+}{\pi_i}}{\pi_i^*} E[J_i] \right) y_i \\
&= \sum_{i \in B_1} y_i + \sum_{i \in B_2} \left(\frac{\pi_i^+}{\pi_i} + 1 - \frac{\pi_i^+}{\pi_i} \right) y_i \\
&+ \sum_{i \in B_3} y_i \\
&= \sum_{i=1}^{N} y_i \\
&= \tau
\end{aligned}
$$

Result: The variance of $\hat{\tau}$ is

$$\begin{aligned} Var[\hat{\tau}] &= \sum_{i=1}^{N} \frac{\pi_i^+(1-\pi_i^+)y_i^2}{\pi_i^2} + \sum_{i\in B} \frac{c_i^2(1-\pi_i^*)y_i^2}{\pi_i^*} \\ &+ \sum_{i=1}^{N}\sum_{k\neq i} \frac{(\pi_{ik}^{++} - \pi_i^+\pi_k^+)y_i y_k}{\pi_i \pi_k} \\ &+ \sum_{i=1}^{N}\sum_{k\in B} \frac{c_k(\pi_{ik}^{+*} - \pi_i^+\pi_k^*)y_i y_k}{\pi_i \pi_k^*} \\ &+ \sum_{i\in B}\sum_{k\in B, k\neq i} \frac{c_i c_k(\pi_{ik}^{**} - \pi_i^*\pi_k^*)y_i y_k}{\pi_i^* \pi_k^*} \end{aligned}$$

where for $i = 1, ..., N,\ k = 1, ..., N$

- π_{ik}^{++} is the joint nonadaptive inclusion probability of units i and k:

$$\pi_{ik}^{++} = 1 - \left(\frac{m_i^+}{N} + \frac{m_k^+}{N} - \frac{m_i^+ + m_k^+}{N}\right)^n$$

- π_{ik}^{+*} is the joint nonadaptive inclusion probability for unit i and adaptive inclusion probability for unit k:

$$\pi_{ik}^{+*} = 1 - \left(\frac{m_i^+}{N} + \frac{m_k^*}{N} - \frac{m_i^+ + m_k^*}{N}\right)^n$$

- π_{ik}^{**} is the joint adaptive inclusion probability for units i and k:

$$\pi_{ik}^{**} = 1 - \left(\frac{m_i^*}{N} + \frac{m_k^*}{N} - \frac{m_i^* + m_k^*}{N}\right)^n$$

Proof:

$$\begin{aligned} Var[\hat{\tau}] &= Var\left[\sum_{i=1}^{N} \frac{y_i}{\pi_i} I_i + \sum_{i\in B} \frac{c_i y_i}{\pi_i^*} J_i\right] \\ &= \sum_{i=1}^{N} \frac{y_i^2}{\pi_i^2} Var[I_i] + \sum_{i\in B} \frac{c_i^2 y_i^2}{(\pi_i^*)^2} Var[J_i] \\ &+ \sum_{i=1}^{N}\sum_{k\neq i} \frac{y_i y_k}{\pi_i \pi_k} Cov[I_i, I_k] \\ &+ \sum_{i=1}^{N}\sum_{k\in B} \frac{y_i c_k y_k}{\pi_i \pi_k^*} Cov[I_i, J_k] \\ &+ \sum_{i\in B}\sum_{k\in B, k\neq i} \frac{c_i c_k y_i y_k}{\pi_i^* \pi_k^*} Cov[J_i, J_k] \end{aligned}$$

where

$$\begin{aligned} Var[I_i] &= \pi_i^+(1-\pi_i^+) \\ Var[J_i] &= \pi_i^*(1-\pi_i^*) \\ Cov[I_i, I_k] &= \pi_{ik}^{++} - \pi_i^+\pi_k^+ \\ Cov[I_i, J_k] &= \pi_{ik}^{+*} - \pi_i^+\pi_k^* \\ Cov[J_i, J_k] &= \pi_{ik}^{**} - \pi_i^*\pi_k^* \end{aligned}$$

Note that the π_{ik}^{+*} and π_{ik}^{++} required to construct a variance estimator are not all known from the final sample.

5. Hansen-Hurwitz-like estimator

The Hansen-Hurwitz-like estimator is

$$\tilde{\tau} = \sum_{k=1}^{N} \frac{f_k^+ y_k}{E[f_k]} + \sum_{k\in B} \frac{c_k f_k^* y_k}{E[f_k^*]}$$

where for $k = 1, .., N$

- f_k is the number of times unit k is included in a sample lacking an adaptive stage.
- f_k^+ is the number of times unit k is in the initial sample within an S_j which does not satisfy the condition.
- f_k^* is the number of times unit k is adaptively sampled over the j draws.

Result: Estimator $\tilde{\tau}$ is unbiased for τ.

Proof: A single draw Horvitz-Thompson-like estimator based on the j^{th} sample unit is

$$\tilde{\tau}_j = \sum_{k=1}^{N} \frac{y_k I_k}{p_k} + \sum_{k\in B} \frac{c_k y_k J_k}{p_k^*}$$

where p_k, p_k^* are single draw versions of π_k,π_k^* given earlier. The sample average of these independent, unbiased estimators over the n sample units is the Hansen-Hurwitz-like estimator.

Result: The variance of the Hansen-Hurwitz-like estimator is

$$\begin{aligned} Var[\tilde{\tau}] &= \frac{1}{n}\left(\sum_{i=1}^{N} \frac{p_i^+(1-p_i^+)y_i^2}{p_i^2}\right. \\ &+ \sum_{i\in B} \frac{c_i^2(1-p_i^*)y_i^2}{p_i^*} \\ &+ \sum_{i=1}^{N}\sum_{k\neq i} \frac{(p_{ik}^{++} - p_i^+ p_k^+)y_i y_k}{p_i p_k} \end{aligned}$$

$$+ \sum_{i=1}^{N} \sum_{k \in B} \frac{c_k (p_{ik}^{+*} - p_i^+ p_k^*) y_i y_k}{p_i p_k^*}$$

$$+ \sum_{i \in B} \sum_{k \in B, k \neq i} \frac{c_i c_k (p_{ik}^{**} - p_i^* p_k^*) y_i y_k}{p_i^* p_k^*} \Bigg)$$

where p_{ik}^{++}, p_{ik}^{+*}, p_{ik}^{**} are the one-draw versions of π_{ik}^{++}, π_{ik}^{+*}, and π_{ik}^{**}.

Proof: Follows since $\tilde{\tau}$ is the sample mean of independent, identically distributed $\tilde{\tau}_j$.

Again, an expression for the variance is available, however an estimator is not available since the required p_{ik}^{+*} and p_{ik}^{**} are not all known.

Result: An unbiased estimator of $Var[\tilde{\tau}]$ is

$$\widehat{Var}[\tilde{\tau}] = \frac{1}{n(n-1)} \sum_{j=1}^{n} (\tilde{\tau}_j - \tilde{\tau})^2$$

Proof: This is simply the sample variance of the $\tilde{\tau}_j$.

6. An alternative adaptive design

A slightly modified adaptive design can be implemented where all randomly selected units in the initial sample sample are treated as if they satisfied the condition. Adaptive sampling is thus initiated near all units with labels in $s_1, ..., s_n$. No further changes are made to the design. If adaptive sampling near an initial sample unit which does not satisfy the condition reveals units which do satisfy the condition, then adaptive sampling proceeds for these units. The additional information provided by this sample design allows estimation of all variances given earlier. The illustrative sample modified for this sampling scheme is shown in figure 3.

Results for these designs are now given, and are largely the same as those for the designs given earlier, except that two additional variance estimators are provided. These variance estimators can be used as required inclusion probabilities are now known for all initial sample units. One effect of the additional information is that $\pi_k = \pi_k^+$ (and hence $c_k = 1$) for $k = 1, ..., N$. Let set A contain labels of units which can be non-adaptively sampled.

Result: An unbiased estimator for population total τ is

$$\hat{\tau}^* = \sum_{k \in A} \frac{y_k I_k}{\pi_k^+} + \sum_{k \in B} \frac{y_k J_k}{\pi_k^*}$$

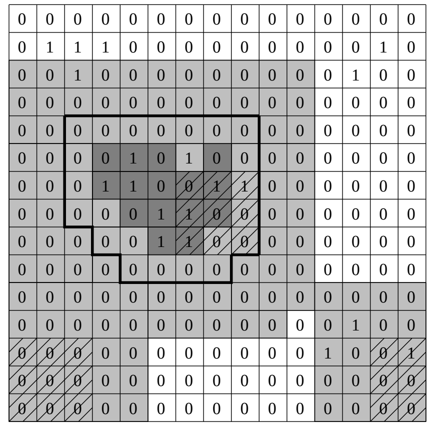


Figure 3: Final adaptive sample for alternative design.

Result: The variance of $\hat{\tau}^*$ is

$$\begin{aligned} Var[\hat{\tau}^*] &= \sum_{i \in A} \frac{(1-\pi_i^+) y_i^2}{(\pi_i^+)^2} + \sum_{i \in B} \frac{(1-\pi_i^*) y_i^2}{\pi_i^*} \\ &+ \sum_{i \in A} \sum_{k \neq i} \frac{(\pi_{ik}^{++} - \pi_i^+ \pi_k^+) y_i y_k}{\pi_i \pi_k} \\ &+ \sum_{i \in A} \sum_{k \in B} \frac{(\pi_{ik}^{+*} - \pi_i^+ \pi_k^*) y_i y_k}{\pi_i \pi_k^*} \\ &+ \sum_{i \in B} \sum_{k \in B, k \neq i} \frac{(\pi_{ik}^{**} - \pi_i^* \pi_k^*) y_i y_k}{\pi_i^* \pi_k^*} \end{aligned}$$

Result: An unbiased estimator of $Var[\hat{\tau}^*]$ for Type-I adaptive samples is

$$\begin{aligned} \widehat{Var}[\hat{\tau}^*] &= \sum_{i \in A} \frac{(1-\pi_i^+) y_i^2}{(\pi_i^+)^2} I_i + \sum_{i \in B} \frac{(1-\pi_i^*) y_i^2}{(\pi_i^*)^2} J_i \\ &+ \sum_{i \in A} \sum_{k \neq i} \frac{(\pi_{ik}^{++} - \pi_i^+ \pi_k^+) y_i y_k}{\pi_i^+ \pi_k^+ \pi_{ik}^{++}} I_{ik}^{++} \\ &+ \sum_{i \in A} \sum_{k \in B} \frac{(\pi_{ik}^{+*} - \pi_i^+ \pi_k^*) y_i y_k}{\pi_i^+ \pi_k^* \pi_{ik}^{+*}} I_{ik}^{+*} \\ &+ \sum_{i \in B} \sum_{k \in B, k \neq i} \frac{(\pi_{ik}^{**} - \pi_i^* \pi_k^*) y_i y_k}{\pi_i^* \pi_k^* \pi_{ik}^{**}} I_{ik}^{**} \end{aligned}$$

where

- I_{ik}^{++} indicates both units i and k non-adaptively sampled.
- I_{ik}^{+*} indicates unit i non-adaptively sampled and unit k adaptively sampled.
- I_{ik}^{**} indicates both units i and k adaptively sampled.

Proof: Result follows since $E[I_{ik}^{++}] = \pi_{ik}^{++}$, $E[I_{ik}^{+*}] = \pi_{ik}^{+*}$, and $E[I_{ik}^{**}] = \pi_{ik}^{**}$.

Result: An unbiased estimator for population total τ is

$$\tilde{\tau}^* = \sum_{k \in A} \frac{f_k^+ y_k}{E[f_k^+]} + \sum_{k \in B} \frac{f_k^* y_k}{E[f_k^*]}$$

Result: The variance of the Hansen-Hurwitz-like estimator is

$$\begin{aligned} Var[\tilde{\tau}^*] &= \frac{1}{n} \left(\sum_{i \in A} \frac{p_i^+(1-p_i^+)y_i^2}{(p_i^+)^2} \right. \\ &+ \sum_{i \in B} \frac{(1-p_i^*)y_i^2}{p_i^*} \\ &+ \sum_{i \in A} \sum_{k \neq i} \frac{(p_{ik}^{++} - p_i^+ p_k^+) y_i y_k}{p_i^+ p_k^+} \\ &+ \sum_{i \in A} \sum_{k \in B} \frac{(p_{ik}^{+*} - p_i^+ p_k^*) y_i y_k}{p_i^+ p_k^*} \\ &+ \left. \sum_{i \in B} \sum_{k \in B, k \neq i} \frac{(p_{ik}^{**} - p_i^* p_k^*) y_i y_k}{p_i^* p_k^*} \right) \end{aligned}$$

Result: An unbiased estimator of $Var[\tilde{\tau}^*]$ for use with Type-I adaptive samples is

$$\begin{aligned} \widehat{Var}[\tilde{\tau}^*] &= \frac{1}{n} \left(\sum_{i \in A} \frac{(1-p_i^+)y_i^2 f_i^+}{(p_i^+)^2} \right. \\ &+ \sum_{i \in B} \frac{(1-p_i^*)y_i^2 f_i^*}{(p_i^*)^2} \\ &+ \sum_{i \in A} \sum_{k \neq i} \frac{(p_{ik}^{++} - p_i^+ p_k^+) y_i y_k f_{ik}^{++}}{p_i^+ p_k^+ p_{ik}^{++}} \\ &+ \sum_{i \in A} \sum_{k \in B} \frac{(p_{ik}^{+*} - p_i^+ p_k^*) y_i y_k f_{ik}^{+*}}{p_i^+ p_k^* p_{ik}^{+*}} \\ &+ \left. \sum_{i \in B} \sum_{k \in B, k \neq i} \frac{(p_{ik}^{**} - p_i^* p_k^*) y_i y_k f_{ik}^{**}}{p_i^* p_k^* p_{ik}^{**}} \right) \end{aligned}$$

where

- f_{ik}^{++} is the number of draws units i and k are both nonadaptively sampled.
- f_{ik}^{+*} is the number of draws units i is nonadaptively sampled and unit k is adaptively sampled.
- f_{ik}^{**} is the number of draws units i and k are both adaptively sampled.

Proof: Result follows since $E[I_{ik}^{++}] = p_{ik}^{++}$, $E[I_{ik}^{+*}] = p_{ik}^{+*}$, and $E[I_{ik}^{**}] = p_{ik}^{**}$.

7. Discussion

This paper has considered adaptive estimators where the adaptive condition is satisfied by the responses of multiple units.

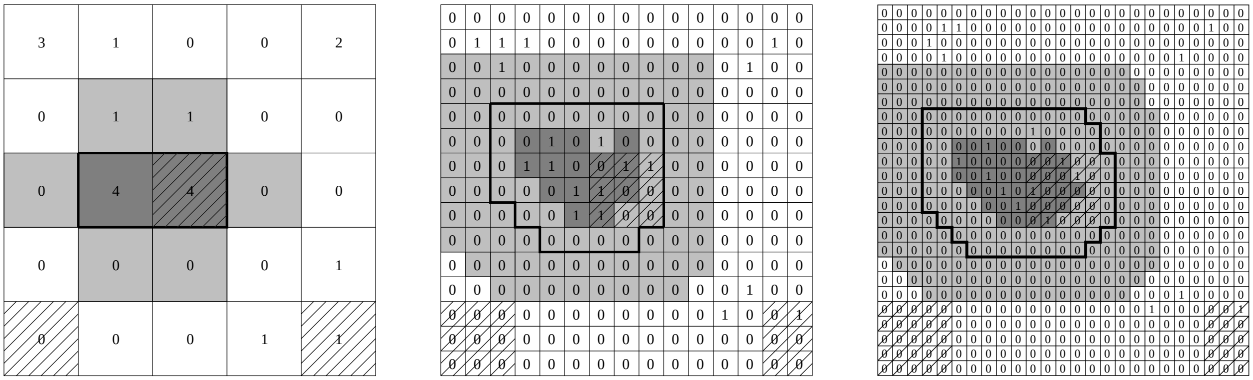
The designs of this paper can be compared to adaptive cluster designs where the condition is satisfied by single units. To simplify the comparison, adaptive cluster design units are given size equal to that of the combined units of a typical label set.

Figure 4 (a) depicts a final adaptive cluster sample of condition $c = 3$ for the illustrative population considered in this paper. At the larger sample unit size, the population consists of 25 sample units with the response for each unit being the sum of the responses over the units for the multiple response design. An initial sample of size $n = 3$ was collected, with two of the units not satisfying the condition, and the remaining unit allowing the adaptive sampling of a network of size 2 units. For the given sample, four units would contribute to estimators (shaded darkly), and six units would be considered edge units. If each unit were divided into nine smaller units, as for the multiple response condition design, then the sampled network would have 18 network units with summed responses equal to 7, and 54 edge units.

Figure 4 (b) depicts the final adaptive sample for the design and example of this paper. The 15 darkly shaded units comprise an adaptive network. The 38 units in the bold-lined region contribute to the estimator with summed responses equal to 10. The adaptively sampled region consists of a total of 106 units.

Figure 4 (c) depicts the final adaptive sample for the design of this paper if unit size were reduced further to 25 units for each unit of the adaptive cluster design in figure 4 (a). For this smaller unit size, and again with a condition $c = 3$ and comparable label sets (i.e., typically of size 25) the darkly-shaded adaptive network contains 40 units. The 117 units in the bold lined region contribute to the estimator with summed responses equal to 10. The adaptively sampled region consists of a total of 291 units.

Measuring area in adaptive cluster units, the final sample of 4 (b) has 11.78 units of adaptively sampled area, a network area of 1.66 units, a total of 4.22 units contributing to estimators, and 7.55 units of edge region. The final sample of 4 (c) has 11.64



(a) (b) (c)

Figure 4: Final adaptive samples of condition $c = 3$ for (a) an adaptive cluster design, (b) a multiple response condition adaptive cluster design with units of 1/9 size, and (c) a multiple response condition adaptive cluster design with units of 1/25 size.

units of adaptively sampled area, a network area of 1.60 units, a total of 4.76 units contributing to estimators, and 6.96 units of edge region.

Comparing the adaptive cluster sample with the other samples it appears that the adaptive cluster design may less fully sample network regions, while also sampling less edge region. Considering only the other samples, it appears that there may be benefits to a decrease in unit size. Simulations are currently underway to allow more definitive statements comparing designs as well as estimator efficiencies.

The designs of this paper have assumed initial stage sampling with replacement. If sampling without replacement were used, then sample units could be partitioned into non-intersecting label sets (for example of size nine like in one adaptive cluster unit of figure 4 (a)). Doing so has the consequence that units can only be either adaptively or non-adaptively sampled. Thus sets A and B of the former sums are disjoint and their union contains all population labels. The estimators reduce to standard adaptive cluster estimators. Any additional information due the smaller unit size (compared to the adaptive cluster design) is lost, as all units within a label set have the same inclusion probabilities.

As with the adaptive cluster designs, it is possible for more than one initial sample to lead to the same final sample, and so the estimators given here are amenable to Rao-Blackwellization. With an adaptive cluster design, this can be done either when an initial sample unit is selected more than once or when more than one initial sample unit lies within a cluster (network plus edge region). For the designs of this paper, Rao-Blackwellization is possible provided that an adaptive network is found. For example, in figure 4 (b), there are 15 ways to obtain the same final sample, not all with the same estimate.

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