Use of Generalized Variance Functions in Multivariate Analysis

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Abstract: In the analysis of complex survey data, goodness-of-fit tests and other model checks often are based on quadratic-form test statistics. The resulting tests generally make explicit or implicit use of variance-covariance matrix estimators. Under some complex sample designs, these matrix estimators are relatively unstable. The can cause serious degradation of the performance of the associated tests. This paper examines the extent to which generalized variance function methods can produce more stable variance-covariance matrix estimators, and thus lead to improved test statistics. Special emphasis is placed on distinctions among three types of error associated with variance-covariance matrix estimators: sampling error, equation error and parametric estimation error. Relationships between the resulting test statistics and Rao-Scott type test statistics are considered. Applications to the U.S. Current Employment Survey and the U.S. Consumer Expenditure Survey are discussed.

1. Introduction

1.1 Multivariate Inference from Complex Survey Data

In the analysis of sample survey data, it is often important to carry out inference for a k-dimensional parameter vector

$$\theta_U = (\theta_{U1}, \ldots \theta_{Uk})' \qquad (1.1)$$

or an associated superpopulation vector,

$$\theta_\xi = (\theta_{\xi 1}, \ldots \theta_{\xi k})' \qquad (1.2)$$

For some general background, see, e.g., Fuller (1975), Rao and Scott (1981, 1984, 1987), Binder (1983), Skinner, Holt and Smith (1989), Korn and Graubard (1990), Pfeffermann (1996) and references cited therein.

In the current discussion, three points will be of principal interest. First, a complex sample design C is used to collect data from which a point estimator $\hat{\theta}_C = (\hat{\theta}_{C1}, \ldots \hat{\theta}_{Ck})'$ is obtained. Let $E_p(\bullet)$ and $V_p(\bullet)$ denote, respectively, the expectation and the variance-covariance matrix of a random vector evaluated with respect to the sample design. We assume that

$$E_p(\theta_C) \cong \theta_U \qquad (1.3)$$

in the sense that the difference $E_p(\theta_C) - \theta_U$ is small relative to other sources of variability.

Second, suppose that the variance-covariance matrix $V_p(\theta_C)$ is nonsingular, and that

$$\{V_p(\theta_C)\}^{-1/2}(\hat{\theta}_C - \theta_U) \to N(0, I_k) \qquad (1.4)$$

in law, where $A^{1/2}$ represents the inverse of the symmetric matrix square root of a symmetric positive definite matrix A, and where the convergence in expression (1.4) refers to the limit of the multivariate distribution induced by the complex sample design C, conditional on a given realized finite population U.

Then we may use the distributional approximation (1.4) to carry out design-based inference for θ_U. For example, given a prespecified vector $\theta_0 = (\theta_{01}, \ldots \theta_{0k})'$, one may test the null hypothesis

$$H_0 : \theta_U = \theta_0$$

using the test statistic

$$T_{C0} = (\hat{\theta}_C - \theta_0)' \left\{ V_p(\hat{\theta}_C) \right\}^{-1} (\hat{\theta}_C - \theta_0) \qquad (1.5)$$

Under H_0 and condition (1.4), T_{C0} is distributed as a chi-square random variable on k degrees of freedom.

Third, in practical applications, we generally do not know $V_p(\hat{\theta}_C)$, but we often can compute a variance-covariance matrix estimator $\hat{V}_p$ (through linearization or replication methods) such that

$$d_p \hat{V}_p \rightarrow \text{Wishart } (d_p, V_p) \qquad (1.6)$$

for some appropriate scalar degrees-of-freedom term d_p, independent of $\hat{\theta}_C$. Then the associated quadratic-form test statistic,

$$\hat{T}_{C0} = (\hat{\theta}_C - \theta_0)' \left(\hat{V}_p \right)^{-1} (\hat{\theta}_C - \theta_0) \qquad (1.7)$$

is distributed as a multiple of an F random variable on k and $d_p - k + 1$ degrees of freedom under H_0.

Fourth, if $d_p - k + 1$ is relatively small, then $\hat{T}_{C0}$ may perform relatively poorly. In addition, in some cases it may be difficult or impossible to compute $\hat{V}_p$ due to confidentiality constraints or limitations on available software. Under such circumstances, one may consider an alternative test statistic,

$$T^*_{C0} = (\hat{\theta}_C - \theta_0)' \left(V^*_p \right)^{-1} (\hat{\theta}_C - \theta_0) \qquad (1.8)$$

where V^*_p is believed to be relatively stable (in a sense specified in Sections 2 and 3), and is computed readily from available data.

1.2 Multivariate Generalized Variance Functions

Properties of the test statistic T^*_{C0}, or of associated test-inversion confidence sets, will depend heavily on the properties of the random matrix V^*_p. The remainder of this paper studies these properties through extensions of ideas developed previously within the literature on univariate generalized variance functions. Section 2 develops a general framework for components of error associated with variance-covariance matrix estimators like $\hat{V}_p$ or V^*_p. Specifically, Subsection 2.1 considers general distinctions among sampling error, equation error and smaller-order parametric estimation error. Subsection 2.2 explores these distinctions further within the context of generalized variance function models. Subsection 2.3 discusses Rao-Scott type adjusted test statistics as special cases of the test statistic (1.8).

Section 3 applies some of the general ideas of Section 2 to a specific problem in the analysis of employment growth rates estimated from the U.S. Current Employment Survey. Of special interest is the fact that in the variance-covariance matrix estimator, there is a nontrivial component of error associated with error in the estimation of the underlying univariate generalized variance function. Section 4 reviews the main ideas of this paper and suggests some possible extensions.

2. Components of Error in Variance-Covariance Matrix Estimators

2.1. Sampling Error, Equation Error and Parametric Estimation Error

Section 1 focused attention on design properties of the point estimator $\hat{\theta}_C$ and the associated variance-covariance matrix estimator $\hat{V}_p$. For example, under conditions (1.4) and (1.6), we have, conditional on a given finite population U,

$$E_p(\hat{\theta}_C)=\theta_U \quad (2.1)$$

$$V_p(\hat{\theta}_C)=V_p \quad (2.2)$$

$$\hat{V}_p=V_p+\varepsilon_p \quad (2.3)$$

where $E_p(\varepsilon_p)=0$. Extension of previous literature on generalized variance functions (e.g., Wolter, 1985, Chapter 5); Johnson and King, 1987; and Valliant, 1987) suggests that one view the finite population U as having been generated by a superpopulation model ξ such that

$$E_\xi \hat{\theta}_U)=\theta_\xi,$$

and

$$E_\xi\left(V_p \mid \theta_\xi, X, \gamma\right)=f(\theta_\xi, X, \gamma) \quad (2.4)$$

where X is a matrix of available auxiliary information (e.g., sample sizes, the coefficient of variation of relevant weights, or other relevant design information), γ is an $r\times 1$ vector of unknown parameters, and $f(\cdot,\cdot,\cdot)$ is a $k\times k$-dimensional matrix function of known form. Then we may define the *equation error*

$$c=V_p-f(\theta_\xi, X, \gamma), \quad (2.5)$$

the difference between the realized random matrix V_p and its superpopulation expectation, $f(\theta_\xi, X, \gamma)$.

In addition, the parameters θ_ξ and γ generally are unknown, and X often is also unknown. Given specific estimators $\hat{\theta}_C$, $\hat{X}$ and $\hat{\gamma}$ (see, e.g., Valliant (1987) for discussion of ordinary least squares and generalized least squares methods for estimation of γ), define the parametric estimation error,

$$b=f(\hat{\theta}_C, \hat{X}, \hat{\gamma})-f(\theta_\xi, X, \gamma), \quad (2.6)$$

Under mild conditions, the error b will depend on both ε_p and c, but generally will be of smaller order of magnitude than either ε_p or c. Now suppose the in computation of the test statistic (1.8), we have used

$$V_p^*=f(\hat{\theta}_C, \hat{X}, \hat{\gamma}), \quad (2.7)$$

Then we may consider the properties of the test statistic (1.8) for three separate cases.

Case 1: Negligible equation error and estimation error. Suppose that $V_p^{-1}c$ and $V_p^{-1}b$ are both of small order of magnitude. Then the misspecification effect associated with the use of V_p^* in place of V_p is negligible, and the test statistic (1.8) will follow approximately (evaluated with respect to the sample design) a noncentral chi-square distribution on k

degrees of freedom, and with noncentrality parameter equal to

$$(1/2)(\theta_U-\theta_0)'\left\{f(\theta_\xi,X,\gamma)\right\}^{-1}(\theta_U-\theta_0)$$

Case 2: Negligible equation error and nontrivial estimation error. Suppose that $V_p^{-1}c$ is of small order of magnitude, but that $V_p^{-1}b$ is not small. Thus, the estimation error b depends primarily on sampling error, ε_p, rather than on equation error; and V_p is approximately equal to $f(\theta_\xi,X,\gamma)$. Suppose further that for some $d_b>0$,

$$d_b f(\hat{\theta}_C,\hat{X},\hat{\gamma})\rightarrow \text{Wishart}\left[d_b, f(\theta_\xi,X,\gamma)\right],$$

and is approximately independent of $\hat{\theta}_C$. (This final condition would hold, for example, if the function $f(\theta,X,\gamma)$ does not depend on θ. Then standard arguments (e.g., Korn and Graubard, 1990) indicate that, evaluated with respect to the design, the test statistic (1.8) is distributed approximately as a scalar multiple of a noncentral F random variable. The relative operating characteristics of the test statistics (1.8) and (1.7) will then depend primarily on the relative values of d_p and d_b.

Case 3: Negligible estimation error and nontrivial equation error. Suppose that $V_p^{-1}b$ is of small order of magnitude, but that $V_p^{-1}c$ is not small. Then two results are of practical interest. First, conditional on a given realization of the finite population U from the superpopulation ξ, use of V_p^* as an approximation to V_p would lead in general to a nontrivial associated misspecification effect matrix,

$$(V_p^*)^{-1/2}V_p(V_p^*)^{-1/2}\approx I_k+(V_p^*)^{-1/2}c(V_p^*)^{-1/2}$$

Thus, if one wished to use V_p^* (e.g., for stability reasons) and to remain within a design-based framework, the performance of (1.8) would depend on the eigenstructure of

$$(V_p^*)^{-1/2}c(V_p^*)^{-1/2}$$

and may in general be problematic.

Under Case 3, a possible alternative would be to consider the use of the test statistic (1.8) in a restricted form of $p\xi$ inference, i.e., inference with respect to both the design and model. Specifically, recall that evaluation a variance-covariance matrix with respect to both the design and superpopulation sources of variability leads to the expression,

$$V_{p\xi}(\hat{\theta}_C)=E_\xi\left\{V_p(\hat{\theta}_C)\right\}+V_\xi\left\{E_p(\hat{\theta}_C)\right\} \quad (2.8)$$

Furthermore, assume that

$$\left\{V_{p\xi}(\hat{\theta}_C)\right\}^{-1}V_\xi\left\{E_p(\hat{\theta}_C)\right\}$$

is negligible. (This would occur, for example, under a standard superpopulation model with independent and identically distributed variates, a negligible sample fraction, and additional regularity conditions.) Then $V_{p\xi}(\hat{\theta}_C)$ is approximately equal to $E_\xi\left\{V_p(\hat{\theta}_C)\right\}$, which by expression (2.4) is equal to $f(\theta_\xi,X,\gamma)$. Thus, under the assumption of negligible estimation error in Case 3, $V_p^*=f(\hat{\theta}_C,\hat{X},\hat{\gamma})$ is approximately equal to $V_{p\xi}(\hat{\theta}_C)$.

If we also assume that the distribution of

$$\left\{V_{p\xi}(\hat{\theta}_C)\right\}^{-1/2}(\hat{\theta}_C-\theta_\xi),$$

evaluated with respect to the $p\xi$ distribution, is approximately $N_k(0, I_k)$, then the test statistic (1.8) has $p\xi$ distribution approximately equal to a noncentral chi-square distribution on k degrees of freedom and with noncentrality parameter

$$(1/2)(\theta_\xi - \theta_0)'\{f(\theta_\xi, X, \gamma)\}^{-1}(\theta_\xi - \theta_0)$$

Note that the abovementioned $p\xi$ approach to Case 3 used several additional assumptions. If these assumptions are not satisfied, then a $p\xi$ approach to Case 3 may be problematic. In particular, if

$$\left\{V_{p\xi}(\hat{\theta}_C)\right\}^{-1} V_\xi \left\{E_p(\hat{\theta}_C)\right\}$$

is not trivial, then $f(\hat{\theta}_C, \hat{X}, \hat{\gamma})$ can seriously underestimate $V_{p\xi}(\hat{\theta}_C)$, and the resulting misspecification effect matrix,

$$\{f(\theta_\xi, X, \gamma)\}^{-1/2} V_{p\xi}(\hat{\theta}_C)\{f(\theta_\xi, X, \gamma)\}^{-1/2}$$

$$= I_k + \{f(\theta_\xi, X, \gamma)\}^{-1/2} V_\xi\{E_p(\hat{\theta}_C)\}\{f(\theta_\xi, X, \gamma)\}^{-1/2}$$

may have one or more eigenvalues substantially greater than unity. Thus, in this setting it is preferable to construct a generalized variance function model that produces a good approximation for $V_{p\xi}(\hat{\theta}_C)$, rather than for $V_p(\hat{\theta}_C)$.

2.2 Links with Univariate Generalized Variance Function Models

The preceding subsection considered the variability of the equation error at a fairly high level of generality, without specific ference to a particular parametric superpopulation model. The application in Section 3 obtains some model identification information through specific distributional assumptions for related univariate generalized variance functions.

For the current discussion, consider a set of J ordered quadruples $(\hat{\theta}_{Cj}, X_j, V_{pj}, \hat{V}_{pj})$, $j = 1, \ldots, J$ associated with J distinct estimands θ_j. In addition, assume that on a logarithmic scale,

$$\ln(\hat{V}_{pj}) = \ln(V_{pj}) + e_{pj} \qquad (2.9)$$

and

$$\ln(V_{pj}) = X_j\gamma + q_j \qquad (2.10)$$

where e_{pj}, $j = 1, \ldots, J$ are independent and identically distributed normal random variables with mean μ_e and variance σ_e^2; q_j, $j = 1, \ldots, J$ are independent and identically distributed normal random variables with mean 0 and variance σ_q^2; and the errors e_{pj} and q_j are mutually independent. Note that the lognormal assumption (2.9) is not consistent with the assumption (1.6) of an approximate Wishart distribution for $d_p\hat{V}_p$. In the current discussion, we will not make direct inferential use of the Wishart assumption (1.6), but we will assume that on the original scale, $d_p\hat{V}_{pj}/V_{pj}$ has the same first and second moments as a chi-square random variable on d_p degrees of freedom, i.e.,

$$E_p(\hat{V}_{pj}/V_{pj}) = 1 \qquad (2.11)$$

and

$$V_p(\hat{V}_{pj}/V_{pj}) = 2/d_p \qquad (2.12)$$

Routine results for the lognormal distribution (e.g., Casella and Berger, 1990, p. 628) and additional algebra show that

$$\mu_e = (-1/2)\ln(1 + 2/d_p)$$

and $\sigma_e^2 = \ln(1 + 2/d_p)$. In addition, under the assumption that the quadruples $(\hat{\theta}_{Cp}, X_j, V_{pj}, \hat{V}_{pj})$ are independent, ordinary least squares regression of $\ln(\hat{V}_{pj})$ on X_j leads to consistent estimators $\hat{\gamma}$ of γ and $(M\hat{S}E)$ of $\sigma_q^2 + \sigma_{ep}^2$. In addition,

$$\hat{\sigma}_q^2 = (M\hat{S}E) - \ln(1 + 2/d_p)$$

provides a consistent estimator of σ_q^2 under the assumptions listed above; and

$$V_{pj}^* = \exp(M\hat{S}E/2 + X_j\hat{\gamma})$$

is a consistent estimator of $f(\theta_\xi, X, \gamma) = E_\xi\{V_p(\hat{\theta}_C)\}$.

Also, additional algebra shows that ,

$$V_\xi\{V_{pj} - f(\theta_\xi, X_j, \gamma)\} = \{f(\theta_\xi, X_j, \gamma)\}^2 \exp(\sigma_q^2 - 1).$$

can be estimated by $\{V_{pj}^*\}^2 \exp(\hat{\sigma}_q^2 - 1)$. As noted above, we are using lognormal assumptions for both the sampling error e_{pj} and the equation error q_j. Nonetheless, analysts often summarize the stability of a variance estimator through a moment-based "degrees of freedom" calculation, which in this case would be,

$$\hat{d}_q = 2/\exp(\hat{\sigma}_q^2 - 1) \qquad (2.13)$$

In this case, expression (2.13) represents the uncertainty in V_{pj}^* as a predictor of the random variable V_{pj}, which is subject to the equation error q_j on the logarithmic scale. Finally, consider again Case 2, in which one has negligible equation error and nontrivial estimation error. Then additional routine algebra leads to the Satterthwaite-type "degrees of freedom" estimator,

$$\hat{d}_b = 2\left[\left\{(1/2, X_j)\hat{V}(\hat{\sigma}_q^2, \hat{\gamma})(1/2, X_j)'\right\}^{-1} - 1\right] \qquad (2.14)$$

Expression (2.14) represents the uncertainty in V_{pj}^* as an estimator of $f(\theta_\xi, X_j, \gamma) = V_{pj}$ under Case 2. Consequently, comparison of d_p, $\hat{d}_q$ and $\hat{d}_b$ can provide a rough indication of the relative magnitudes of, respectively, the sampling errors, equation error effects and estimation error effects considered in Cases 1 through 3.

2.2. Misspecification Effect Matrices and Rao-Scott Adjusted Test Statistics

Section 2.1 discussed misspecification effect matrices within the context of the equation error model (2.5). The original work by Rao and Scott (1981, 1984, 1987) on quadratic-form test statistics focused principal attention on approximations for the distribution of a test statistic under a null hypothesis associated with the parameter vector θ. We note, however, that to some degree, the adjusted matrices used in the Rao-Scott test statistics can be viewed as simple multivariate variance function estimators.

3. Application to Data from the U.S. Current Employment Survey

We applied the principal ideas of Section 2 to estimates of total employment for a large number of domains covered by the U.S. Current Employment Survey (CES). For some background on the Current Employment Statistics Survey, see American Statistical Association (1994), Werking (1997), Butani, Harter and Wolter (1997), Butani, Stamas and Brick (1997), West, Kratzke and Grden (1997). For the present discussion, four features are of principal interest. First, domains were defined by the intersection of three factors:

Industry $i = 1,\ldots,I$; $I = 6$ (Mining; mining and construction combined; construction; durables goods manufacturing; nondurable goods manufacturing; and wholesale trade. For some areas, mining and construction are combined, while for other areas, they are treated as distinct industries.)

Area $a = 1,\ldots,A$ ($A = 272$ metropolitan areas in the United States).

Month $t = 1,\ldots,12$ (January through December, 2000).

To reflect the Industry$\times$Area$\times$Month structure used to define the domains of interest, the estimand subscript j used in Section 2 will be replaced by the triple subscript (i,a,t). For instance, for industry i, area a, and month t, we have

y_{iat} = True total employment;

$\hat{y}_{iat}$ = A direct (weighted link relative) estimator of y_{iat}, based only on data from industry i and area a.

$\hat{V}_{iat}$ = An estimator of the design variance of $\hat{y}_{iat}$, computed through standard fractionally weighted methods of balanced repeated replication.

We explored several possible versions of the logarithmic model (2.10) and ultimately selected one with a relatively large number of predictors:

$$\ln(\hat{V}_{piat}) = \gamma_{0i} + \gamma_{1i}\ln(y_{ia0}) + \gamma_{2i}\ln(n_{ia0}) + \gamma_{3i}\ln(t) + e_{piat} + q_{iat} \qquad (3.1)$$

where $(\gamma_{0i},\gamma_{1i},\gamma_{2i},\gamma_{3i})$ are vectors of nonrandom coefficients that are allowed to vary across industries; y_{ia0} is the nominal true total employment in the intersection of industry i and area a during a benchmark month 0; and n_{ia0} is the number of sample units (unemployment insurance accounts) selected from industry i and area a.

Second, the original sample design stratified the population by state, industry and size class. Within a given state $\times$ industry $\times$ size cell, units were selected through a systematic sampling method that implicitly stratified by geographical area. Consequently, it is reasonable to treat the errors as independent across industry $\times$ area combinations. However, except for attrition and a small amount of sample rotation, the same units were included in the sample in each month, so one could not reasonably assume independence of the error terms across months.

Third, within a given industry $\times$ area combination, the principal relevant stratification factor was size class. The original sample design provided eight size classes, but for purposes of variance estimation, the three largest size classes were collapsed, yielding an effective number of strata equal to six. In parallel with this, the balanced repeated replication method used eight distinct sets of fractional replicate weights. Thus, customary arguments would assign no more than $d_p = 6$ degrees of freedom to the direct variance estimator $\hat{V}_{iat}$. Also, the current analysis excluded domains that had less than or equal to twelve responding units in a given month. Consequently, a total of 5160 domains had estimates $\hat{V}_{iat}$ available for use in fitting model (3.1).

Finally, additional modeling results not detailed here indicated that the magnitudes of equation errors in this case were small relative to other sources of variability, corresponding to an estimate of d_q greater than 100, while estimation error had a nontrivial effect on the random variability of V^*_{iat}, corresponding to moderate average degrees-of-freedom terms $\hat{d}_b$ = 20.8 for durable goods, $\hat{d}_b$ = 27.4 for nondurable goods and $\hat{d}_b$ = 56 for wholesale trade. Thus, the remainder of this analysis will follow Case 2 in Section 2.1.

3.2 Comparison of Industry-Specific Growth Rates Across Areas Within States

In development of small domain estimation methods for the domains (iat), one important issue was the comparison of period-specific growth rates for a given industry across metropolitan areas in a given state. If the observed data were consistent with a null hypothesis of equal growth rates, then it might be reasonable to consider use of a synthetic estimator based on state-level growth rates within the given industry. Conversely, if the observed data were not consistent with an assumption of homogeneous growth rates, then a more refined small domain estimator would need to be used.

To study this, consider a set of A areas and define the vector $y_{i\cdot t} = (y_{i1t}, \ldots, y_{iAt})'$; define $\hat{y}_{i\cdot t} = (\hat{y}_{i1t}, \ldots, \hat{y}_{iAt})'$ similarly; and let M_i be an $(A-1)\times A$-dimensional matrix with i-th row equal to M_{ia}, where M_{ia} has its a-th column equal to $y^{-1}_{ia0} - (\sum_{b=1}^{A} y_{ib0})^{-1}$ and all other columns equal to $-(\sum_{b=1}^{A} y_{ib0})^{-1}$. Thus, $M_{ia} y_{i\cdot t}$ is equal to the difference between the growth rate from month 0 to month t in area a, compared with the corresponding aggregate growth rate taken across all A areas in the state. Thus, a null hypothesis of homogeneous growth rates may be written,

$$H_0 : M_i y_{i\cdot t} = 0_{(A-1)\times 1}$$

If one knew the true design variance of $\hat{y}_{i\cdot t}$, then one could test H_0 with the nominal pivotal quantity

$$T_{M0} = (M_i\hat{y}_{i\cdot t} - 0)'\{M_i V_p(\hat{y}_{i\cdot t}) M_i'\}^{-1}(M_i\hat{y}_{i\cdot t} - 0) \quad (3.2)$$

Under H_0 and additional regularity conditions, T_{M0} is distributed (with respect to the sample design) approximately as a central chi-square random variable on $A-1$ degrees of freedom.

Because of the very limited number of degrees of freedom, $d_p = 6$, associated with the direct variance estimator $\hat{V}_{iat}$, substitution of the terms $\hat{V}_{iat}$ into expression (3.2) was inadvisable. Instead, in keeping with the reasoning and results presented at the end of section 3.1, we computed

$$T^*_{M0i} = (M_i\hat{y}_{i\cdot t})'\{M_i V^*_p(\hat{y}_{i\cdot t}) M_i'\}^{-1}(M_i\hat{y}_{i\cdot t}) \quad (3.3)$$

where $V^*_p(\hat{y}_{i\cdot t}) = diag\{V^*_p(\hat{y}_{i1t}), \ldots, V^*_p(\hat{y}_{iAt})\}$

and $V^*_p(\hat{y}_{iat})$ is computed as described in Section 2.2, based on model (3.1). The reasoning presented at the end of Section 3.1 then indicates that under H_0 and additional regularity conditions,

T^*_{M0i} is distributed (with respect to the sample design) approximately as

$$(\hat{d}_{bi} - A + 2)^{-1}\hat{d}_{bi}(A-1)F(A-1, \hat{d}_{bi} - A + 2),$$

where $F(A-1, \hat{d}_{bi}-A+2)$ is a central F random variable on $A-1$ and $\hat{d}_{bi}-A+2$ degrees of freedom. Table 1 reports numerical values of T^{*}_{M0i} and related quantities for three industries: durable goods manufacturing, nondurable goods manufacturing and wholesale trade for the state of Pennsylvania. Note that the applicable number of metropolitan areas, A, varied across industry due to the exclusion of area × industry combinations that had less than or equal to 12 responding sample units. Note especially that for all three industries, the computed test statistic T^{*}_{M0i} is small relative to the corresponding critical value, indicating that we do not have sufficient evidence to reject the null hypothesis of constant growth rates across metropolitan areas.

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Table 1: Tests for Homogeneity of Growth Rates Across Areas, By Industry. Pennsylvania, June 2000

Industry	A	$\hat{d}_{bi}$	T^*_{M0i}	Critical value
Durables	11	20.8	7.1	48.6
Nondurables	9	27.4	2.4	26.2
Wholesale	4	56.2	2.7	8.6