Loss Function Analysis for A.C.E. Revision II Estimates of Census 2000 Coverage Error

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1. BACKGROUND

This paper discusses the use of confidence intervals and loss function analyses to evaluate the Census Bureau's revised estimates of coverage error in Census 2000 from the Accuracy and Coverage Evaluation Survey or A.C.E.(U. S. Census Bureau 2003). The original A.C.E. estimates in March 2001 indicated a 1.18 percent undercount in the Census 2000 population size of 281,421,906. The Census Bureau discovered that undetected duplicate enumerations in the census were a major source of error in the A.C.E. estimates and in October 2001 produced the A.C.E. Revision Preliminary estimates, which indicated the net undercount was 0.06 percent (Thompson, Waite, Fay 2001, Mule 2002). The latter estimates included adjustments to account for duplicate census enumerations and other enumeration sample (E-sample) measurement errors detected by the Measurement Error Reinterview (Raglin and Kresja 2001) and the Matching Error Study (Bean 2001). Subsequently, the Census Bureau developed the A.C.E. Revision II estimates, which included an adjustment for correlation bias and improved adjustments for measurement error in the E- sample and in the population sample (P-sample) and produced a revised estimate of -0.49 percent undercount.

The A.C.E. Revision II estimates are subject to both nonsampling error and sampling error. Two methods of summarizing the relative accuracy of the Census and the A.C.E. Revision II are confidence intervals for the net undercount rate and loss function analyses that estimate the overall difference in accuracy between the A.C.E. estimates and the unadjusted census estimates of population size (or level) and population shares. We form the confidence intervals for net undercount rate using estimates of variance and net bias for the census coverage correction factors. In the loss function analysis, we estimate loss by the weighted Mean Squared Error (MSE), with the weight of the reciprocal of the census count for levels and the reciprocal of the census share for shares. We estimate the aggregate loss for levels and shares for states, counties, and places across the nation and for counties and places within state.

These methods for evaluating the accuracy of the census and an adjustment of the census have been used previously (Mulry and Spencer 1993, 2001; CAPE 1992).

2. METHODOLOGY

The construction of the confidence intervals incorporates both sampling and nonsampling error. Since A.C.E. Revision II incorporates most of the data available on the biases in the original A.C.E., the remaining components of bias are relatively few and include error due to inconsistent reporting of variables used in poststratification (Bench 2002), error due to using the inmovers to represent the movers in the PES-C formulation of the dual system estimator (DSE) (Keathley 2002), and error in the identification of duplicate enumerations in the census as measured by administrative records (Bean and Bauder 2002). No estimate of bias from imperfect estimates of correlation bias was available for use. The estimate of the variance in A.C.E. Revision II used three error components, each represented by replicates (Mulry and ZuWallack 2002). These components are the sampling error, the error due to the choice of the missing data model (Kearney 2002), and the error due to the choice of model for correcting for P-Sample cases that matched census enumerations outside the A.C.E. search area, which was based on the model selected and three alternative models (Davis 2002). Note that the sampling error component includes the contribution of variance from adjustment of the DSE for estimates of measurement error in the A.C.E.

2.1 Confidence Intervals

Confidence intervals that incorporate the net bias as well as the variance for the net undercount rate $\hat{U}$ provide a method for comparing the relative accuracy of the census and the A.C.E. Revision II estimates. We construct the intervals by estimating the net bias and

[1]This paper reports the results of research and analysis undertaken by Census Bureau staff. It has undergone a review more limited in scope than that given to official Census Bureau publications. This report is released to inform interested parties of research and to encourage discussion of work in progress.

variance in the census coverage correction factor for each poststratum. Then we can estimate the bias $\hat{B}$ and variance $\hat{V}$ in the net undercount rate $\hat{U}$ and form the 95% confidence interval for the net undercount rate for a poststratum or a group of poststrata by

$$(\hat{U} - \hat{B} - 2\sqrt{\hat{V}},\ \hat{U} - \hat{B} + 2\sqrt{\hat{V}}).$$

Since $\hat{U} = 0$ corresponds to no adjustment of the census, one comparison of the relative accuracy of the census and the A.C.E. Revision II estimates is based on an assessment of whether the confidence intervals for the evaluation poststrata cover 0 and $\hat{U}$. (See section 2.3 for limitations in the scope of $\hat{V}$ and $\hat{B}$.)

2.2 Loss Functions

The loss function analysis uses the estimated bias and variance to estimate an aggregate expected loss for the census and the A.C.E. Revision II for population levels and shares for counties and places across the nation and within state. The loss function is the weighted squared error, which also may be described as the weighted Mean Squared Error (MSE). The weight for both the census loss and the A.C.E. Revision II loss calculation is the reciprocal of the census count (for levels) or the reciprocal of the census share (for shares). The motivation for the selection of the groupings of areas for the loss functions was the potential use of the coverage error estimates in the postcensal estimates program.

2.3 Limitations

The estimated bias $\hat{B}$ does not account for all the sources of bias and may fail to accurately estimate the nonsampling biases that were included. For example, this could be a problem for some of the estimates derived from Census and Administrative Records Study (CARDS) (Bean and Bauder 2002). There were discrepancies between CARDS and another evaluation of the identification of census duplicates, the Clerical Review of Census Duplicates (CRCD) (Byrne, Beaghen, and Mulry 2002). Due to time limitations, estimates of ratio-estimator bias were not included. More important is that estimates of correlation bias used in the A.C.E. Revision II are assumed to be without error.

The estimated variance in the A.C.E. Revision II estimates $\hat{V}$ may not account for all the sources of variance or may not account for the included nonsampling error components well, especially for error from choice of model for accounting for duplicates.

Synthetic error, which is not directly included in the loss function analysis, may arise from two sources. One source of synthetic error involves correcting the individual post-stratum estimates for errors estimated at more aggregate levels (such as the corrections for correlation bias and coding errors). Another source of synthetic error is variations of census coverage within post-strata (something not captured by synthetic application of post-stratum coverage correction factors for specific areas). To assess whether omission of resulting synthetic biases from the loss function analysis tilted the comparisons in one direction or another, analyses based on artificial populations that simulated patterns of coverage variation within post-strata were done. These analyses, although limited in some respects (Griffin 2002), did not in general change the loss function results, though they had some limitations. Synthetic error is believed to be more important for smaller areas whose estimates are being compared. Thus, any limitations of the loss functions regarding synthetic error would be more important in analyses of accuracy for small places or counties than for large places or counties.

The construction of the bias-corrected confidence intervals and the loss function analysis excludes consideration of the following errors: synthetic estimation error, response error and coding error in A.C.E. Revision II P-Sample residency and match status and E-Sample correct enumeration status (e.g., conflicting cases), response error and coding error in A.C.E. Revision II P-Sample mover status, error in Demographic Analysis sex ratios for correlation bias estimation, error due to the model used to estimate correlation bias from Demographic Analysis sex ratios, error due to the model for estimating the effect of E-Sample cases with duplicates outside the A.C.E. search area.

The effect of omitting a variance component (if the corresponding error is uncorrelated with other random effects) would be to overstate the accuracy of the A.C.E. Revision II estimate and to understate the accuracy of the census. The effects of neglecting bias components are more difficult to predict for two reasons: (1) positive biases may cancel with negative biases, and (2) omitting biases affects the estimates of accuracy of both the A.C.E. Revision II estimates and the census. The direction of the effect of omitted biases on the comparison of accuracy depends on the sign of a weighted sum of products of neglected biases and expected values of the undercount estimates (Mulry and Spencer 2001, p.6). The limitation of omitted biases does not predictably tilt the loss function analysis to “favor” either the A.C.E. Revision II estimates or the census estimates in the comparisons of accuracy.

3. RESULTS

3.1 Confidence Intervals

Table 1 shows the variance components for the A.C.E. Revision II estimates of net undercount rate at the national level and for owners and renters. The sampling variance was estimated using an alternative variance estimator that treats the correlation bias correction factor as a scalar. Table 2 displays 95 percent confidence intervals for the net undercount rate for groups defined by race/Hispanic ethnicity and tenure. Recall that the census corresponds to an undercount rate of 0.

Table 1. Variance components for the A.C.E. Revision II estimates of undercount rate (percent)

group	UC Rate	SE* samp	SE impute model	SE dup model
Total	-0.49	0.19	0.10	0.18
Owner	-1.25	0.19	0.07	0.16
Renter	1.14	0.35	0.18	0.21

*The standard error for sampling uses an alternative variance estimator.

Neither the census nor A.C.E. Revision II estimate for Non-Hispanic Blacks lies within the interval. (Remember that both the interval and the A.C.E. Revision II DSE are adjusted for estimated correlation bias.). The reason for the relatively large estimated bias in the DSE is unclear – additional tabulations by enumeration and residency status by domain would indicate whether it arises from the effect of undetected duplicates in the P-Sample or the E-Sample. For example, duplications of erroneous enumerations in the E-Sample would tend to bias the A.C.E. Revision II estimate downward, and if the evaluation found large numbers of such cases, a large $\hat{B}$ could occur. The interval does include the Demographic Analysis estimate of a 2.78 percent undercount rate for Blacks (Robinson and Adlaka 2002). The census estimate for Non-Hispanic Whites does not lie within the interval although the A.C.E. Revision II estimate does. The intervals for all the other domains cover both the census and the A.C.E. Revision II estimate.

Neither the census nor the A.C.E. Revision II estimates for Black Owners and for Black Renters lie within the 95 percent confidence intervals. Additional tabulations by enumeration and residency status by domain and tenure could shed light on the reason for the large estimates of bias in the DSEs. The intervals for all other groups include the A.C.E. Revision II estimate.

The census numbers for all Owners, NonHispanic White Owners, Hispanic Owners, and Asian Owners do not fall within their intervals. The intervals for the other groups do encompass the census with the exception of Black Owners and Black Renters mentioned in the previous paragraph.

Table 2. 95 % Confidence Intervals for Undercount Rate (percent)

Domain & tenure group	UC rate	bias-corrected UC rate	SE (UC)	Lower bound	Upper bound
Total US	-0.49	-0.33	0.28	-0.89	0.22
Owner	-1.25	-0.89	0.26	-1.41	-0.37
Renter	1.14	0.86	0.44	-0.02	1.74
AIAN on Res	-0.88	-1.41	2.29	-6.00	3.18
Owner	-0.74	-1.49	2.54	-6.58	3.60
Renter	-1.17	-1.23	2.48	-6.19	3.73
AIAN off Res	0.62	-0.49	1.43	-3.35	2.37
Owner	-1.53	-2.91	1.95	-6.81	0.98
Renter	3.54	2.79	2.21	-1.62	7.20
Hispanic	0.71	-0.18	0.49	-1.16	0.80
Owner	-1.08	-1.63	0.54	-2.70	-0.55
Renter	2.35	1.15	0.66	-0.17	2.47
Non-Hisp Black	1.84	3.56	0.42	2.72	4.40
Owner	0.56	2.83	0.49	1.84	3.81
Renter	3.06	4.27	0.56	3.16	5.39
NHPI	2.12	0.30	2.23	-4.16	4.76
Owner	0.67	-1.82	3.33	-8.47	4.84
Renter	3.64	2.49	2.91	-3.33	8.32
Asian	-0.75	-0.83	0.68	-2.19	0.54
Owner	-1.71	-1.73	0.85	-3.44	-0.02
Renter	0.68	0.53	0.96	-1.40	2.45
Non- Hisp White	-1.13	-1.04	0.27	-1.58	-0.50
Owner	-1.46	-1.19	0.26	-1.70	-0.68
Renter	-0.07	-0.57	0.48	-1.53	0.38

3.2 Loss Functions

The loss function analyses are available for all groups except the within-state shares for all places, an analysis that was planned but not completed. The analyses were based on the estimates $\hat{B}$, $\hat{V}$, and $\hat{U}$ as discussed in section 2. Tables 3 and 4 contain results.

The results indicate smaller expected loss for the DSE than the census for all of the shares considered, and smaller expected loss for all of the levels except for all places with population greater than 100,000.

Table 3. Loss function results shares

Geographic Group	No. of Areas	Cen Loss x1000	DSE Loss x1000	Cen Loss / DSE Loss
St Share All counties	3141	1.716	0.590	2.908
US Share Places > 25,000 and < 50,000	595	0.060	0.016	3.665
US Share Places > 50,000 and <100,000	322	0.054	0.014	3.851
US Share Places > = 100,000	223	0.035	0.009	3.783
US Share All states	51	0.023	0.005	4.442

For insight, consider the following totals for all places with population of at least 100,000 as estimated by the census, the A.C.E. Revision II DSE, and the Target, which is equal to the DSE minus its estimated bias:

Census	71,829,465
A.C.E. Revision II	71,967,488
Target	71,512,212.

Comparison of the census and the target shows a *net overcount* in the census for these areas, but the excess of the DSE over the target and the census indicates that the DSE estimated a *net undercount*. Thus, the analysis indicates that DSE has either overestimated the number of census misses or underestimated the number of duplicates or both. A tabulation of the results from the Census and Administrative Records Study (Bean and Bauder 2002) would determine if it suggests that the A.C.E. Revision II missed large numbers of duplicates in these areas. When CARDS finds duplicates for erroneous enumerations, the effect in the estimation is to increase the correct enumeration rate.

To gain further insight into the loss function results for levels for places with population of at least 100,000, we examined loss function results when these places were separated into two groups, those with population between 100,000 and 1 million and those with population of at least 1 million. We found that the loss functions still indicated smaller expected loss for the DSE for levels for both groups of counties. For levels for places with population between 100,000 and 1 million, the ratio of census loss to DSE increased almost to 1. However, for levels for places with population over 1 million, the ratio of census loss to DSE loss was much smaller than the ratio for levels for places with population of at least 100,000. This indicates that the bulk of the error in the A.C.E. Revision II for places with population of at least 100,000 appears to lie in the nine (9) places with population of at least 1 million. Additional tabulations would aid in explaining the result.

Table 4. Loss function results levels

Geographic Group	No. of Areas	Cen Loss	DSE Loss	Cen Loss / DSE Loss
Counties<= 100,000	2617	15514	3730	4.158
Counties >= 100,000	524	21810	9258	2.355
Counties > 100,000 & < 1 million	490	16779	5726	2.929
Counties > = 1 million	34	5031	3531	1.424
Places > 25,000 and < 50,000	595	2785	966	2.883
Places > 50,000 and <100,000	322	2537	1070	2.371
Places > = 100,000	223	3251	4271	0.761
Places > 100,000 and < 1 million	214	2573	2671	0.963
Places > = 1 million	9	678	1600	0.424

Loss function analyses were also carried out under the assumption that the modeling of the correction for P-Sample cases that matched census enumerations outside the A.C.E. search area was without error. The latter assumption served to increase the estimate of census loss and decrease the estimate of DSE loss, but the findings were not qualitatively different than the results discussed above.

We also examined the loss function results when the Targets include only the bias due to inconsistent reporting of poststratification variables, which is very near zero. Under the assumption that the remaining variance components are the only errors, this loss function analysis shows that the A.C.E. Revision II estimate has less error than the census for levels and shares for all groups considered, even for levels for places with population of at least 100,000.

4. CONCLUSIONS

Results of evaluations performed on the A.C.E. Revision II estimates provided data to estimate bias (systematic error) and variance (random error) for use in constructing bias-corrected confidence intervals and in a loss function analysis. The sources of bias were more limited than for previous dual system estimates because data that previously were used to estimate bias in the original A.C.E. were incorporated into the A.C.E. Revision II estimates in order to correct for major errors discovered in the March 2001 A.C.E. estimates. However, the adjustments to reduce bias did put upward pressure on the variance. Using the A.C.E. evaluation data in formation of the A.C.E. Revision II created higher quality estimates because of the correction of major errors in the A.C.E. estimates. Nevertheless, although the evaluations do account for the variance arising from the corrections for bias, the corrections for bias in the A.C.E. Revision II estimates may themselves be subject to bias, the magnitude of which has not been quantified. This is particularly true for the corrections for correlation bias and for P-Sample cases that matched census enumerations outside the A.C.E. search area.

The correlation bias correction used the ratio of males to females at the national level from Demographic Analysis. Since Demographic Analysis uses data from vital records, the information about racial groups is limited to Blacks and Non-Blacks. The correction assumed the same error rate within a group of poststrata defined by Black and Non-Black and the age groups Different models that provide the same fit to the data could have been used to allocate the estimated correlation bias among the poststrata. However, there were no data that would indicate which model had the least bias. Therefore the model with the least variance was chosen.

A major concern is the puzzling inconsistency between the A.C.E. Revision II and the DA estimates of coverage rates for children aged 0-9. More research is needed to understand the cause of this difference in the estimates. The loss function analysis did not address this discrepancy.

Any limitations of the loss functions regarding synthetic error are expected to be more important when comparing small places or counties than for large places or counties. The use of different poststrata for the E and P samples may have reduced synthetic error for small areas (relative to the synthetic error when the poststrata are constrained to be identical, as in the Census Bureau's previous applications of dual system estimation). However, the correction factors were extreme for some poststrata, leading to increased concern about synthetic error in the areas with population concentrations in such poststrata. For example, undercount rates lower than -10% (i.e., overcounts rates greater than 10%) were estimated for 107 of the 19,269 places in the U. S.,) all 107 had populations under 10,000 and 76 had populations under 100. Undercount rates greater than 5% were estimated for 15 places, all with populations under 10,000 and for which 9 had populations under 100. (U. S. Census Bureau 2003)

The evaluations detected only a small amount of bias in the A.C.E. Revision II estimate of the net undercount rate at the national level, -0.16 percent. The small bias that does seem to be present appears to arise from error in the identification of duplicates; the effects of the error due to inconsistent post-stratification variables and the error due to using inmovers to estimate movers appear very small.

Judging from the bias-corrected 95-percent confidence intervals, both the census and the A.C.E. Revision II estimates are too low for Non-Hispanic Blacks and both Non-Hispanic Black Owners and Renters. The intervals show the census is too high for Non-Hispanic Whites, Owners, White Owners, and Hispanic Owners. All other census and A.C.E. Revision II estimates are covered by their bias-corrected 95-percent confidence intervals. The source of most of the bias estimate is the CARDS evaluation of the identification of duplicates.

The loss function analysis examines the relative accuracy by using the estimates of sampling variance and nonsampling bias and variance to estimate the aggregate expected loss for the census and the A.C.E. Revision II for levels and shares for counties and places across the nation and within state. The analyses indicated that the A.C.E. Revision II is more accurate than the census for every loss function considered with the exception of levels for places with population of at least 100,000. The bulk of the error in the A.C.E. Revision II for places with population of at least 100,000 appears to lie in the 9 places with population of at least

1 million. More research is needed to understand the one exceptional result.

The validity of the conclusions to be drawn from the loss function analysis depends on the quality of the estimates of components of error in the A.C.E. Revision II, and some of those components are not accurately quantified. The pressure point seems to be the bias estimates. If one assumed that the A.C.E. Revision II estimates were unbiased and had variances as estimated, the loss function analyses would find that the A.C.E. Revision II estimates are more accurate than the census for all groupings considered, even for levels for places with population of at least 100,000.

The major source of estimated bias in the A.C.E. Revision II is the estimation of census duplicates. (Imperfect adjustments for correlation bias could also be a major source of bias in the A.C.E. Revision II estimates but no quantification of the error contribution was available.) There are two evaluations of the estimates of census duplicates, CARDS (Bean and Bauder 2002) and CRCD(Byrne, Beaghen, and Mulry 2002). The estimates of bias used in the loss function analysis are based on CARDS. There are some discrepancies in findings from CARDS and CRCD. If these differences were resolved, one or more of the conclusions from the outcome of the loss function analysis could change. Further analyses assuming larger amounts of bias or a different distribution of the bias would increase the knowledge of the limitations of the data.

In summary, when viewing the results of the loss function analysis, one must keep the assumptions and limitations in mind, as well as realize that the effect of any omitted biases could be in either direction (increasing or decreasing the estimate of the relative accuracy of the census versus the A.C.E. Revision II estimates). While the loss function evaluations suggest the superiority of the A.C.E. Revision II estimates, concerns do remain about whether the bias estimates used in the loss function analysis are of sufficient quality to assure the correctness of the results.

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