## Chain Drift in Experimental Air Travel Price Index Series

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## 1. Introduction

Since air travel prices were deregulated in 1978, airlines have offered a variety of fares—often carrying extensive restrictions—for the same basic services. The internet now provides consumers with service and fare options that literally change by the minute, reflecting airlines' efforts to compete in a challenging market and to fill all seats on every flight. These factors, together with a host of special discount programs (e.g., frequent flyer awards, credit card points), greatly complicate the task of estimating price movements for commercial air travel.

The Bureau of Labor Statistics publishes both a consumer price index (CPI) and a producer price index (PPI) for airfares. Due to a tight publication schedule, BLS collects the price quotes from the SABRE system (an airline ticket reservation system) for the CPI and from airline pricing departments for the PPI. There is no guarantee that these "list" prices were actually paid by any purchasers. Moreover, the price sources do not include all the special discount fares available to consumers. Thus these index series suffer a bias relative to an index series based on transaction prices.

While the magnitude of the "list price" bias of the official airfare indexes is difficult to estimate, the Bureau of Transportation Statistics (BTS), in collaboration with BLS, is developing a new, more extensive set of price index series for air travel. The new indexes will be based on data from BTS' Passenger Origin and Destination (O&D) Survey, described in the next section. One issue that must be addressed for nearly every new index estimation system is the choice of an appropriate index "chaining" interval. In Section 3 below, we describe the index chaining process, and, in Section 4, we discuss an empirical study in which we compare alternative chaining intervals for the BTS airfare indexes. Section 5 provides an overview of current BTS plans related to the new index series. Detailed formulas and graphical results appear in the appendices.

### 2. Price Indexes and the O&D Survey

A price index is a measure of change in the value of a monetary unit. In estimating an "all items" price index, government statistical agencies generally compute a large number of "sub-indexes" indexes indicating change in the monetary unit's purchasing power with regard to specific, usually narrowly defined, categories of goods or services. Thus we may compute a separate price index for, say, washing machines, breakfast cereals, or, as in the case in point, airline tickets.

To compute an airfare sub-index for the CPI, BLS selects a sample of airline itineraries, with sampling probabilities proportional to expenditure shares, from a frame provided by BTS. The frame is a subset of the itinerary-level data that BTS collects from the airlines through the Passenger Origin and Destination. BLS then prices the selected trips each month, gathering price data from the SABRE system and combining these with expenditure share weights computed using data from the O&D Survey and the Consumer Expenditure Survey. (For more information on the BLS method, see http://www.bls.gov/cpi/cpifacaf.htm.)

The O&D Survey sample is a quarterly 10% sample of airline tickets used by passengers for travel during the reference quarter. Tickets purchased but not used are out of scope of the survey. Each airline ticket issued by the reporting carriers (which include all major domestic airlines) is identified by a unique serial number. If the number ends in "0," the ticket is in the O&D Survey sample, and the issuing airline reports itinerary information on the sample ticket to BTS. Data items collected include trip route, class of service (e.g., coach, first class), and transaction fare including taxes. Since the serial numbers assigned to airline tickets are essentially random, we treat the O&D Survey sample as a simple random sample of itineraries flown.

The O&D Survey's scope encompasses all itineraries having some U.S. component, i.e., every itinerary that includes a flight to or from a domestic airport is in scope. Although BTS collects O&D Survey data from foreign as well as domestic air carriers, confidentiality constraints currently prevent the use of the foreign carrier data for statistical purposes. Thus BTS estimates price indexes based on data from domestic carriers only.

When goods and services are sampled for the purpose of estimating a price index, the sample items generally remain in sample over an extended time period (e.g., two years) unless they're taken off the market by the retailer. The stable sample allows comparison of prices across time for identical items. Ratios of prices in different time periods for individual items (often called "price relatives") are the building blocks of the traditional price index formulas. In the O&D Survey, however, the sam-

<sup>\*</sup>Research for this paper was performed at the Bureau of Labor Statistics (BLS). Opinions expressed are those of the author and do not constitute policy of the BLS or the BTS.

pling is performed independently for each quarter. Since the itineraries selected in a given quarter may not "match" those selected for a previous or subsequent quarter, we developed and tested a two-stage process for matching *categories* of itineraries across quarters and comparing *average* prices within categories across time. The ratio of average prices for different time periods is called a *unit value index*. These sub-indexes form the bases of all of our index estimators.

## 3. Price Index Formulas and Index Chaining

Price index literature provides many formulas that can serve as "targets" for a price index estimator. Different formulas rely on different assumptions regarding the behavior of purchasers. In the "fixed market basket" approach, we assume that purchasers continue to buy the same fixed set of items regardless of changes in relative prices. For this approach, we can simply select a collection of goods and services (in fixed quantities) and track the total price of the "market basket" across time. The fixed market basket idea underlies the Laspeyres index

$$L_{t_1,t_2} = \frac{\sum_{j=1}^{N} q_{j,t_1} p_{j,t_2}}{\sum_{j=1}^{N} q_{j,t_1} p_{j,t_1}} = \sum_{j=1}^{N} w_{j,t_1} \left(\frac{p_{j,t_2}}{p_{j,t_1}}\right),$$
(3.1)

where, for each item j in the population of N goods and services,  $p_{j,t_k}$  and  $q_{j,t_k}$  represent the price and quantity purchased, respectively, in time period  $t_k$ for  $k \in \{1,2\}$ , and  $w_{j,t_k} = q_{j,t_k}p_{j,t_k}/\sum_j q_{j,t_k}p_{j,t_k}$ , the expenditure share for item j in period  $t_k$ . In our airfare index application, as noted above, we cannot compute the individual price ratios  $p_{j,t_2}/p_{j,t_1}$  for the Laspeyres formula. We use instead the unit value index  $u_{c,t_1,t_2}$  as defined in Appendix A for a given category c of airline itineraries or flights. The resulting Laspeyres estimator takes the form

$$\widehat{L}_{t_1, t_2} = \sum_{c \in C_{t_1, t_2}} w_{c, t_1} u_{c, t_1, t_2}, \qquad (3.2)$$

where  $C_{t_1,t_2}$  is the collection of itinerary (or flight) categories populated by sample records in both time periods  $t_1$  and  $t_2$ . A second index based on the fixed market basket approach is the Paasche index, estimated in our application as

$$\widehat{P}_{t_1,t_2} = \left(\sum_{c \in C_{t_1,t_2}} \frac{w_{c,t_2}}{u_{c,t_1,t_2}}\right)^{-1}.$$
 (3.3)

In the "Cost of Living Index" (COLI) approach to index calculation, we seek to estimate the change in the cost of a fixed level of consumer satisfaction or "utility," rather than the change in the cost of a fixed collection of goods and services. Motivated by economic utility theory, this approach allows for the possibility that, as relative prices change, consumers may revise their market baskets to obtain a constant level of utility across time. (For more on index numbers and the COLI approach, see, for example, Diewert 1987 or Reinsdorf 1998.) Two index formulas based on the COLI approach are the Fisher and Törnqvist formulas, whose estimators may be defined, respectively, as follows:

 $\widehat{F}_{t_1,t_2} = \sqrt{\widehat{L}_{t_1,t_2}\widehat{P}_{t_1,t_2}}$ 

and

$$\widehat{T}_{t_1,t_2} = \prod_{c \in C_{t_1,t_2}} u_{c,t_1,t_2}^{\left(w_{c,t_1}+w_{c,t_2}\right)/2}.$$
 (3.5)

(3.4)

Unlike the Laspeyres and Paasche indexes the Fisher and Törnqvist formulas involve expenditure share weights  $(w_{j,t_k})$  from both periods  $t_1$  and  $t_2$ . By using this additional data (which is unavailable in many applications), we may account for consumer substitution behavior, avoiding the uncertainty associated with assuming a fixed market basket. (For more on estimating a COLI from survey data, see Shapiro and Wilcox 1997 or Dorfman, Leaver, and Lent 1999.) The extent to which consumers change their buying behavior in response to changes in relative prices can be quantified by the "elasticity of substitution." A zero elasticity is equivalent to a fixed market basket.

Price index *chaining* is estimating long-term price changes as products of shorter-term changes ("links"). For example, suppose the price of a widget moves as shown below for time periods 1 through 4:

Time:	1	2	3	4
Price:	1.00	0.55	1.10	1.10

A direct measure the change between periods 1 and 4 is

$$I_{1,4} = \frac{1.10}{1.00} = 1.10.$$

The corresponding chained measure is

$$I'_{1,4} = I_{1,2}I_{2,3}I_{3,4}$$
  
=  $\left(\frac{0.55}{1.00}\right)\left(\frac{1.10}{0.55}\right)\left(\frac{1.10}{1.10}\right) = 1.10.$ 

In practice, the intermediate links  $I_{t-1,t}$  may be *estimated* changes for a *category* of items; in this case, we generally have  $I_{1,4} \neq I'_{1,4}$ .

For practical reasons, index chaining is widely used by government statistical agencies in computing price indexes. Both the universe of available goods and the sample of items used for index computation are in constant flux; new products are introduced daily, and the sample is routinely rotated to keep pace. Short-term changes may therefore be measured more accurately than long-term changes: the samples for two consecutive months, for instance, contain many more comparable items than the samples for two months one year apart. Some chained index estimators, however, are subject to systematic biases relative to their direct counterparts (see, for example, Szulc 1983). The magnitude and direction of the "chain drift" depend on the index aggregation formula, the economic behavior of the purchasing population, and the properties of the sample survey data used in estimation.

The theory of index chaining was expounded by Divisia (1925), who used integral calculus to formulate a chained index with arbitrarily short links. Richter (1966) presented invariance axioms for a variety of index numbers, including price indexes. More recently Forsyth (1978) and Forsyth & Fowler (1981) concluded that the choice of chaining interval (or link length) came down to a choice between "transitivity and representativity." Direct indexes provide the former, since they are independent of the pattern of price change between the two reference periods of interest. Chained indexes, however, provide more representativity: their shorter links, being less affected by factors such as the flow of goods in and out of the market, are stronger indicators of actual price change.

# 4. Empirical Study of Chaining Intervals

In the airfare index application we consider here, the trade-off involved in the index chaining decision is further complicated by an unconventional index estimation method. Since the O&D Survey sample itineraries are selected independently every quarter, and since airline itineraries are customized items, we cannot match every sample itinerary (by means of unit value indexes, as explained above), to an "identical" itinerary in a preceding or succeeding quarter. Our matching procedures, described in Lent and Dorfman 2003, result in about 90% to 95% of itinerary segments (i.e., individual flights) being matched across consecutive quarters. Matching itineraries across non-consecutive quarters, however, reduces this percentage. When we matched fourth-quarter data sets for consecutive years, for example, we found that the number of segments matched was roughly 75% to 80% of the number matched in consecutive quarters. Air carriers constantly enter and exit service markets, causing a steady reduction in the percentage of itineraries matched as the time between the quarters considered increases.

Study Methods. Because the matching percentages were unacceptably low for quarters more than one year apart, we compared quarterly chained index series to their annually chained counterparts, noting that annually chained indexes are less susceptible to drift. The goal of our empirical study<sup>\*</sup> was to evaluate the severity of the chain drift in the quarterly chained indexes under the Laspeyres, Paasche, Fisher, and Törnqvist formulas given above. (See Appendix A for detailed index formulas based on the different chaining intervals.) We used O&D Survey data from the first quarter of 1995 to the first quarter of 2001.

The charts appearing in Appendix B show index series that are representative of the overall study results. Many of the series display a "break" in the 1997-98 period, which coincides with a change in O&D Survey reporting procedures and thus does not necessarily indicate price change. The break is useful, however, for examining the relative effects of extreme valued subindexes on the aggregate indexes computed by the different formulas.

Study Results. Figures 1 through 4 display quarterly and annually chained series, based on the Fisher, Törnqvist, Laspeyres, and Paasche formulas for all classes of service and all geographic areas combined. At this highest level of aggregation, we see no evidence of chain drift in the Fisher and Törnqvist series (Figures 1 and 2): differences between the quarterly and annually chained series are barely noticeable. Moreover, differences between the Fisher and Törnqvist series can be traced primarily to the 1997-98 break mentioned above, which affects the Törnqvist series more than the Fisher. (See Lent 2002 for a discussion of the reasons for this.) The quarterly chained Laspeyres series (Figure 3), however, is clearly subject to a slight upward drift relative to the annually chained series, while the quarterly chained Paasche (Figure 4) shows a slight downward drift. The mild quality of these drifts indicates a low but positive elasticity of substitution between the unit value categories. Elasticity estimates computed for this application are generally quite low: although consumers may readily substitute between the services of different air carriers, they generally will not substitute one travel destination for another.

Figure 5 provides a comparison of the annually chained Fisher, Laspeyres, and Paasche series, which are virtually indistinguishable from one another. Differences between the quarterly chained

<sup>\*</sup>For the purposes of the study, we performed only itinerary-level (first stage) matching, since previous studies indicated that, for the air travel categories examined, the index series relying only on itinerary-level matching differed little from those produced through both itinerary- and segment-level matching. See Lent and Dorfman (2003) for a discussion of the matching procedures.

versions of the three series (Figures 1, 3, and 4) can thus be attributed largely to chain drift in the Laspeyres and Paasche indexes. Though the chain drift in these indexes could be corrected through the use of a combination of quarterly and annually chained links, such a procedure would complicate index computation, and we would expect the corrected Laspeyres and Paasche series to closely track the quarterly chained Fisher.

The remaining series shown Appendix B are all based on the Fisher formula. Figures 6 through 8 show quarterly and annually chained Fisher index series for three "class of service" categories: restricted coach class, unrestricted coach class, and unrestricted business class. A ticket is considered restricted if the fare paid carries any restrictions on service, e.g., the passenger may be required to book the trip two weeks in advance or stay at the destination over a Friday or Saturday night. For the largest service category, restricted coach (Figure 6), there is no discernible difference between the quarterly and annually chained Fisher series. For the second-largest category, unrestricted coach (Figure 7), slight differences are noticeable, but we see no systematic upward or downward drift in the quarterly chained series relative to the annually chained series. Figure 8 shows the series for unrestricted business class service, one of the smallest categories. Here the quarterly chained series shows a pronounced separation from the annually chained series at the 1997-98 break. Since the two series continue roughly parallel to each other after the break, however, we again have no evidence of chain drift in the Fisher index estimates.

One of the advantages of the O&D Survey indexes over the official CPI is the greatly increased sample size, which allows estimation of index series for states and even for moderately large cities. Figure 9 in Appendix B shows quarterly and annually chained Fisher series for itineraries originating in two specific cities, New York and Minneapolis. For trips originating in the New York/Newark area, we see no difference between the quarterly and annually chained series. For Minneapolis—a considerably smaller city—the differences are discernible, but they occur primarily at the 1997-98 break and provide no evidence of systematic drift. We conclude that chain drift is not a concern for the quarterly chained Fisher series, even at this fairly low level of aggregation.

## 5. Future Plans for the O&D Survey Index

Lent (2002) examined the effects of extreme price values on the O&D Survey index series and concluded that the Fisher index formula was more robust to outliers than was the comparable Törnqvist formula. The study described above leads us to further conclude that a quarterly chained Fisher index would be relatively robust to chain drift. BTS is therefore planning to put the O&D Survey index series into regular production using a quarterly chained Fisher index formula. As indicated above, the index series produced will provide a high degree of geographic detail (e.g., city level indexes), allowing airline industry analysts and government policy makers to examine the effects of airline industry events on a large number of geographic groups.

The primary limitation of the planned index series is the time lag currently needed to compile and edit the O&D Survey data. Because of the four to five month lag, the O&D Survey indexes cannot be produced in time to be included as components of the BLS All-items CPI. Moreover, the current O&D Survey data are categorized only by reference quarter, while the CPI is published monthly. In the future, however, BTS plans to collect more detailed, timely airfare data, which may eventually allow monthly O&D Survey air travel price indexes to be produced on a more timely basis.

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# Appendix A: Formulas for Quarterly and Annually Chained Indexes

A measure of relative change in the price of a particular item j between quarters  $t_1$  and  $t_2$  is the price ratio  $p_{j,t_2}/p_{j,t_1}$ , where  $p_{j,t_k}$  represents the price of item j at time  $t_k$  for  $k \in \{1, 2\}$ . Since each quarterly O&D Survey sample is independently drawn, we cannot match each individual itinerary with an identical one in the following (or previous) quarter and compute individual price ratios. We therefore compute *unit value indexes* (defined below) for itineraries (in the first stage) within each unit value category  $c \in C_{t_1,t_2}$ , where  $C_{t_1,t_2}$  is the collection of categories populated by sample itineraries in quarters  $t_1$  and  $t_2$ . (In the second matching stage, the categories are defined in terms of characteristics of individual flights. For definitions of the firststage categories and other details of the estimation method, see Lent and Dorfman 2003.)

Let  $q_{j,t_k}$  be the quantity of item j purchased in period  $t_k$ , and let

$$q_{c,t_k} = \sum_{j \in c} q_{j,t_k}$$

The unit value index estimator for category c is defined as

$$u_{c,t_1,t_2} = \frac{\sum_{j \in c} q_{j,t_2} p_{j,t_2} / q_{c,t_2}}{\sum_{j \in c} q_{j,t_1} p_{j,t_1} / q_{c,t_1}}.$$

In words, the unit value index is the average price of an item in category c during quarter  $t_2$  divided by the average price of an item in category c during quarter  $t_1$ .

For quarterly chained indexes, we always use two consecutive quarters as the reference periods in the calculation of unit value indexes, i.e.,  $t_2-t_1 = 1$ . Thus, for example, we compute a unit value index measuring price change between the third and fourth quarters of 1999 for each category  $c_q \in C_{99Q4,00Q1}$  as follows:

$$u_{c_q,99Q3,99Q4} = \frac{\sum_{j \in c_q} q_{j,99Q4} p_{j,99Q4} / q_{c_q,99Q4}}{\sum_{j \in c_q} q_{j,99Q3} p_{j,99Q3} / q_{c_q,99Q3}}.$$

By contrast, the annually chained indexes are based on unit value indexes computed for reference quarters one *year* apart, i.e.,  $t_2 = t_1 - 4$ . To maximize the number of annual "links" we can compute from the available data, we use the first quarter of each year as an annual reference period. Thus, for example, we compute unit value indexes measuring price change between the first quarter of 1999 and the first quarter of 2000 for each category  $c_a \in C_{99Q1,00Q1}$ :

$$u_{c_a,99Q1,00Q1} = \frac{\sum_{j \in c_a} q_{j,00Q1} p_{j,00Q1} / q_{c_q,00Q1}}{\sum_{j \in c_a} q_{j,99Q1} p_{j,99Q1} / q_{c_q,99Q1}}.$$

Data availability dictates that we may have  $C_{99Q4,00Q1} \neq C_{99Q1,00Q1}$ , i.e., the collection of categories for which quarterly unit value indexes are estimated may differ from the collection for which annual unit value indexes are estimated.

Once the unit value index estimates are computed for all  $c \in C_{t_1,t_2}$ , we treat them as price ratios in the standard index formulas. For  $k \in \{1,2\}$ , let

$$w_{c,t_k} = \frac{\sum_{j \in c} p_{j,t_k} q_{j,t_k}}{\sum_{c \in C_{t_1,t_2}} \sum_{j \in c} p_{j,t_k} q_{j,t_k}},$$

the expenditure share for category  $c \in C_{t_1,t_2}$  during time period  $t_k$ . (Note that  $w_{c,t_k}$  depends on  $C_{t_1,t_2}$ and would be more clearly denoted by  $w_{C(t_1,t_2),t_k}$ . For ease of notation we leave this dependence implicit; for typographical clarity, we also use the notation  $tk, k \in \{1, 2\}$ , in place of  $t_k$  in some of our subscripts.) We estimate the Laspeyres, Paasche, Fisher, and Törnqvist indexes using these components and the formulas given in Section 3.

The differences between the quarterly chained and annually chained series lie in the relative values of  $t_1$  and  $t_2$ . For the quarterly chained indexes,  $t_2 = t_1 - 1$  for all aggregate indexes, e.g., we compute

$$\widehat{F}_{99Q4,00Q1} = \sqrt{\widehat{L}_{99Q4,00Q1}\widehat{P}_{99Q4,00Q1}}.$$

Similarly, for the annually chained indexes,  $t_2 = t_1 - 4$ , and we compute

$$\widehat{F}_{99Q1,00Q1} = \sqrt{\widehat{L}_{99Q1,00Q1}\widehat{P}_{99Q1,00Q1}}.$$

To estimate long-term indexes with base quarter 95Q1, we set all index values equal to 100 for the base quarter. For each subsequent quarter  $t_k$ , we compute long-term indexes by multiplying the long-term index for the previous quarter  $t_k - 1$  (or, for annually chained indexes,  $t_k - 4$ ) by the value of the short-term index  $\widehat{I}_{t_k-1,t_k}$  (or  $\widehat{I}_{t_k-4,t_k}$ ), where  $\widehat{I} \in \{\widehat{L}, \widehat{P}, \widehat{F}, \widehat{T}\}$ . That is, for quarterly chained indexes,

$$\widehat{I}_{95Q1,t_k}^{(q)} = \widehat{I}_{95Q1,t_k-1}^{(q)} \widehat{I}_{t_k-1,t_k}^{(q)} = 100 \prod_{i=1}^{Q_k} \widehat{I}_{t_i-1,t_i}^{(q)},$$

where  $Q_k$  denotes the number of quarters between 95Q1 and  $t_k$ . Similarly, for annually chained indexes,

$$\widehat{I}_{95Q1,t_k}^{(a)} = \widehat{I}_{95Q1,t_k-4}^{(a)} \widehat{I}_{t_k-4,t_k}^{(a)}$$

$$= 100 \prod_{l=1}^{A_k} \widehat{I}_{t_l-1,t_l}^{(a)},$$

where  $A_k$  is the number of years between 95Q1 and  $t_k$ , and  $t_l$  denotes the first quarter in year l.

## **Appendix B: Figures**







Figure 3 Quarterly and Annually Chained Preliminary Laspeyres Series for All Classes of Service Combined, 95Q1=100

















Figure 8 Quarterly and Annually Chained Preliminary Fisher Series for Unrestricted Business Class Service, 95Q1=100



