Using Isotonic Regression to Smooth State-Level Variance Estimates from a National Complex-Design Survey

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1. Introduction

In large-scale government surveys, sample designs based upon complex probability sampling methods are typically used. The resulting data are frequently analyzed using so-called design-based or randomization inference and not with strong model-based assumptions about sampling distributions. Such design-based techniques are considered an objective approach to data analysis, and are often used in policy making. The reader is referred to Särndal et al. (1992, ch. 1) for basic ideas about design-based and model-based survey design inference. For design-based analyses to be performed, design features (e.g, strata, sampling clusters, selection weights) must be identified, but because of confidentiality concerns, design features on public-use micro data are only released in a coarse or masked form, thus restricting study to national domains.

The National Health Interview Survey (NHIS), a complex clustered sample of about $40,000$ households, follows such a data release strategy. The 50 states and District of Columbia are in fact sampling strata for the NHIS, but state and sub-state geographical identifiers are not released to the public. To partially satisfy external needs for statistics on smaller geographical domains, a sponsoring agency, like the National Center for Health Statistics (NCHS), the sponsor of the NHIS, can produce internally many basic statistics, e.g., estimated means and proportions, for the geographical domains not accessible to the public. These statistics can then be released with a measure of reliability, usually an estimate of standard error or coefficient of variation. In this paper we focus upon the state level geographical domains, but the methods discussed can be applied to other subnational domains.

Given the power of modern computer machinery, the computation of complex-design means and associated standard errors is relatively inexpensive, and at least conceptually, state estimates and their standard errors could be internally computed and released to the public. A major concern, however, is that many state domain statistics have unstable and/or biased design-based variance estimators. At NCHS the "institutionalized" production method for producing standard errors for means and proportions is the Taylor-linearization method implemented with commercially available software. Such a production method works well for national level domains, but at the state level the sampling units from which the variance estimator is constructed, e.g., primary and secondary unit clusters, may be few in number, thus resulting in an unstable variance estimator. For example, several states while having NHIS samples in excess of 200 households, have relatively few clusters, thus resulting in a variance estimator with a small associated degrees of freedom. Furthermore, as Särndal et al. (1992, sec. 5.5) point out, the Taylor linearization method has a tendency to underestimate variances for "small" samples. Part of this problem is due to the substitution of estimated expectations for true values in the linearized forms. Empirical evidence based on NHIS state tabulations suggests that this is a problem for about half of the states.

Any agency-produced report on state estimates would most likely be targeted to an audience focused on the first-order estimates. In-depth discussion on topics of variance estimator bias and stability would most likely not be appropriate to such an audience. To meet such needs, a reasonable strategy might be to keep the direct design-based first-order estimates, but smooth out the design-based variances using a modeling approach.

2. Models for Smoothing Variances

First, we provide a general mathematical framework for our problem. Suppose that D is a complex design and for a characteristic x on subdomain d of state s let $\hat{p}_{sdx}$ be the usual design-based estimator of proportion for the true population value p_{sdx}. The usual estimator takes the form $\sum w_i x_i I_i(sd) / \sum w_i I_i(sd)$

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where $I_i(d) = 1(0)$ if unit i is in state domain sd, and w_i are the sampling weights with possible adjustments.

We assume that $\hat{p}_{sdx}$ is an unbiased estimator, $E_D(\hat{p}_{sdx}) = p_{sdx}$, and the estimator has design variance $\text{Var}_D(\hat{p}_{sdx})$ and variance estimator $\hat{v}_{sdx}$. As discussed in the introduction, the variance estimator for a target state domain may be both unacceptably unstable and biased when its definition is a function of relatively few sample clusters. By "smoothing" a collection of estimated variances subject to some realistic structural model, we can often reduce the impact of these deficiencies.

The simplest model that is frequently used to smooth the variances is the design-effects model. Here, for our purposes, the design-effect, *deff*, for a characteristic x on state domain unit sd is defined as $\text{deff}_{sdx} = \text{Var}_D(\hat{p}_{sdx}) \cdot \left(\frac{p_{sdx}(1-p_{sdx})}{n_{sd}}\right)^{-1}$, where n_{sd} is the expectation of observed sample size on state domain sd. This is the ratio of complex variance to that of a variance from a hypothetical simple random sample of size n_{sd} on domain sd.

A more general complex-design variance parametric model as used in Johnson and King (1987) is

$$\text{Var}_D(\hat{p}_{sdx}) = k \cdot \left(\frac{p_{sdx}(1-p_{sdx})}{n_{sd}}\right)^{\beta} \quad (1.1)$$

for characteristic x and domain sd. Note, if $\beta = 1$, then this model is equivalent to the universal design effect model, and if $\beta > 1$ this model implies that deff_{sdx} is an increasing function of the simple random sample variance, $\frac{p_{sdx}(1-p_{sdx})}{n_d}$. While a simple parametric model may perform well for large domains sd, a set of diverse domains may need more domain-specific parameters, say β_{sd} and k_{sd} to explain the variances. Wolter (1985, ch. 5) discusses this topic of generalized variance functions (GVFs) in great detail.

Instead of using the parametric GVF modeling approach to smoothing the variances, we will impose a non-parametric functional form that makes use of what we feel are some natural monotonic relations among state effective sample sizes, and the structural form of the true variances.

To establish the structure, we first note that if (a, b) and $w(a, b)$ represent a point and an associated non-negative weight on a rectangular grid, then a function $f(a, b)$ is said to be monotonic on the rectangular grid if $a_1 \leq a_2$ and $b_1 \leq b_2$ imply $f(a_1, b_1) \leq f(a_2, b_2)$ whenever $w(a, b) > 0$. That is, for any fixed point (a, b) in the grid with positive weight, all f points to the upper right are equal or larger than $f(a, b)$ and f points to the lower left are equal or smaller than $f(a, b)$. Other f points have no relation with (a, b). This model is referred to as an *isotonic regression* model on a two dimensional grid. The parametric function of equation (1.1) satisfies such conditions for $a = p_{sdx}(1 - p_{sdx})$ and $b = \frac{1}{n_{sd}}$. The reader may refer to Robertson, et al. (1988) for a discussion of such structures.

To define a monotonic grid structure for the problem at hand, we start with $\mathcal{A} = \{s, d, x\}$ as representing a select set of states, s, state domains, d, and characteristics, x. Corresponding to $\mathcal{A}$ are $\hat{p}_{sdx}$, p_{sdx}, $\hat{v}_{sdx}$ and $\text{Var}_D(\hat{p}_{sdx})$ as defined earlier. To each $\hat{v}_{sdx}$ corresponds a measure of its stability, e.g., degrees of freedom, which we will express as a weight w_{sdx}. Any modeling requires some commonalities on the distributional properties of the elements of $\mathcal{A}$. As discussed in Wolter (1985, ch. 5.2 and 5.3) there is a lack of rigorous theory for GVF procedures and considerable care must be taken in grouping the statistics under consideration. With these caveats in mind, we express some distributional assumptions about $\mathcal{A}$:

C.1 General properties of $\mathcal{A}$

i. The estimators can be considered independent from state-to-state.

ii. The state domain sample sizes, $\hat{n}_{sd}$, are considered as approximately fixed from sample to sample. Thus, $\hat{n}_{sd} \doteq E_D(\hat{n}_{sd}) \equiv n_{sd}$.

C.2 Within-state properties of $\mathcal{A}$:

i. The design effect is only a function of the characteristic x, $\text{deff}_{sdx} = \text{deff}_{sx}$ for state domain d.

ii. If state domains d_1 and d_2 satisfy $\frac{p_{sd_1x}(1-p_{sd_1x})}{n_{sd_1}} \leq \frac{p_{sd_2y}(1-p_{sd_2y})}{n_{sd_2}}$, then $\text{Var}_D(\hat{p}_{sd_1x}) \leq \text{Var}_D(\hat{p}_{sd_2y})$

Condition C.2 (i.) requires that if a given characteristic is considered over several different state domains then we expect the sampling and weighting to have about the same impact on variances regardless of domain. Condition C.2 (ii.) requires the order imposed on the variances by a hypothetical simple random sample is preserved with the complex design; such a constraint is also implicit in equation (1.1) for $\beta \geq 0$

C.3 Between-state properties of $\mathcal{A}$:

First, assuming that conditions C.1 and C.2 hold and letting

$\tilde{n}_s = \max\{n_{sd} |\, d$ a domain in state $s\}$ we can express

$$\begin{aligned}\mathrm{Var}_D(\hat{p}_{sdx}) &= \left[\frac{\tilde{n}_s}{n_{sd}} p_{sdx}(1-p_{sdx})\right] \cdot \left[\frac{\mathrm{deff}_{sx}}{\tilde{n}_s}\right] \\ &\equiv [b_{sdx}] \cdot [a'_{sdx}] \qquad (1.2)\end{aligned}$$

We see that from a purely theoretical view that the function Var_D is monotonic on the grid subset $\{(a'_{sdx}, b_{sdx})\}$. Obviously, the value of a' directly depends upon the value of Var_D which is the target of estimation. This form of a' suggests, however, a method for establishing an ordering useful in practice. Using sampling design information and prior knowledge of the variables of interest, we will attempt to capture the orderings of the $\frac{\mathrm{deff}_{sx}}{\tilde{n}_s}$ for $\mathcal{A}$ in a somewhat coarser form than that defined by a'. We focus on forming classes of states ordered by a measure of effective sample sizes.

i. The elements of $\mathcal{A}$ are such that there exist classes of *distinct* states, $\mathcal{S}_1 \preceq \mathcal{S}_2 \ldots \preceq \mathcal{S}_K$, defined such that if $s_1 \in \mathcal{S}_1$ and $s_2 \in \mathcal{S}_2$ then it is conjectured that $\frac{\mathrm{deff}_{s_1x}}{\tilde{n}_{s_1}} \leq \frac{\mathrm{deff}_{s_2y}}{\tilde{n}_{s_2}}$ holds whenever $b_{s_1d_1x} \leq b_{s_2d_2y}$.
For such an ordering we would define $a_1 \leq a_2 \leq \ldots \leq a_K$ to correspond to one dimension of the grid.

ii. Within a class $\mathcal{S}_i$ if $s_1 \neq s_2$ then it is conjectured that $\frac{\mathrm{deff}_{s_1x}}{\tilde{n}_{s_1}} \leq \frac{\mathrm{deff}_{s_2y}}{\tilde{n}_{s_2}}$ holds whenever $b_{s_1d_1x} \leq b_{s_2d_2y}$ subject to the class $\mathcal{S}_{i-1}$ and $\mathcal{S}_{i+1}$ constraints.

Combining conditions C.2 and C.3 we have the monotonicity of the variance on the grid. Establishing such a set of a's is part of the modeling process, and the degree of success strongly depends upon how well the characteristics and domains have been grouped in forming the set $\mathcal{A}$. We discuss a modeling technique to define the a's in the next section.

Now, while the b's are based upon unknown parameters, we would estimate those quantities with the sample proportions $\hat{p}_{sdx}$. This is the same type of strategy that one uses with a parametric model of equation(1.1). In practice, once we model the a's we will have the grid points $\{(a, \hat{b}_{sdx})\}$ along with the estimated design-based variances $\hat{v}_{sdx}$ and weight function $w_{sdx} > 0$ for observed points.

With this information we determine the closest monotonic function, v_{ab}, to the estimated variances by minimizing $\sum_{a,b}(\hat{v}_{sdx} - v_{ab})^2 w_{sdx}$ subject to monotonicity on the grid points (a, b). This order-restricted least squares problem is discussed in Robertson et al. (1988), and algorithms and programs for this specific least squares problem can be found in Bril et al. (1984) and Qian and Eddy (1996).

3. Application to NHIS State estimates of Variance

Now, the NHIS design, documented in Botman et al. (2000), can be thought of as a multistage cluster sampling design. While state weights are currently not produced for the NHIS, a state-level weight would typically have three components: inverses of probabilities selection, non-response adjustment, and a poststratification adjustment to Census control totals specific to each state. We now outline the methods used for NHIS modeling.

3.1 Simplified NHIS design

Available to us were the universe of PSUs along with all first-stage strata and Census projected tabulations by race-ethnic distribution within each universe PSU. All first-stage probabilities of selection, marginal and joint, for PSUs were available, but while the second-stage substrata and sampling rates were well-defined, only limited second stage clustering information was available within the universe PSUs. Similarly, higher level universe information was not available. Given this information we conceptualized the NHIS as a two-stage design, having all the original first-stage information, but we treated the within-PSU sampling as reasonably modeled by a simple random sample from three person-level substrata: Hispanics, Non-Hispanic blacks and all others. Three differential sampling rates, somewhat consistent with observed NHIS sampling rates for these three groups, were used. Furthermore, three within-substrata design effects were used to help account for the higher levels of sampling for which information was not available.
Next, considering a finite population unit j specified by igj within substratum g of PSU i, we assume this unit has overall sampling weight w_g if selected. These sampling weights satisfy the structural relation
$w_g = \frac{M_{ig}}{\pi_i m_{ig}}$, $g = 1, 2, 3$, where π_i is the first-stage selection probability, and M_{ig}, m_{ig} are the number of universe and sample units respectively.

Now, for a domain d we will treat units in substratum ig belonging to d as a sampling substratum,

$ig \cap d$, instead of treating d as a random characteristic. We will assume that the number of universe units, $M_{ig\cap d}$, is known. Under this assumption, we may absorb the index d into the index g, say $g \equiv g \cap d$ to simplify notation of expressions.

With these basic structures a D-design unbiased estimator for arbitrary characteristic z on a domain d for a given PSU i is of the form

$$\hat{Z}_{di} = \sum_{g=1}^{3} \frac{M_{ig}}{m_{ig}} \sum_{j=1}^{m_{ig}} z_{igj}$$

and the form of a generic stratum estimator is

$\hat{Z}_d = \sum_i \delta_i \frac{\hat{Z}_{di}}{\pi_i}$ where δ_i is the inclusion variable for PSU i.

We see that $\hat{Z}_d$ is just a 2-stage Horvitz-Thompson estimator, which has mean
$E_D(\hat{Z}_d) = \sum_i \sum_{g=1}^{3} \sum_{j=1}^{M_{ig}} z_{igj}$
and has approximate variance (assuming negligible 2^{nd}-stage sampling fractions)

$$\begin{aligned} \mathrm{Var}_D(\hat{Z}_d) &\doteq \sum_{i>k} (\pi_i \pi_k - \pi_{ik}) \left(\frac{Z_{di}}{\pi_i} - \frac{Z_{dk}}{\pi_k} \right)^2 \\ &+ \sum_{i=1}^{N} \frac{1}{\pi_i} \sum_{g=1}^{3} \frac{M_{ig}^2}{m_{gi}} S^2(z_{ig}) \cdot \gamma_g \qquad (1.3) \end{aligned}$$

where S^2 is the population variance at level ig, and γ_g is a modeled within-PSU sampling design effect.

For the super population process, ξ, we assume all design information including domains of interest is fixed on each universe unit igj, but that ξ generates a variable z_{igj} to each unit. The totality of points provides a finite population universe. Thus the form of equation (1.3) remains the same for all z realizations. Our target statistics are proportions, so we attempt to define ξ as emulating the unit components of a linearized proportion. For our purposes we assumed a simple random effects model, $z_{igj} = \alpha_i + e_{igj}$ where the variables are independent with zero means and respective variances σ_α^2 and σ_e^2.

Taking expectations with respect to the ξ variable greatly simplifies the usual finite population form of variance expressed in equation (1.3) . Using first and second moments we can express:

$$E_\xi(V_D(\hat{Z}_d)) \doteq \sum_i \left(\sigma_\alpha^2 (\textstyle\sum_g M_{ig})^2 + \sigma_e^2 (\sum_g M_{ig}) \right) \left(\frac{1-\pi_i}{\pi_i}\right) + \sigma_e^2 \sum_i \sum_g M_{ig} w_g \gamma_g$$

where the first and second terms are the ξ-expectations of the between-variance and within-variance terms, respectively.

Calculations done in the same spirit show that if $\hat{m}_d$ is the sample size on domain d then $E_\xi E_D(\hat{m}_d) = E_D(\hat{m}_d) = \sum_{g=1}^{3} \frac{M_{gd}}{w_g}$, where M_{gd} is the universe total of $g \cap d$ values, and for a simple random sample of size $E_D(\hat{m}_d)$:

$$E_\xi \mathrm{Var}_{\mathrm{SRS}}(\hat{Z}_{srs,d}) \doteq M_d / E_D(\hat{m}_d) \times \left(\sigma_\alpha^2 (M_d - (\textstyle\sum_i M_{id})^2 / M_d) + \sigma_e^2 (M_d) \right) \quad (1.4),$$

where M_d is the universe of d values and M_{id} is the universe of d values on PSU i.

The ratio of equations (1.3) and (1.4) can be used to define a superpopulation mean design effect, $\mathrm{deff}_{\xi sd}$, when applied at the state level s. Of the parameters in the definition of $\mathrm{deff}_{\xi sd}$ only the γ_g's and the ratio $\frac{\sigma_e^2}{\sigma_\alpha^2}$ are flexible for defining the magnitude. Our approach was to define these 4 parameters based upon our understanding of the national NHIS and then apply these values to the states. For example, the γ_g's could be defined as $1.10, 1.10, 1.05$ as within-PSU design effects for the three classes of substrata: Hispanic, black and other, to reflect the greater variability observed in the NHIS of minority sample weights compared to non-minority sample weights. The ratio $\frac{\sigma_e^2}{\sigma_\alpha^2}$ can be defined to give an expected 5% between-component of variance at the national level. Examples are discussed in the next section. Also, if the domains d in $\mathcal{A}$ are selectively chosen so that $\frac{M_{id}}{M_{id'}} \doteq k(d, d')$, a constant depending only on d and d', then $\mathrm{deff}_{\xi sd} \doteq \mathrm{deff}_{\xi sd'}$ thus making condition C.2 (i.) reasonable.

As a final ξ-application, we assess the weight associated with each $\hat{v}_{sdx}$ as a Satterwaith-type degrees of freedom, computed by the ratio

$df_{\xi sd} = \frac{2(E_\xi E_D(\hat{v}_{sdz}))^2}{E_\xi \mathrm{Var}_D(\hat{v}_{sdz})}$ where here, we treated the variance estimator as a two-stage Yates-Grundy-Sen form using totals from the first-stage PSU sampling and second-stage sampling units. This form is consistent with the form used by the production software. For our simplified structures, the second-stage sampling units were treated as clusters of second-stage units discussed when presenting the first-order approximations. Furthermore, we assumed normal distributions of the ξ to facilitate computations of the second moments, and d chosen so that $df_{\xi sd}$ was a function of the state and not of the domain d.

3.2 Numerical results

As mentioned earlier, no "official" state statistics have yet been produced from the NHIS for public dissemination. For this paper we used a 1997 NHIS adult sample database of 10 health variables and several state domains for which some experimental poststratified state-level proportions and corresponding standard errors, produced using the SUDAAN software, were available.

Since the adult sample selects just one adult per family, we hypothesized that the effects of weighting and clustering should be similar for male, female and both combined. Also the relation of the last section: $\frac{M_{id}}{M_{id'}} \doteq k(d, d')$, a constant, seems somewhat reasonable for these domains, so these three domains were considered for smoothing.

First, we determined values for $\mathrm{deff}_{\xi s}$ that provided effective sample sizes that were somewhat consistent with the observed data. Past experience and knowledge of the design structures suggested that most NHIS sampling variation would result from the within-PSU sampling. We chose the parameters γ_g's and $\frac{\sigma_e^2}{\sigma_\alpha^2}$ as given in section 4.1. These 4 parameters applied to formulas in section 4.1 resulted in a national $\mathrm{deff}_\xi = 1.19$, and the states' ξ-design effects fluctuated about this value. The $\mathrm{deff}_{s\xi}$ and resulting state effective sample sizes are given in Table 1. Note, that the ordering induced by ξ-smoothing has an impact only if the nominal sample sizes tend to be close in magnitude. Of the 10 states with the largest nominal sample sizes, only Pennsylvania and Illinois were reversed in order. In general, increasing a state's race/ethnic populations increases the variation in sampling weights thus increasing the design effect. Furthermore, states with a large nonselfrepresenting component will tend to have larger clustering effects thus increasing the design effect. These two factors result in the state fluctuations about the national 1.19 figure. If the effective sample sizes were close, then the states were combined to the same $\mathcal{S}_i$ class. This combining operation was somewhat subjective, and reduces the impact of the overall ordering constraints.

The ξ-stability measure of the variance estimator, the degrees of freedom, $df_{\xi s}$, is highly dependent upon the non-selfrepresenting allocation of sample within the state. The degrees of freedom parameter provided in Table 1. should only be considered as a relative weight for the least squares fitting.

We used the general principles of variable selection for $\mathcal{A}$ as discussed by Wolter (1985 ch. 5) to identify good candidate health variables for smoothing. Based upon empirically observed commonalities required by conditions C.2 and C.1., our final set $\mathcal{A}$ consisted of 5 prevalence variables: former smoker, reported hearing loss, reported asthma, reported overweight, and reported obesity. As discussed in the introduction, many small states have negatively biased estimates of standard error. Because of the extreme nature of this bias, we modified the directly computed variances to a reasonable adjustment: $\max\{\frac{p_{sdx}(1-p_{sdx})}{n_{sd}}, \hat{v}_{sdx}\}$. For comparison purposes we also fit the log version of the two parameter model of equation (1.1) using ordinary least squares as suggested by Johnson and Kingman (1987). The fit of the isotonic model was assessed in part by considering standardized residuals. In Table 2. some selected results for three different sizes of states are provided. For large sample states the direct estimates of variance usually are monotonic on the grid points, so little smoothing is necessary. This was the case for the state of Texas. For medium sample size states like Virginia or small sample size states like Mississippi, many violations of the grid order were observed, so more smoothing is required. The comparison parametric fitting resulted in a $\beta = 0.982$ and $r^2 = .92$, which resulted in a design effect that tended to decrease as the state sample size decreased. Such a relation would be difficult to justify by an analysis of the state design structures. All the states exhibited a very high degree of smoothing with this parametric model. Contrasting these two approaches one can see that the isotonic regression smoothing is more data-driven than model-driven. We feel that the main advantage of the nonparametric isotonic regression approach to smoothing is that the directly computed design-based variance estimates are only modestly changed for the larger states while the smaller states have estimated variances forced to have magnitudes consistent with design structures. This approach is less extreme than the parametric GVF modeling.

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Table 1: NHIS State-Level ξ -based parameters

State	order	observed sample size	design effect	effective sample size	degrees of freedom	State	order	observed sample size	design effect	effective sample size	degrees of freedom
US	0	36115	1.19	30340	995	US	0	36115	1.19	30340	995
CA	1	4305	1.21	3545	324	OK	25	455	1.16	390	12
TX	2	3030	1.27	2395	66	KY	25	445	1.16	385	11
NY	3	2560	1.17	2200	150	SC	26	410	1.18	350	20
FL	4	2120	1.20	1770	72	OR	26	410	1.19	345	9
PA	5	1555	1.12	1390	64	IA	27	380	1.22	310	5
IL	6	1580	1.18	1340	51	KS	27	360	1.17	310	9
OH	7	1350	1.13	1200	40	MS	28	375	1.29	290	4
MI	8	1260	1.14	1105	37	AR	29	330	1.22	270	6
NJ	9	1105	1.14	975	186	NM	30	350	1.40	250	3
GA	10	950	1.21	785	22	NE	31	250	1.20	205	4
MA	11	855	1.10	775	72	UT	32	215	1.14	190	15
NC	12	915	1.20	765	22	WV	32	225	1.17	190	4
VA	13	875	1.16	750	25	HI	33	165	1.13	145	1
MO	14	785	1.17	670	12	ME	34	160	1.15	140	4
IN	15	770	1.17	655	18	NH	34	150	1.09	135	10
MN	16	705	1.14	620	17	NV	35	155	1.19	130	7
WI	17	715	1.18	605	12	ID	35	170	1.28	130	2
WA	18	680	1.17	580	20	RI	35	145	1.11	130	20
TN	19	670	1.18	570	14	DC	36	125	1.13	110	16
AL	20	645	1.19	545	12	MT	37	110	1.20	95	2
MD	20	605	1.12	540	51	DE	38	90	1.10	85	14
AZ	21	630	1.22	520	30	SD	39	90	1.16	75	2
LA	22	585	1.17	500	15	ND	39	85	1.15	75	2
CT	23	520	1.12	460	22	WY	40	65	1.23	55	1
CO	24	530	1.22	435	14	VT	41	55	1.12	50	3
						AK	42	45	1.19	35	1

Table 2: State estimated standard errors and design effects: direct, isotonic, and parametric model

State domain	variable	observed sample	$\hat{p}$	direct stderr	isotonic stderr	parametric stderr	direct deff	isotonic deff	parametric deff
Texas									
adult	asthma	3050	7.4	0.57	0.57	0.57	1.45	1.45	1.41
female	asthma	1750	7.3	0.69	0.69	0.74	1.22	1.22	1.40
male	asthma	1300	7.6	0.91	0.91	0.87	1.53	1.53	1.39
adult	obese	2950	21.4	0.91	0.91	0.89	1.44	1.44	1.39
female	obese	1650	20.1	1.09	1.09	1.15	1.24	1.24	1.37
male	obese	1250	22.9	1.35	1.35	1.38	1.32	1.32	1.37
Virginia									
adult	asthma	850	10.2	1.13	1.19	1.20	1.22	1.34	1.37
female	asthma	500	11.5	1.59	1.63	1.67	1.24	1.30	1.36
male	asthma	400	8.8	1.96	1.82	1.70	1.80	1.56	1.36
adult	obese	850	17.4	1.19	1.63	1.52	0.83	1.57	1.36
female	obese	500	19.5	1.78	2.11	2.10	0.96	1.35	1.34
male	obese	350	15.0	1.78	2.11	2.14	0.93	1.30	1.34
Mississippi									
adult	asthma	400	7.7	1.41	1.41	1.61	1.05	1.05	1.36
female	asthma	200	6.3	1.32	2.11	1.91	0.64	1.65	1.35
male	asthma	150	9.5	2.26	3.18	2.71	0.93	1.84	1.33
adult	obese	350	23.2	2.34	2.55	2.57	1.11	1.32	1.34
female	obese	200	26.6	1.92	3.31	3.53	0.39	1.17	1.32
male	obese	150	19.4	4.91	4.35	3.65	2.38	1.86	1.32