Monte Carlo Study on the Successive Difference Replication Method for Non-Linear Statistics

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Abstract:

Replication methods for variance estimation with complex survey data have been widely used in practice. The Successive Difference Replication Method (SDRM) is a replication method for variance estimation developed based on a systematic sample. Even though this method has been consistently used in several large government surveys; however, only a few literatures about the SDRM are available.

The SDRM variance estimator was developed by Robert E. Fay based on the variance for a systematic sample proposed by Wolter (Wolter 1984). The basic theoretical development of the SDRM (Wolter 1984, Fay and Train 1995) was described in terms of linear estimators (simple mean and total). Here, our work focuses on the use of SDRM for non- linear statistics, such as ratio, correlation coefficient, regression coefficient, and median. The SDRM variance estimates are evaluated against the variance estimates computed through the Taylor Series method and the Jackknife method using Monte Carlo simulation under several different stratified populations. An empirical example is given using the 1993 National Survey of College Graduate (NSCG) data.

1. Introduction

The SDRM variance estimator is one of the replication methods of variance estimator that so far has been successfully applied by federal government statistical agencies, such as the U.S. Bureau of Census. The Bureau widely uses the SDRM variance estimation for large scale surveys such as the National Survey of College Graduate (NSCG), the Current Population Survey (CPS), and the 2000 Census - Long Form survey.

This method was developed by Robert E. Fay (Fay and Train 1995) based on the variance estimator proposed by Wolter (Wolter 1984) for a systematic sample, where the variability is calculated based on the squared differences between neighbouring samples. The SDRM variance estimator utilizes orthogonal Hadamard matrix to create the replicates. Fay and Train (1995) showed that the estimator is analytically equivalent to the original variance calculated through differencing neighbouring samples. Fay and Train (1995) presented the development of this method based on a linear estimator. In this paper we will extend the use of this method for non-linear estimators.

The SDRM variance estimator has not much been studied especially for complex statistics, neither has it been compared with other replication variance estimators. This paper focuses on such comparison particularly for estimating non-linear estimators, such as ratio estimator, correlation coefficient, regression coefficient, and medians. Using Monte Carlo studies, the variance estimates computed through the SDRM will be compared with the variance estimates computed through the Jackknife method. In the comparison we will also include variance estimates calculated through the standard linearization (Taylor Series) method.

Section 2 will present theoretical background of the SDRM variance estimator, and describe how the replicates are constructed based on a Hadamard matrix. Section 3 presents an empirical comparison of the SDRM variance estimates with the variance estimates computed through the Taylor Series and Jackknife methods using the 1993 NSCG data. Section 4 describes the simulation setting, i.e. the finite pseudo-populations generated, the sampling design used, and the statistics under study; as well as the results. Section 5 summarizes the study.

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2. The Successive Difference Replication Method

One of the variance estimators proposed by Wolter (1984) for estimating variance of a mean estimator, $\overline{y} = (1/n)\sum y_i$, from a systematic sample is the variance estimator based on differences of two consecutive observations as

$$v_2(\overline{y}) = (1-f)(1/n)\sum_{i=2}^{n}(y_i - y_{i-1})^2/2(n-1)$$

where f is the sampling fraction n/N. Even though this estimator is not unbiased, the estimator performs reasonably well for different systematic samples drawn from several different population models. Similar estimator for a total $\hat{y} = \sum_{i=1}^{n} y_i^*$, where $y_i^* = (N/n)y_i$, can be expressed as

$$v_2(\hat{y}) = (1-f)n\sum_{i=2}^{n}(y_i^* - y_{i-1}^*)^2/2(n-1).$$

Notice that the estimator is computed based on $(n-1)$ pairs of neighboring samples.

Fay and Train (1995) modified this estimator by adding a difference between the first and the last observations, i.e.

$$v_{2m}(\hat{y}) = (1/2)(1-f)\left[(y_n^* - y_1^*)^2 + \sum_{i=2}^{n}(y_i^* - y_{i-1}^*)^2\right]$$

and showed that a replication version of v_{2m} can be developed, which will be explained in the following sections. This replication variance estimator is then called the SDRM variance estimator.

2.1 SDRM Variance Estimator

The SDRM utilizes a $k \times k$ Hadamard matrix H to create the replicates, where in this case k is the number of replicates used in a SDRM. Let $H = \{a_{ir}\}$ be a Hadamard matrix of order k where k is an integer multiplication of 4, and for now suppose $k \geq n+2$.

For $i < n$, the replicate factor f_{ir} is defined as

$$f_{ir} = 1 + 2^{-3/2}a_{i+1,r} - 2^{-3/2}a_{i+2,r},$$

and for $i = n$

$$f_{nr} = 1 + 2^{-3/2}a_{n+1,r} - 2^{-3/2}a_{2,r}.$$

Let the rth replicate estimate of $\hat{y}$ be calculated as

$$\hat{y}_r = \sum_{i=1}^{n} f_{ir}y_i^*.$$

The SDRM variance estimator is calculated as

$$v_{r2m}(\hat{y}) = (4/k)(1-f)\sum_{r=1}^{k}(\hat{y}_r - \hat{y})^2.$$

Fay and Train (1995) showed that this replicate variance estimator is equivalent to $v_{2m}(\hat{y})$. Fay and Train claimed that even though this estimator is not unbiased for simple random samples the bias is relatively small, even for relatively small n.

Without loss of generality, under a complex sample design the SDRM variance estimator for $\hat{\theta}$ can be defined as (ignoring the finite population correction factor)

$$v_{r2m}(\hat{\theta}) = (4/k)\sum_{r=1}^{k}(\hat{\theta}_r - \hat{\theta})^2$$

where $\hat{\theta}_r$ is the point estimator calculated from the rth replicate. The formula can be extended to include the finite population correction factor.

2.2 Construction of the Replicates

The replicates are constructed by first assigning pairs of rows in H to each sample case, that is started from assigning row 2 and 3 to the first observation, then assigning rows 3 and 4 to the second observation, and so on until assigning rows $n+1$ and $n+2$ of H to the last observation. This assignment gives a unique set of pairs of rows in H. However, when n is large and much greater than k, the assignment will run out of rows of H before all observations can be uniquely paired with two-rows in H. To solve this problem, SDRM implements a row assignment algorithm as follows.

Since the original variance estimator is calculated based on the differences between two successive sample cases, prior to rows assignment, sample cases must be sorted in a manner similar to the order of the cases at the time of sampling. Then, choose an H of order k such that $(k-1)$ is a prime number. Never use/assign row 1 in H because in most Hadamard matrix it contains all +1's. We start by assigning rows 2 and 3 of H to the first sample case. Then, assign rows 3 and 4 to the second sample case, assign rows 4 and 5 to the third sample case, $\cdots$, and so on until we reach row k. This two-rows assignment from row 2 to k is called a cycle. The first cycle is done with row increment of 1, i.e. assigning rows 2 and 3, rows 3 and 4, rows 4 and 5, $\cdots$, and so on. If $n > k$, at this point the assignment is started over to the second row (again skip the first row), i.e. assigning rows k and 2 to the next sample case, then repeat the cycle but now with row increment 2, i.e. assigning rows 2 and 4, rows 4 and 6, rows 6 and 8, $\cdots$, and so on. The cycle with row increment 2 should be done twice. The cycle with row increment 3 are done three times, the cycle with row increment 4 are done four times, $\cdots$ and so on until all sample cases are paired with two rows in H. The row increment and cycles are re-set back to 1 after we reach increment 10 (Finamore 2002).

The rth column in H corresponds to the rth replicate of SDRM. For the ith sample case ($i = 1, \cdots, n$) and the rth replicate ($r = 1, \cdots, k$) calculate the replicate factor, f_{ir},

$$f_{ir} = 1 + [2^{-3/2} h_{(R1i,r)}] - [2^{-3/2} h_{(R2i,r)}],$$

where subscripts $R1i$ and $R2i$, respectively, denote the row number for the first and second rows from the assignment described above for the ith case; $h_{(R1i,r)}$ is the entry of the $(R1i, r)$th cell of H assigned to sample case i; and similarly $h_{(R2i,r)}$ is the entry of the $(R2i, r)$th cell of H assigned to sample case i. Since the value of $(h_{(R1i,r)}, h_{(R2i,r)})$ will be one of $(1, 1), (-1, 1), (1, -1), (-1, -1)$, then the value of f_{ir} will be one of three values: 0.3, 1.0, 1.7. By the end of this process, each sample case will have k replicate factors. An SDRM replicate is then produced by multiplying y_i with f_{ir}.

3. The 1993 National Survey of College Graduate

The 1993 NSCG is an NSF survey conducted by the U.S. Bureau of Census. The target population of this survey consists of scientists and engineers with at least a bachelor's degree who (as of April 1, 1990) were age 72 or younger. Sampling frame was constructed from the 1990 Decennial Census Long Form sample.

The survey implemented a two-phase stratified random sampling, where at the first phase, sample of the Long Forms was drawn using a stratified systematic sampling, and at the second phase, subsample of the Long Form cases was selected through a stratified design with probability-proportional-to-size systematic selection within strata. For variance estimation the Census Bureau created 160 replicate weights through SDRM.

Using some of the 1993 NSCG survey variables we compared variance estimates of totals, ratios (weighted means/proportions), and medians computed through the SDRM with those computed through the Taylor Series method and the Jackknife method. For the Taylor Series estimation a single phase stratified sample design is assumed, where stratification variable used is the original sampling strata used for the 1993 NSCG. For the Jackknife method, the original 1993 NSCG sample design is approximated by a two-PSUs per stratum design. A Jackknife replicate was produced by randomly deleting one PSU within a stratum. For both the SDRM and Jackknife replicate weights, the weights have been adjusted for nonresponse and poststratification.

The statistics being estimated are listed in Table 1. For each of these statistics we calculated the point estimates and the variance estimates through the Taylor Series, Jackknife, and Successive Difference Replication methods. We then compared the standard errors (i.e. square-root of variances) by calculating the relative differences defined as

$$\frac{\text{stderr}_i - \text{stderr}_{\text{base}}}{\text{stderr}_{\text{base}}} \times 100\%$$

where the Taylor Series standard error was used as the baseline. The results of relative differences are visualized in Figure 1 for (a) total estimator, (b) ratio estimator, and (c) median.

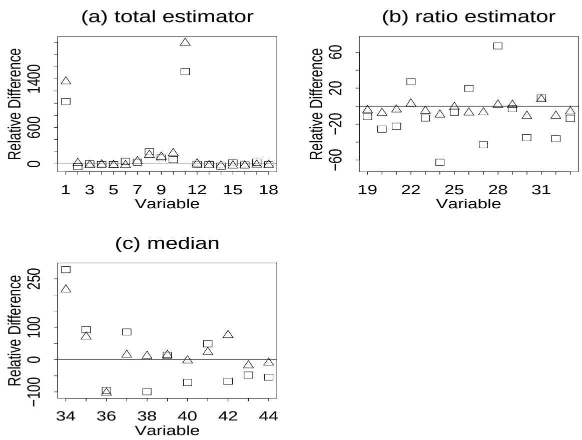

Figure 1: The relative differences of the 1993 NSCG standard error estimates. Taylor Series (TS) was used as the baseline. Legend: □ = Jackknife, △ = SDRM

Based on the 1993 NSCG data, for the total estimation (figure (a)) the three variance estimation methods agree to each other except for variable number 1 (grand total) and 11 (total non-science). For the ratio estimation (figure (b)), both SDRM and Jackknife method produce either smaller or larger standard errors than does the Taylor Series method, with the standard error of the SDRM tends to be closer to those of Taylor Series. For the median

Table 1: Variable Numbers and Labels for the Total, Ratio, and Median Estimators

Total		Ratio		Median	
No.	Variable	No.	Variable	No.	Variable
1	All Level	19	Prop.Unempl, Look for Job	34	Salary All Level
2	Bachelor	20	Prop.Empl, Job-Edu Smwhat Relat	35	Salary Bach.
3	Master	21	Prop.Empl, Job-Edu Related	36	Salary Master
4	Ph.D.	22	Prop.Empl, Job-Edu Not Related	37	Salary Ph.D.
5	Prof. Degree	23	Mean Salary, All Level	38	Salary Prof. Deg.
6	Other Degree	24	Mean Salary, Bach.	39	Salary Other Deg.
7	Physical Science	25	Mean Salary, Master	40	Salary Physical Sci.
8	Math/Comp Science	26	Mean Salary, Ph.D.	41	Salary Math/Comp Sci.
9	Social Science	27	Mean Salary, Prof. Deg.	42	Salary Social Science
10	Engineering	28	Mean Salary, Other Deg.	43	Salary Engineering
11	Non-Science	29	Mean Salary, Physical Science	44	Salary Non-Science
12	Employed	30	Mean Salary, Math/Comp. Scien		
13	Unemployed	31	Mean Salary, Social Science		
14	Unemployed Bach.	32	Mean Salary, Engineering		
15	Unemployed Master	33	Mean Salary, Non-Science		
16	Unemployed Ph.D.				
17	Unemployed Prof. Deg.				
18	Unemployed Other Deg.				

estimation (figure (c)), both SDRM and Jackknife method do not agree with the Taylor Series method. However, there is a tendency that the SDRM and Jackknife method produce about similar standard errors.

4. Monte Carlo Study

To further study the performance of the SDRM for nonlinear estimators and median, we carried out simulation study under several different hypothetical populations. For each population, which will be explained in the next sections, we replicate the sample 1,000 times. We compared the SDRM variance estimator to the Taylor Series and the Jackknife variance estimators based on the population used by Kovar (1985, 1987).

4.1 Hypothetical Populations and Samples

For this simulation we used 30 stratified populations as described in Kovar (1985, 1987). Within each population we have 32 strata and two variables (X, Y), where within stratum $h; h = 1, \cdots, 32$, the (X_h, Y_h) is assumed to be distributed as bivariate normal with parameters $(\mu_{Xh}, \mu_{Yh}, \sigma_{Xh}, \sigma_{Yh}, \rho_{XY})$. The 30 populations are distinguished by the different values of these parameters and the stratum weight $W_h = N_h/N$. For all populations, within each stratum then we generate sample of size two.

4.2 Parameters of Interest and the Estimators

The parameters being estimated are defined as follows: (R = ratio, B = regression coefficient, C = correlation coefficient, M = median)

$$
\begin{aligned}
R &= \mu_y/\mu_x = (\sum_{h=1}^{32} W_h \mu_{yh})/(\sum_{h=1}^{32} W_h \mu_{xh}) \\
B &= \frac{\sum_{h=1}^{32} W_h[\rho\sigma_{xh}\sigma_{yh} + (\mu_{xh}-\mu_x)(\mu_{yh}-\mu_y)]}{\sum_{h=1}^{32} W_h[\sigma_{xh}^2 + (\mu_{xh}-\mu_x)^2]} \\
C &= \frac{\sum_{h=1}^{32} W_h[\rho\sigma_{xh}\sigma_{yh} + (\mu_{xh}-\mu_x)(\mu_{yh}-\mu_y)]}{\{S_{xx}\cdot S_{yy}\}^{1/2}} \\
M &= F^{-1}(1/2)
\end{aligned}
$$

where

$$
\begin{aligned}
S_{xx} &= \sum_{h=1}^{32} W_h[\sigma_{xh}^2 + (\mu_{xh}-\mu_x)^2] \\
S_{yy} &= \sum_{h=1}^{32} W_h[\sigma_{yh}^2 + (\mu_{yh}-\mu_y)^2] \\
F(t) &= \sum_{h=1}^{32} W_h F_h(t),
\end{aligned}
$$

and $F_h(t) = P(X_h < t)$ is the distribution function of X_h.

Let statistics calculated from a stratified sample be defined as follows

$$\begin{aligned}
\bar{t}_1 &= \sum_{h=1}^{32} W_h \sum_{i=1}^{2} x_{hi}/2 \\
\bar{t}_2 &= \sum_{h=1}^{32} W_h \sum_{i=1}^{2} y_{hi}/2 \\
\bar{t}_3 &= \sum_{h=1}^{32} W_h \sum_{i=1}^{2} x_{hi}y_{hi}/2 \\
\bar{t}_4 &= \sum_{h=1}^{32} W_h \sum_{i=1}^{2} x_{hi}^2/2 \\
\bar{t}_5 &= \sum_{h=1}^{32} W_h \sum_{i=1}^{2} y_{hi}^2/2.
\end{aligned}$$

Then the estimates of R, B, C, M are calculated respectively by

$$\begin{aligned}
\widehat{R} &= \bar{t}_2/\bar{t}_1 \\
\widehat{B} &= (\bar{t}_3 - \bar{t}_1\bar{t}_2)/(\bar{t}_4 - \bar{t}_1^2) \\
\widehat{C} &= (\bar{t}_3 - \bar{t}_1\bar{t}_2)/[(\bar{t}_4 - \bar{t}_1^2)(\bar{t}_5 - \bar{t}_2^2)]^{1/2} \\
\widehat{M} &= F_n^{-1}(1/2)
\end{aligned}$$

where $F_n(t) = \sum_{h=1}^{32} W_h F_{nh}(t)$ and $F_{nh}(t) = \sum_{i=1}^{2} I(x_{hi} \leq t)/2$.

4.3 Variance Estimators

Let $\widehat{\theta}$ denotes the point estimate of interest computed from the full sample (in this work $\widehat{\theta} = \widehat{R}, \widehat{B}, \widehat{C}, \widehat{M}$). For $\widehat{R}, \widehat{B}, \widehat{C}$, the Taylor Series variance estimator is respectively given as follow:

$$v_T(\widehat{\theta}) = \sum_{h=1}^{32} W_h^2 \sum_{i=1}^{2} (e_{hi} - \bar{e}_h)^2/2$$

where

$$\begin{aligned}
e_{hi}(\widehat{R}) &= (y_{hi} - \widehat{R}x_{hi})/\bar{t}_1 \\
e_{hi}(\widehat{B}) &= [(2\bar{t}_1\widehat{B} - \bar{t}_2)x_{hi} - \bar{t}_1 y_{hi} + x_{hi}y_{hi} \\
&\quad -\widehat{B}x_{hi}^2]/s_x^2 \\
e_{hi}(\widehat{C}) &= [(\widehat{C}\bar{t}_1 s_y/s_x - \bar{t}_2)x_{hi} + (\widehat{C}\bar{t}_2 s_x/s_y - \bar{t}_1)y_{hi} \\
&\quad +x_{hi}y_{hi} - (\widehat{C}s_y/2s_x)x_{hi}^2 - (\widehat{C}s_x/2s_y)y_{hi}^2 \\
&\quad -(\widehat{C}s_x/2s_y)y_{hi}^2]/s_x s_y
\end{aligned}$$

where $s_x^2 = \bar{t}_4 - \bar{t}_1^2$ and $s_y^2 = \bar{t}_5 - \bar{t}_2^2$. For the median, we used Woodruff (1952) variance estimator obtained by inverting the $(1-\alpha)100\%$ confidence interval for $\widehat{M}$, i.e.

$$v_T(\widehat{M}) = \left\{ \frac{F_n^{-1}(.5 + z_{\alpha/2}s_p) - F_n^{-1}(.5 - z_{\alpha/2}s_p)}{2z_{\alpha/2}} \right\}^2$$

where $s_p^2 = \sum_{h=1}^{32} W_h^2 F_{nh}(.5)[1 - F_{nh}(.5)]/(n_h - 1)$, and $z_{\alpha/2}$ is the $(1-\alpha/2)$ quantile of standard normal distribution. In this simulation we used $\alpha = 0.05$.

In this study a Jackknife replicate was created based on deleting the second observation within a stratum. Let $\widehat{\theta}_k$ denotes the corresponding point estimate calculated from the kth replicate. The Jackknife variance estimator is defined as

$$v_J(\widehat{\theta}) = \sum_{k=1}^{32} (\widehat{\theta}_k - \widehat{\theta})^2.$$

For the SDRM we chose a Hadamard matrix of order (32×32) to create the 32 SDRM replicates. This matrix can be found in Wolter (1985). The SDRM variance estimator is given as follow:

$$v_S(\widehat{\theta}) = (4/32) \sum_{k=1}^{32} (\widehat{\theta}_k - \widehat{\theta})^2.$$

4.4 Evaluation Methods

The simulation was carried out with 1,000 replications. The mean squares error (MSE) of the estimator $\widehat{\theta}$ is computed as

$$MSE(\widehat{\theta}) = \sum_{r=1}^{1000} (\widehat{\theta}_r - \theta)^2/1000.$$

For the rth replicate ($r = 1, ...1000$), we then calculated the variance estimates under the jth method ($j = 1, 2, 3$), $v_{jr}(\widehat{\theta})$, as given in Section 4.3. To study the bias and stability of the jth variance estimators, respectively we used relative variance and relative stability as follows

$$\begin{aligned}
\textit{Rel. Var}(v_j) &= \frac{\sum_{r=1}^{1000} v_{jr}/1000}{MSE}, \\
\textit{Stab}(v_j) &= \frac{[\sum_{r=1}^{1000} (v_{jr} - MSE)^2/1000]^{1/2}}{MSE}.
\end{aligned}$$

A value of relative variance equal to or close to 1 indicates that the variance estimator is less unbiased. A relative variance value greater than 1 indicates that the variance estimator is positively bias. On the other hand, a value smaller than 1 indicates that the variance estimator is negatively bias.

The value of relative stability is calculated as a variability of the variance estimates to the MSE. Thus, the smaller the value, the more stable the variance estimator.

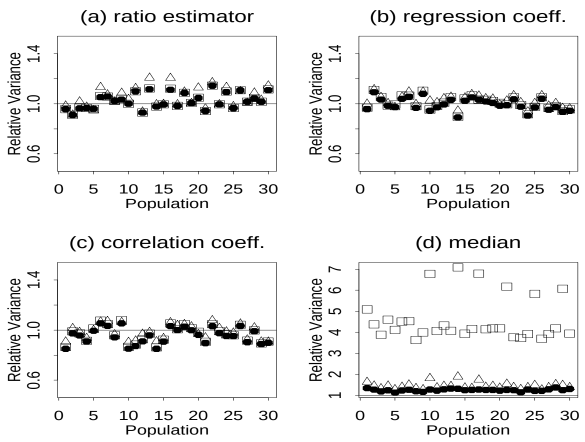

Figure 2: The relative variance for (a) ratio, (b) regression, (c) correlation, (d) median, for the 30 populations. Legend: ● = TS, □ = Jackknife, △ = SDRM

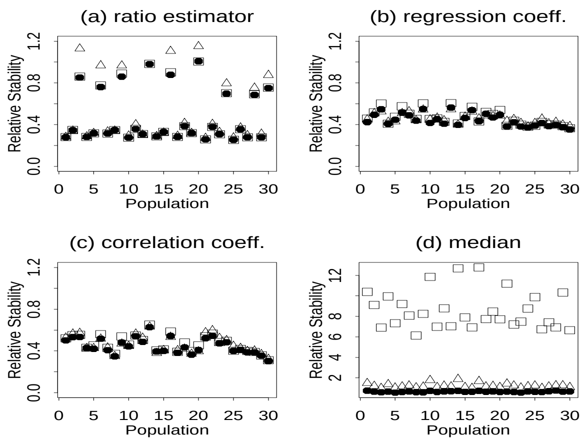

Figure 3: The relative stability for (a) ratio, (b) regression, (c) correlation, (d) median, for the 30 populations. Legend: ● = TS, □ = Jackknife, △ = SDRM

4.5 Simulation Results

Figure 2 presents the relative variance for (a) ratio estimator, (b) regression coefficient, (c) correlation coefficient, and (d) median, for each of the 30 populations. The horizontal line across the y-axis at point 1.0 can be used as a baseline to indicate the biasness of the variance estimator. The closer the relative variance to this line, the more unbiased is the estimator.

Figure 3 presents the relative stability for (a) ratio estimator, (b) regression coefficient, (c) correlation coefficient, and (d) median, for each of the 30 populations.

Based on the relative variance and relative stability, for non-linear statistics such as the ratio estimator (figure (a)), regression coefficient (figure (b)), and the correlation coefficient (figure (c)), the three methods seem to agree to each other. It is known that the Jackknife and Taylor Series variance estimators are consistent and asymptotically equivalent for linear and smooth non-linear statistics (Shao 1996). Thus based on our limited simulation, for these statistics the SDRM variance estimator is comparable to the Jackknife and Taylor Series estimators.

For the median estimator, there is a substantial variation between the SDRM and the Jackknife, with the SDRM is closer to the Taylor Series than the Jackknife. The SDRM is comparable to the Taylor Series method; while the Jackknife method produces positive bias and unstable variance estimates.

5. Conclusion

The Taylor Series, Jackknife and the Successive Difference Replication methods perform equivalently for the variance of total estimator. The SDRM for non-linear statistics is comparable to the Taylor Series and Jackknife methods. Moreover, the SDRM for a non-smoth function such as median is comparable to the Taylor Series method.

This study was based on a specific finite population and/or a limited setting of the simulation. Different setting or finite population may result in different conclusions. Further investigation under different simulation setting (sample design, underlying population, etc.) need to be carried out to get a complete understanding of the performance of the Successive Difference Replication Method. Theoretical analytical investigation can also give a clear picture of this method.

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