Evaluation of Inferential Performance Through Confidence Bounds for Power Curves

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1. Introduction

In the analysis of complex survey data, one often use a nominal pivotal quantity

$$t_0 = v^{-1/2}(\widehat{\theta} - \theta_0) \tag{1}$$

in a test of the null hypothesis $H_0 : \theta = \theta_0$ vs. $H_1 : \theta \neq \theta_0$, where θ is the univariate parameter of interest and v is an estimator of the variance of the approximate distribution of $\widehat{\theta}$. The test will reject H_0 if

$$|t_0| > z, \tag{2}$$

where z is the $(1 - \alpha/2)$ quantile of standard normal distribution. Under a complex sample design and regularity conditions $v^{-1/2}(\widehat{\theta} - \theta_0) \xrightarrow{\mathcal{L}} N(0, 1)$ (e.g., Skinner, et al. 1989, Subsection 2.14) provided $E(\widehat{\theta}) = \theta_0$; the approximate distribution of $n^{1/2}(\widehat{\theta} - \theta_0)$ is normal with mean equal to zero and variance equal to V; and $nv \to V$ in probability. However, due to limitations of the sample design, the data collection process or proposed estimation methods, test procedures based on t_0 may be affected by: (1) bias of the point estimators, (2) inflation in the variance of the point estimator, and (3) bias of the variance estimator. These limitations can have a serious effect on the power curve of the test, and the coverage rates and mean width of associated confidence intervals. To study this issues, we will develop several results on the asymptotic properties of t_0. In addition, the corresponding power curves and confidence bounds for the power curves are developed. Examples will be given based on the Consumer Expenditure Survey data.

2. Properties of the Nominal Pivotal Quantity under Moderate Deviations from Ideal Conditions

Under the assumption that $\widehat{\theta}$ is unbiased for θ, we may rewrite the hypothesis as

$$H_0 : E(\widehat{\theta}) = \theta_0 \ \text{ vs. } \ H_1 : E(\widehat{\theta}) \neq \theta_0\,, \tag{3}$$

where the expected value is evaluated with respect to the sample design. The following lemma shows the asymptotic properties of the nominal pivotal quantity under moderate deviations from standard idealizied conditions.

Lemma 1. *Assume* $V^{-1/2}n^{1/2}[\widehat{\theta} - E(\widehat{\theta})] \xrightarrow{\mathcal{L}} N(0, 1)$, *and for some constants* $a, b,$ *and* c,
$C_1 = n^{1/2}|E(\widehat{\theta}) - \theta_0| = O(n^{-a})$,
$C_2 = [E(v)/V]^{1/2} = 1 + O(n^{-b})$,
$C_3 = [n\, v/E(v)]^{-1/2} = 1 + O_p(n^{-c})$.
Then, for t_0 *defined in (1), consider the following four cases.*

Case (1): *If* $a, b, c > 0$, *then* $t_0 \xrightarrow{\mathcal{L}} N(0, 1)$.
Case (2): *If* $a, c > 0$, *and* $b = 0$, *then* $[E(v)/V]^{1/2}\, t_0 \xrightarrow{\mathcal{L}} N(0, 1)$.
Case (3): *If* $b, c > 0$, *and* $a = 0$, *then* $t_0 - V^{-1/2}n^{1/2}[E(\widehat{\theta}) - \theta_0] \xrightarrow{\mathcal{L}} N(0, 1)$.
Case (4): *If* $a = b = 0$, *and* $c > 0$, *then* $[E(v)/V]^{1/2}\{t_0 - V^{-1/2}n^{1/2}[E(\widehat{\theta}) - \theta_0]\} \xrightarrow{\mathcal{L}} N(0, 1)$.

Case (1) shows that if we have unbiased point estimator and a variance estimator that is approximately unbiased and relatively stable, then t_0 is distributed asymptotically as a $N(0, 1)$ random variable. Case (2) shows that if we use a variance estimator that has a nontrivial relative bias, the distribution of t_0 will have a variance that will be inflated (deflated) depending on the value of $V/E(v)$. This will affect the significance levels of the hypothesis tests and coverage rates for associated confidence interval as discussed in Skinner, et al. (1989, page 29). Under Case (3), when $b, c > 0$, and $a = 0$, the bias $[E(\widehat{\theta}) - \theta_0]$ has the same order of magnitude as the standard error, i.e. $O(n^{-1/2})$, and it will produce a horizontal shift in the distribution of t_0. Moreover, if $b, c > 0$, and $a < 0$, then the point estimation bias is of larger order of magnitude than that of the standard error. Thus, the bias will dominate the asymptotic properties of t_0 and the test will have a type II error that converges to 1. Section 3 will present more detailed discussion of the issues of associated power of the test. Under Case (4), we have both a nontrivial bias $[E(\widehat{\theta}) - \theta_0]$ and bias in variance estimator. Notice that in addition to Lemma 1, if $a, b > 0$, and $c = 0$, we have limited stability in variance estimator v. The test statistic t_0 is no longer approximately distributed as a normal random variable. If dvV^{-1} follows a chi-square distribution with d degrees of freedom, independent of $\widehat{\theta}$, then t_0 has a t distribution on d degrees of freedom. This will lead to issues of small degrees of freedom problems in complex surveys (see, for example, Korn and Graubard 1990; Eltinge, Parsons, and Jang 1997).

3. Power of the Test

The power of a statistical test is defined as the probability that the test leads to rejection of H_0 given a specific true value of the parameter θ. A power curve may display power to reject H_0 for different possible true values θ defined in the H_1, or display the power to reject H_0 for a fixed value of the parameter θ specified in the point alternative hypothesis H_1 but with different possibly wrong null values θ_0. The curve displays the sensitivity of the test in distinguishing between specific null and alternative hypothesis value of θ.

Power curves provide a useful graphical tool to explore the inferential effects of point estimation bias, variance estimation bias, and variance estimator instability. Bias in a point estimator leads to a corresponding horizontal shift in the associated power curve. A positive (negative) bias in a variance estimator will produce a downward (upward) vertical shift in the power curve. Moreover, inflation in the true variance of a point estimator will attenuate the slope of the power curve at a given horizontal distance from θ_0.

In this section we consider power of the test given in (3) with the test statistic and the decision rule as given in (1) and (2), respectively.

Lemma 2. *Assuming that $V^{-1/2}n^{1/2}[\widehat{\theta} - E(\widehat{\theta})] \xrightarrow{\mathcal{L}} N(0,1)$ and define $\gamma(\theta_0) = pr(|t_0| > z \mid \theta_0)$, the power of the nominal level-α hypothesis test (3) with test statistic defined in (1) and decision rule given in (2), for various possible values of θ_0. Then $\gamma(\theta_0) = 1 - \Phi(B_U) + \Phi(B_L) + o(1)$, where*

$$(B_L, B_U) = \left(V^{-1/2}n^{1/2}[E(\widehat{\theta}) - \theta_0] \pm z\,[E(v)/V]^{1/2}[v/E(v)]^{1/2}\right). \qquad (4)$$

and $\Phi(\cdot)$ is the standard normal distribution function.

For the cases outlined in Lemma 1, we will now discuss the approximation and estimation of the associated power functions.

3.1 The Effect of Bias in Point Estimators

We first consider the power of a test that uses a biased point estimator, and an approximately unbiased and relative stable variance estimator.

Corollary 3.1. *Assume conditions C_1, C_2 and C_3 for some $b, c > 0$, and $a = 0$. In addition, assume that $V^{-1/2}n^{1/2}[\widehat{\theta} - E(\widehat{\theta})] \xrightarrow{\mathcal{L}} N(0,1)$. Then the power function of the nominal level-α hypothesis test of $H_0 : E(\widehat{\theta}) = \theta_0$ with test statistic defined in (1) and decision rule given in (2), for various possible values of θ_0 is given as*

$$\gamma(\theta_0) = 1 - \Phi(B_U) + \Phi(B_L) + O(max(n^{-b}, n^{-c})),$$

where

$$(B_L, B_U) = \left\{V^{-1/2}n^{1/2}[E(\widehat{\theta}) - \theta_0] \pm z\right\}. \qquad (5)$$

The estimated of the true power curve (5) may be obtained by direct subtitution of $\widehat{\theta}$ for its expectation and v_0 for V, so that $\widehat{\gamma}(\theta_0) = 1 - \Phi(\widehat{B}_U) + \Phi(\widehat{B}_L)$, where

$$(\widehat{B}_L, \widehat{B}_U) = \left\{v_0^{-1/2}n^{1/2}[\widehat{\theta} - \theta_0] \pm z\right\}. \qquad (6)$$

Suppose now we have another point estimator $\widetilde{\theta}$, say, for θ. Under the assumption that $V^{-1/2}n^{1/2}[\widetilde{\theta} - E(\widehat{\theta})] \xrightarrow{\mathcal{L}} N(0,1)$, the estimated power for testing $H_0 : E(\widetilde{\theta}) = \theta_0$ is given as $\widehat{\gamma}(\theta_0) = 1 - \Phi(\widehat{B}_U) + \Phi(\widehat{B}_L)$, where

$$(\widehat{B}_L, \widehat{B}_U) = \left\{v_0^{-1/2}n^{1/2}[\widetilde{\theta} - \theta_0] \pm z\right\}. \qquad (7)$$

Overlaid power curves for testing $H_0 : E(\widehat{\theta}) = \theta_0$ and $H_0 : E(\widetilde{\theta}) = \theta_0$ can be used to explore the effect of possible bias in a point estimator to the power. For example, under assumption that $\widetilde{\theta}$ is an unbiased estimator for θ_0, the horizontal shift of the power curve (6) relative to the power curve (7) indicates that $\widehat{\theta}$ is biased for θ_0. If the bias $[E(\widehat{\theta}) - \theta_0]$ is of a larger order of magnitude than that of the standard error, i.e. condition C_1 with $a < 0$, then the test will have a type II error that converges to 1.

3.2 The Effect of Bias in Variance Estimators

Consider two different estimators of the true variance V. First, let $v = v_0$ be a consistent estimator of V. For example, v_0 may be a design based variance estimator constructed using the linearization method or related replication methods (see, for example, Shao 1996). Also, consider a second variance estimator $v = v_1$ that may be biased. For example, v_1 may not fully account for all features of sample design. In keeping with Case (2) and (4) of Lemma 1, now consider a hypothesis test when we have not only a nontrivial bias in point estimator, but also a nontrivial bias in its variance estimator. An example of such a case arises when we use a biased variance estimator v_1.

Corollary 3.2. *Assume conditions C_2 and C_3 for some $c > 0, b = 0$, and assume that $|E(\widehat{\theta}) - \theta_0| \le Mn^{-1/2-a}$ for some $M > 0$ and $a = 0$. In addition, assume that $V^{-1/2}n^{1/2}[\widehat{\theta} - E(\widehat{\theta})] \xrightarrow{\mathcal{L}} N(0,1)$. Then the power of the test of the null hypothesis (3) is given as $\gamma(\theta_0) = 1 - \Phi(B_U) + \Phi(B_L) + O_p(n^{-c})$, where $(B_L, B_U) =$*

$$\left(V^{-1/2}n^{1/2}[E(\widehat{\theta}) - \theta_0] \pm z[E(v)/V]^{1/2}\right). \qquad (8)$$

A bias in a variance estimator will shift the original power curve vertically. In this work we define a positive bias if $E(v)/V > 1$ and the negative bias if $E(v)/V < 1$. A positive bias will produce a downward vertical shift, reflecting the conservative property of the associated test. On the other hand, a negative bias will produce an upward vertical shift, reflecting the fact that the associated test has a type I error in excess of its nominal level α. For example, for a level-α test, at $E(\widehat{\theta}) = \theta_0$ if the variance estimator bias is positive, then $\gamma(\theta_0) < \alpha$. On the other hand, if the bias is negative, $\gamma(\theta_0) > \alpha$.

Direct substitution of $\widehat{\theta}$ for its expectation, v_1 for $E(v)$, and v_0 for V leads to a point estimator of $\gamma(\theta_0)$, $\widehat{\gamma}(\theta_0) = 1 - \Phi(\widehat{B}_U) + \Phi(\widehat{B}_L)$, where

$$(\widehat{B}_L, \widehat{B}_U) = \left(v_0^{-1/2}n^{1/2}(\widehat{\theta} - \theta_0) \pm z\sqrt{v_1/v_0}\right). \qquad (9)$$

Overlaid power curves of (6) and (9) can be used to explore the effect of bias in variance estimator to the power. A vertical shift of power (9) relative to (6) indicates that v_1 is biased for V. In addition, if $v_0 < v_1$, then the slope of estimated power function (6) is steeper than that of (9).

4. Confidence Bounds for a Power Curve

Note that in the work given above, we are assuming that our power is a function only of some known parameters, and the unknown parameter θ. For example, in Subsection 3.1 for the power given in expression (5) we are assuming that $\gamma(\theta)$ is a function of a known variance V and the unknown parameter $E(\widehat{\theta})$.

Let Θ_C be a $(1-\omega)100\%$ confidence set for θ. Then a corresponding $(1-\omega)100\%$ confidence set for $\gamma(\theta)$ is

$$\left\{\min_{\theta\in\Theta_C}[\gamma(\theta)]\ ,\ \max_{\theta\in\Theta_C}[\gamma(\theta)]\right\} = (\gamma_L\ ,\ \gamma_U), \text{ say.} \qquad (10)$$

Therefore, for a fixed null hypothesis,

$$P[\gamma(\theta) \in (\gamma_L\ ,\ \gamma_U)] \ge P[\theta \in \Theta_C] \ge 1 - \omega$$

due to the definitions of $(\gamma_L\ ,\ \gamma_U)$ and Θ_C.

4.1 Unbiased Variance Estimator

Consider the special case defined by the test $H_0 : E(\widehat{\theta}) = \theta_0$ where we assume that our variance estimator is unbiased and stable. Recall from Corollary 3.1 that under these conditions, $\gamma(\theta_0) \doteq 1 - \Phi(B_{\theta U}) + \Phi(B_{\theta L})$, where $(B_{\theta L}, B_{\theta U}) = b_\theta \pm z$ and $b_\theta = V^{-1/2}n^{1/2}[E(\widehat{\theta}) - \theta_0]$. Suppose we have a design based $(1-\omega)100\%$ confidence set Θ_C for $E(\widehat{\theta})$. For example, under the usual normal approximations, Θ_C can be calculated as $\widehat{\theta} \pm z_{(1-\omega/2)}\sqrt{v_0}$. In addition, let

$$B_C = \{b_\theta : \theta \in \Theta_C\} = (b_{\theta L}, b_{\theta U})$$

be the corresponding confidence set for b_θ. Application of (10) leads to the confidence interval for $\gamma(\theta)$

$$(\gamma_L, \gamma_U) = \left\{\min_{b_\theta\in B_C}[1 - \Phi(B_{\theta U}) + \Phi(B_{\theta L})]\ ,\right.$$
$$\left.\max_{b_\theta\in B_C}[1 - \Phi(B_{\theta U}) + \Phi(B_{\theta L})]\right\}.$$

Arguments similar to those for the power function of a two-sided uniformly most powerful unbiased level α test (Lehmann 1986, pp. 101-103) show that $\gamma(\theta) = 1 - \Phi(b_\theta + z) + \Phi(b_\theta - z)$ is not monotone in b_θ. However, for $b_\theta \in (-\infty, 0)$, $\gamma(\theta)$ is strictly decreasing; and for $b_\theta \in (0, \infty)$, $\gamma(\theta)$ is strictly increasing. In particular, $\lim_{b_\theta\to-\infty}\Phi(b_\theta \pm z) = 0$ and $\lim_{b_\theta\to+\infty}\Phi(b_\theta \pm z) = 1$, so $\gamma(\theta) \uparrow 1$ as $|b_\theta| \to \infty$, and $\gamma(\theta)$ will reach its minimum value of α at $b_\theta = 0$. Thus, an approximate $(1-\omega)100\%$ confidence interval for $\gamma(\theta)$ as defined in (10) can be constructed as follows

(a) If $(b_{\theta L}, b_{\theta U}) \ni 0$, $\gamma_L = \alpha$, and

$$\gamma_U = \max[1 - \Phi(b_{\theta U} + z) + \Phi(b_{\theta U} - z)\ , \ 1 - \Phi(b_{\theta L} + z) + \Phi(b_{\theta L} - z)].$$

(b) If $b_{\theta U} < 0$, then $\gamma_L = 1 - \Phi(b_{\theta U} + z) + \Phi(b_{\theta U} - z)$ and $\gamma_U = 1 - \Phi(b_{\theta L} + z) + \Phi(b_{\theta L} - z)$.

(c) If $b_{\theta L} > 0$, then $\gamma_L = 1 - \Phi(b_{\theta L} + z) + \Phi(b_{\theta L} - z)$ and $\gamma_U = 1 - \Phi(b_{\theta U} + z) + \Phi(b_{\theta U} - z)$.

Figure 1 shows confidence bounds of power curve with several values of confidence levels under the assumption that the variance estimator used in the test is unbiased and relatively stable.

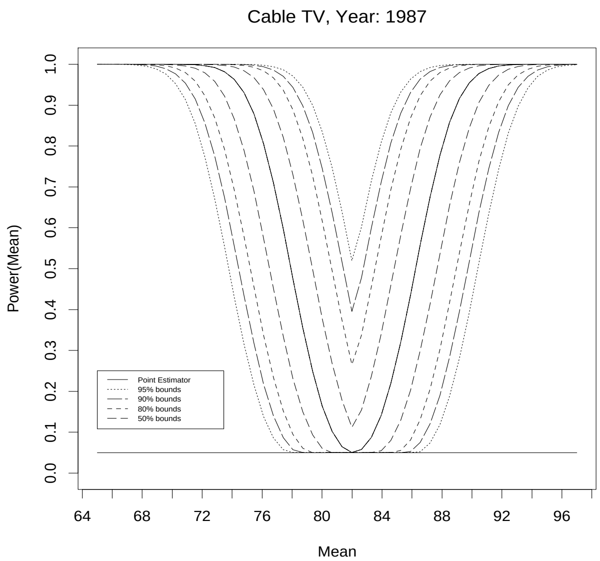

Figure 1: The 95%, 90%, and 50% confidence bounds on power curve of the $\alpha = 5\%$ test.

4.2 Biased Variance Estimator

Now consider the hypothesis test (3) where we have bias in the variance estimator v; for example, $v = v_1$. Recall from Corollary 3.2, $\gamma(\theta_0) \doteq 1 - \Phi(B_{\theta U}) + \Phi(B_{\theta L})$, where $(B_{\theta L}, B_{\theta U}) = b_\theta \pm z[E(v)/V]^{1/2}$. In addition, assume that we have a second variance estimator $\tilde{v}$ that is approximately unbiased for V. Such an example is $\tilde{v} = v_0$. In addition to condition in Lemma 1, we assume condition C_4 as follows

$$C_4 = \{n\tilde{v}\}^{-1/2} E(\tilde{v})^{1/2} = 1 + O_p(n^{-d}) \text{ for some constant } d.$$

Now consider two cases,

Case (I): Assume conditions C_2, C_3, and C_4 such that $b, c, d > 0$, so that the variability in v and $\tilde{v}$ is small relative to the variability in $\widehat{\theta}$. Then a $(1-\omega)100\%$ confidence interval for $\gamma(\theta_0)$ follows from a $(1-\omega)100\%$ confidence set for $E(\widehat{\theta})$ and from expression (8). Similar to those in Subsection 4.1, let Θ_C be the $(1-\omega)100\%$ design based confidence set for $E(\widehat{\theta})$, and let $B_C = \{b_\theta : \theta \in \Theta_C\} = (b_{\theta L}, b_{\theta U})$ be the corresponding confidence set for b_θ. Then, an algorithm similar to that for the confidence set in Subsection 4.1 can be constructed as follows

(a) If $(b_{\theta L}, b_{\theta U}) \ni 0$, then
$\gamma_L = 1 - \Phi(+z\sqrt{v/\tilde{v}}) + \Phi(-z\sqrt{v/\tilde{v}})$,
and

$$\gamma_U = \max\Big[1 - \Phi(b_{\theta U} + z\sqrt{v/\tilde{v}}) + \Phi(b_{\theta U} - z\sqrt{v/\tilde{v}})\ , \ 1 - \Phi(b_{\theta L} + z\sqrt{v/\tilde{v}}) + \Phi(b_{\theta L} - z\sqrt{v/\tilde{v}})\Big].$$

(b) If $b_{\theta U} < 0$, then
$\gamma_L = 1 - \Phi(b_{\theta U} + z\sqrt{v/\tilde{v}}) + \Phi(b_{\theta U} - z\sqrt{v/\tilde{v}})$
and
$\gamma_U = 1 - \Phi(b_{\theta L} + z\sqrt{v/\tilde{v}}) + \Phi(b_{\theta L} - z\sqrt{v/\tilde{v}})$.

(c) If $b_{\theta L} > 0$, then
$\gamma_L = 1 - \Phi(b_{\theta L} + z\sqrt{v/\tilde{v}}) + \Phi(b_{\theta L} - z\sqrt{v/\tilde{v}})$
and
$\gamma_U = 1 - \Phi(b_{\theta U} + z\sqrt{v/\tilde{v}}) + \Phi(b_{\theta U} - z\sqrt{v/\tilde{v}})$.

Case (II): Assume conditions C_2, C_3, and C_4 such that $b, c > 0$, but assume that $\tilde{v}$ has a nontrivial amount of variability. Then a confidence interval for $\gamma(\theta_0)$ is developed based on confidence set of vector of parameters $\psi = (\theta, V)'$. Suppose $\Psi_{1-\omega}$ is an approximate $(1-\omega)100\%$ confidence set for ψ treating v as known and equal to $E(v)$. Then, a confidence interval for $\gamma(\theta_0)$ is defined as

$$(\gamma_L\ ,\ \gamma_U) = \left\{\min_{\psi \in \Psi_{1-\omega}} [\gamma(\theta)]\ ,\ \max_{\psi \in \Psi_{1-\omega}} [\gamma(\theta)]\right\}. \quad (11)$$

Since ψ contains two parameters θ and V, then we have three possible confidence sets for $\Psi_{1-\omega}$ based on dependency/independency of θ and V. First, let $\Theta_{1-\omega} = [\widehat{\theta} \pm z_{(1-\omega/2)}\tilde{v}^{1/2}]$ be the usual normal approximation $(1-\omega)100\%$ confidence interval for θ. Second, if we assume that $d\tilde{v}V^{-1}$ is distributed as a chi-square random variable on d degrees of freedom, then $pr[c^{-1}_{1-\omega/2,d}\ d\ \tilde{v} < V < c^{-1}_{\omega/2,d}\ d\ \tilde{v}] = 1 - \omega$, where $c_{1-\omega/2,d}$ is the $(1-\omega/2)$ quantile of a chi-square distribution on d degrees of freedom. Thus, the corresponding $(1-\omega)100\%$ confidence interval for V is $\mathcal{V}_{1-\omega} = (c^{-1}_{1-\omega/2,d}\ d\ \tilde{v}\ ,\ c^{-1}_{\omega/2,d}\ d\ \tilde{v})$.

Now, three possible confidence sets for $\Psi_{1-\omega}$ can be computed as follows.

Case (IIa): Assume that $n^{1/2}(\widehat{\theta} - \theta) \sim N(0\ ,\ V)$

independent of $V^{-1}d\,\tilde{v} \sim \chi^2_d$. Then

$$\Psi^{(I)}_{1-\omega} = \{(\theta, V) : \theta \in \Theta_{(1-\omega)^{1/2}} \text{ and } V \in \mathcal{V}_{(1-\omega)^{1/2}}\}$$

where $\Theta_{(1-\omega)^{1/2}}$ and $\mathcal{V}_{(1-\omega)^{1/2}}$ are $(1-\omega)^{1/2}100\%$ confidence sets for θ and V respectively.

Case (IIb): If we can not reasonably assume that $\tilde{v}$ is independent of $\widehat{\theta}$, then we can construct the Bonferroni confidence set

$$\Psi^{(B)}_{1-\omega} = \{(\theta, V) : \theta \in \Theta_{1-\omega/2} \text{ and } V \in \mathcal{V}_{1-\omega/2}\} .$$

Case (IIc): When d is large, we may assume that $n^{1/2}[(\widehat{\theta}, \tilde{v})' - (\theta, V)'] \sim N(0, \Omega)$. Given an approximately unbiased estimator $\widehat{\Omega}$ of Ω, we may construct a Scheffé type confidence set $\Psi^{(S)}_{1-\omega} =$

$$\{(\theta, V) : [(\widehat{\theta}, \tilde{v}) - (\theta, V)]n\widehat{\Omega}^{-1}[(\widehat{\theta}, \tilde{v}) - (\theta, V)]' \\ \leq (2d)/(d-1)F^{\omega}_{2,d-1}\}$$

where $F^{\omega}_{2,d-1}$ is the upper ω point of an F distribution with 2 and $(d-1)$ degrees of freedom. Then, the confidence interval for $\gamma(\theta_0)$ follows from (11).

5. Test of the Expected Value of the Mean Estimators for the Consumer Expenditure Survey

Empirical example in this paper is motivated based on the work in Eltinge, Sukasih, and Weber (2002), where the authors proposed a model-assisted estimator for the annual means of group expenditures (known as the six digit Universal Classification Code or UCC) from the U.S. Consumer Expenditure Survey (CES) data. CES collects the data through two modes of data collection, known as the diary and the interview, respectively. Some UCCs are collected through the interview only, some others are collected through the dairy only, and still some others are collected through both the diary and the interview. For some UCCs that are collected through both the diary and the interview, data are available from five potential data sources: (1) interview data for the most recent record month, (2) interview data for the second recent record month, (3) interview data for the most distant record month, (4) diary data from the first week, and (5) diary data from the second week of data collection. However, the currently published estimates are based only on the diary data, where the variances are calculated through the Balanced Repeated Replication (BRR) method.

Eltinge, et al. (2002) proposed a generalized linear model that models the relationship between estimated means for the five data sources and the true mean value. Under this model, the point estimator (and its variance) that combined all five data sources can be calculated through the generalized least-squares method. In addition, since CES provides replicates data, then the variance for the proposed estimator can also be calculated through the BRR method. Thus, for our empirical example here, two different point estimators (let's call them as the *direct estimator* and the *estimated generalized least-squares* (or EGLS) estimator) are available for comparison. In addition, for the EGLS estimator there are two variance estimators available, i.e. based on EGLSE and BRR methods.

5.1 The Effect of Bias in Point Estimators

Under the assumption that the two diaries are unbiased, let $\widetilde{\theta}$ be the direct estimator calculated as a simple mean of two diaries. In addition, let $\widehat{\theta}$ be the EGLS estimator. Let $v(\widetilde{\theta})$ and $v(\widehat{\theta})$, respectively, denotes the variance computed through the BRR method.

The estimated power of the test

$$H_0 : E(\widehat{\theta}) = \theta_0 \tag{12}$$

using test statistic $t_1 = [v(\widehat{\theta})]^{-1/2}(\widehat{\theta} - \theta_0)$, can be overlaid with the estimated power of the test

$$H_0 : E(\widetilde{\theta}) = \theta_0 \tag{13}$$

using test statistic $t_2 = [v(\widetilde{\theta})]^{-1/2}(\widetilde{\theta} - \theta_0)$. Under the assumption that $\widetilde{\theta}$ is unbiased for θ_0, if $\widehat{\theta}$ is significantly biased, we will see a significant horizontal shift in the estimated power curves of (12) relative to those of (13).

Figure 2 represents overlaid power curves of testing (12) and (13). In Figure 2 the dashed lines represent the power curves for testing $H_0 : E(\widehat{\theta}) = \theta_0$. In addition, the solid lines represent the power curves for testing $H_0 : E(\widetilde{\theta}) = \theta_0$. Under the assumption that $\widetilde{\theta}$ is unbiased for θ_0, then for Cable TV (figures (B) and (C)), estimator $\widehat{\theta}$ shows some pronounced bias, indicated by horizontal shift of the power curves for testing $H_0 : E(\widehat{\theta}) = \theta_0$ relative to the power curves for testing $H_0 : E(\widetilde{\theta}) = \theta_0$. On the other hand, for Men's Accessories and Sewing Patterns (figures (A) and (D)), there are no substantial bias effects of $\widehat{\theta}$ to the power curves.

5.2 The Effect of Bias in Variance Estimators

Two possible variance estimators may be used to test $H_0 : E(\widehat{\theta}) = \theta_0$ with test statistic $t_0 = (\widehat{\theta} - \theta_0)/v^{1/2}$. We may use variance estimator $v = v_0$ computed through the BRR method, or variance estimator $v =$

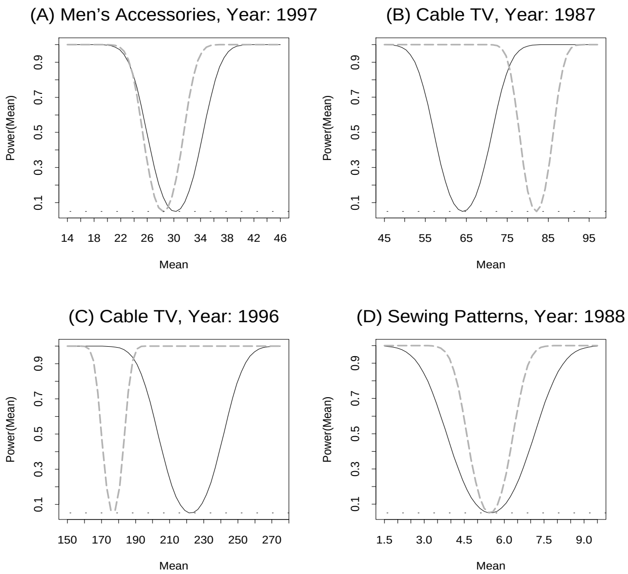

Figure 2: Power curves of testing H_0 with unbiased and stable variance estimator, where point estimator is unbiased (A and D), negative biased (B), and positive biased (C).

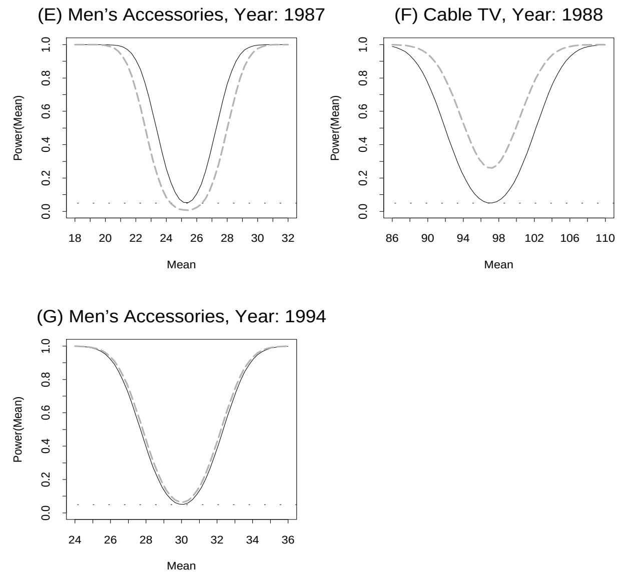

Figure 3: Power curves of testing H_0 with positive biased variance estimator (E), negative biased variance estimator (F), and unbiased variance estimator (G).

v_1 computed directly through the generalized least squares method. However, under assumption that v_0 is unbiased, then the use of v_1 may results in vertical shift (upward or downward) of the power curve as explained in Subsection 3.2.

Figure 3 represents the overlaid estimated power curves of $H_0 : E(\widehat{\theta}) = \theta_0$ with test statistic $t_0 = (\widehat{\theta} - \theta_0)/v^{1/2}$, using $v = v_0$ and $v = v_1$, separately. The solid lines is calculated with $v = v_0$, i.e. under the assumption that v_0 is the unbiased variance estimator, whereas the dashed lines is calculated with $v = v_1$. Figures (E) shows an downward vertical shift, indicating a positive bias of v_1 relative to v_0. On the other hand, figures (F) shows an upward vertical shift, indicating a negative bias of v_1 relative to v_0. However, in figures (G) v_1 may not have a significant bias effect relative to v_0.

6. References

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