A Bayesian Model for a Proportion with Nonignorability

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1. INTRODUCTION

Recently, nonresponse rates have been increasing in many surveys, making the nonresponse problem more and more important. There has been much activity in estimating survey nonresponse. The main difficulty in modeling of nonresponse is in building a sensible relation between the respondents and the nonrespondents. This is especially important when there is very limited information from the nonrespondents. While the method of ratio estimation is simple, it treats the respondents and the nonrespondents symmetrically, and therefore, it is inaccurate when respondents and nonrespondents actually differ. For many surveys the units are households, and the response is binary. Thus, we propose a method to estimate the proportion of households possessing a characteristic (e.g., doctor visits) using a Bayesian method which allows linking respondents and nonrespondents and pooling of data across areas.

The National Health Interview Survey (NHIS) is one of the surveys conducted by the National Center for Health Statistics to assess some aspect of the health status of the U.S. population. By estimating the proportion of households with at least one doctor visit in the past two weeks, we can have some information about the household doctor visits for the U.S. population. The main issue we consider here is how to account for the bias due to nonresponse in the NHIS. Nonresponse arises mainly from refusals, non contacts, those households with language difficulties, and others. Thus, there may be differences between respondents and nonrespondents. The ratio method, used previously for the NHIS, assumes that the proportion of the characteristic for the respondents and the nonrespondents is the same. Therefore, the ratio method will be inaccurate for the situations in which respondents and nonrespondents differ. We use the NHIS data to demonstrate our method to estimate the proportion of households with at least one doctor visit in the past two weeks.

Little and Rubin (1987) describe two types of models which differ according to the ignorability of nonresponse. In the ignorable model the distribution of the variable of interest for a respondent is the same as the distribution of that variable for a nonrespondent with the same values of covariates. In addition, the parameters in the distributions of the variable and the response must be distinct. All other models are nonignorable. We consider a model that centers a nonignorable model on an ignorable one. In this model an odds ratio (the odds of a household doctor visit among the responding households versus the odds of a household doctor visit among all households) is used to control the extent of nonignorability, and thereby in the Bayesian approach inducing uncertainty about ignorability. We will call this family of models the expansion model because when the odds ratio is unity, the model is ignorable.

Little and Rubin (1987) distinguish between two classes of models for missing data. In the selection approach the hypothetical complete data are modeled, and a model for the nonresponse mechanism is added conditional on the hypothetical data (Heckman 1976). In the pattern mixture approach the population is stratified into two patterns, respondents and nonrespondents, each being modeled separately and the final answer is obtained by a probabilistic mixture of these two. The selection approach is more natural for our problem because it links the respondents and nonrespondents directly and all parameters are identified, albeit weakly. A "borrowing of strength" across states leads to improved inference.

The NHIS data are collected from the fifty states and the District of Columbia. For each area (a state or the District of Columbia) there are counts on the number of households, the number of responding households, and the number of household doctor visits. Like most nonresponse models, many of the parameters in our expansion model are weakly identifiable. Thus, in the spirit of small area estimation, our expansion model "borrows strength" across the areas. Stasny (1991) used a hierarchical Bayes nonignorable selection model to study victimization in the National Crime Survey. She used a Bayes empirical Bayes approach, in which maximum likelihood estimates are substituted for the unknown hyperparameters. Nandram and Choi (2001) provide several extensions of this work.

Our objective is to describe the expansion model, to show how to fit it using a full Bayesian method, to apply it to the NHIS unweighted data on doctor visit in the past year, and to assess its properties.

2. NHIS DATA

One of the variables we use in the NHIS is the number of doctor visits by the members of an entire household in the past two weeks. It is standard practice to use the binary variable, *doctor visit*, which is 0 if the number of doctor visits by all members of a household is 0, and 1 otherwise.

The NHIS nonresponse can be classified mainly as refusals, non contacts and those households with language difficulties, and others. They may arise nonrandomly. For example, the refusal problem may be confined to some special groups such as recent immigrants, who are not representative households, and therefore, nonresponse from this source can be considered nonrandom nonresponse. We observed that the average NHIS nonresponse rate was about 2-3 percent until the 1980's; it has been increasing annually and reached 8-13 percent in 1999.

The NHIS frame is basically a two stage stratified sample survey design of probability proportional to population size (pps). The first stage is the selection of primary sampling units within strata with pps design, and the second stage is the selection of segments with equal probability. All the sample households in each segment are interviewed. Thus, we assume that the sample design does not cause any selection bias beyond the nonresponse bias we address here. Also, it is not unreasonable to treat the states as exchangeable in our model and to assume that this design feature does not interfere with our model approach.

Weighting in the NHIS is a multi-stage scheme, and one of the stages is ratio adjustment for nonresponse at the segment level. This ratio is the proportion of all sample persons to the respondents in the segment. This estimate is adequate when respondents and nonrespondents are similar. However, this method can fail badly when these two groups differ according to important characteristics which an investigator wants to study. We address nonignorable nonresponse problems by expanding the method of random weighting, and the Bayesian method is introduced as a possible alternative to impute the NHIS nonresponse.

We analyze the nonresponse unweighted data for the 10 states from the 50 states and the District of Columbia which have varying rates of nonresponse and visits among respondents in the 1995 household survey. The 10 states are the ones with smallest response rates (at least 7.2% nonrespondents). These states are Colorado, Delaware, District of Columbia, Florida, Louisiana, Maryland, Nevada, New York, South Carolina ,and West Virginia (see Table 1).

Table 1: **NHIS 1995 data by state**

st	y	$r-y$	$n-r$	$\hat{p}_{obs}$	$\hat{p}_{seg}$	$\hat{\delta}$
CO	144	385	62	.27	.27	.90
DE	37	63	12	.37	.37	.89
DC	31	66	14	.32	.32	.87
FL	706	1542	219	.31	.31	.91
LA	186	389	54	.32	.32	.91
MD	223	446	69	.33	.33	.91
NE	60	113	14	.35	.34	.93
NY	860	1962	278	.31	.30	.91
SC	153	311	43	.33	.33	.92
WV	68	143	22	.32	.32	.91

NOTE: 10 states with least 7.2% observed nonresponse rate

In Table 1 we present n_i, r_i ,and y_i which are the numbers of sampled households, responding households and households with doctor visits, respectively, over the past year. The fourth column has the observed proportion $\hat{p}_{obs,i} = y_i/r_i$ of responding households with at least one doctor visit. Hawaii and Maine reported the highest $\hat{p}_{obs,i}$ of doctor visits with 38% for each of these states. Colorado and Oregon reported the lowest $\hat{p}_{obs,i}$ of doctor visits with 27% for each of these states.

The fifth column shows the proportion $\hat{p}_{seg,i} = y_i/r_i$ of responding households with at least one doctor visit by a weighted average over the segments where the weights are inversely proportional to the sample size within segments based on the assumption of ignorability nonresponse. For these states the observed rates $\hat{p}_{obs,i} = y_i/r_i$ and the rates $\hat{p}_{seg,i} = y_i/r_i$ based on the segments are very similar within the segments. This is true because the proportions of households with doctor visits are very similar within the segments.

Finally, the sixth column also has the observed proportion $\hat{\delta}_i = r_i/n_i$ of responding households by state unlike the weighted average of the segment data. The response rates range from 0.87 in the District of Columbia to 0.99 in Idaho.

3. METHODOLOGY FOR NONIGNORABLE NONRESPONSE

In this section we describe our model, show how to fit it and how to make inference about the parameters. Our analysis uses all 50 states and the District of Columbia, but we present only 10 states in Table 1. Let ℓ be the number of areas. We assume that a sample of n_i households is taken from the i^{th} area, $i = 1, \ldots, \ell$.

Let the binary characteristic be $y_{ij} = 1$ if at least one member of household j in area i visited doctor's office and $y_{ij} = 0$ otherwise. The response variable $r_{ij} = 1$ if household j in area i is a respondent and $r_{ij} = 0$ otherwise. We use a probabilistic structure to model y_{ij} and r_{ij}, and this is the expansion model.

Let $r_i = \sum_{j=1}^{n_i} r_{ij}$ be the number of households with respondents and $y_i = \sum_{j=1}^{n_i} y_{ij}$ the number of households with at least one doctor visit, and $n_i - r_i$ is the number of nonrespondents.

3.1 Expansion Model

The expansion model for nonignorable nonresponse is

$$y_{ij} \mid p_i \overset{iid}{\sim} \text{ Bernoulli } (p_i)$$

$$r_{ij} \mid \pi_i, \gamma_i, y_{ij} = 1 \overset{iid}{\sim} \text{ Bernoulli } (\gamma_i \pi_i)$$

$$r_{ij} \mid \pi_i, \gamma_i, y_{ij} = 0 \overset{iid}{\sim} \text{ Bernoulli } (\pi_i). \quad (1)$$

The γ_i are the ratios of the odds of success (doctor visit) among respondents to the odds of success (doctor visit) among all individuals in the i^{th} area. The γ_i show the extent of nonignorability of the nonrespondents and, in fact, incorporate the uncertainty about ignorability into the model. If $\gamma_i = 1$, the model becomes ignorable and there is no difference between respondents and nonrespondents.

The parameters of interest are γ_i, δ_i and mainly p_i where δ_i is the probability of responding in area i and is given by

$$\delta_i = \pi_i \{\gamma_i p_i + (1 - p_i)\}. \quad (2)$$

Assuming all areas are similar, we take the parameters (p_i, π_i, γ_i) to have a common distribution. This assumption is useful because it helps in the estimation for the parameters such as π_i and γ_i and, therefore, δ_i, that are weakly identified by the data.

For p_i, we take parameters

$$p_i \mid \mu_1, \tau_1 \overset{iid}{\sim} \text{ Beta } (\mu_1 \tau_1, (1 - \mu_1)\tau_1). \quad (3)$$

Note that $E(p_i \mid \mu_1, \tau_1) = \mu_1$ and $var(p_i \mid \mu_1, \tau_1) = \mu_1(1-\mu_1)/(\tau_1+1)$. This reparameterization is useful because the parameters μ_1 and τ_1 are approximately orthogonal.

We wish to center the γ_i at unity (i.e., center on an ignorable model). It is possible to do so by assuming that the γ_i have a common mean of unity. Thus, one can assume that $\gamma_i \mid \nu \overset{iid}{\sim} \Gamma(\nu, \nu)$, $\gamma_i > 0$, where $E(\gamma_i \mid \nu) = 1$ and $Var(\gamma_i \mid \nu) = 1/\nu$. Thus, we can center our expansion model on an ignorable model with γ_i fluctuating about unity with a standard deviation $1/\sqrt{\nu}$ a priori. But there is the issue that $0 < \gamma_i \pi_i < 1$.

Thus, we assume that the parameters (π_i, γ_i) are jointly independent with

$$\pi_i \mid \mu_2, \tau_2 \overset{iid}{\sim} \text{ Beta } (\mu_2 \tau_2, (1 - \mu_2)\tau_2)$$

and

$$\gamma_i \mid \nu, \pi_i \overset{iid}{\sim} \text{Gamma}(\nu, \nu), \quad (4)$$

$0 < \gamma_i < 1/\pi_i$ and $0 < \pi_i < 1$.

Thus, the joint prior distribution for (p_i, π_i, γ_i) is the product of the densities in $p_i \mid \mu_1, \tau_1 \overset{iid}{\sim}$ Beta $(\mu_1 \tau_1, (1 - \mu_1)\tau_1)$ and $p(\pi_i, \gamma_i \mid \mu_2, \tau_2, \nu)$.

For a full Bayesian analysis, prior distributions are needed for the hyperparameters μ_1, τ_1, μ_2, τ_2 and ν. Thus, we take $\mu_r \overset{iid}{\sim} Beta(1, 1)$, $r = 1, 2$. That is, uniform proper prior densities are used for μ_1 and μ_2. We also use proper prior distributions for τ_1, τ_2 and ν. These prior distributions are similar to the uniform shrinkage proper prior distributions. Specifically, we take $p(\nu) = 1/(\nu+1)^2$, $\nu \geq 0$ and $p(\tau_r) = 1/(\tau_r + 1)^2, \tau_r \geq 0$, $r = 1$, 2 with independence over μ_1, τ_1, μ_2, τ_2 and ν. These prior distributions discourage the posterior modal estimates of τ_1, τ_2 and ν to be on the boundary of the parameter space which will affect inference.

It is pertinent to compare the model of Stasny (1991) with ours. We note that in Stasny's model $\pi_{i0} \equiv \gamma_i \pi_i$ and $\pi_{i1} \equiv \pi_i$ in (1). The prior density in (3) is similar. The key difference is that while prior densities are assigned to γ_i and π_i in our model, they are assigned to π_{i0} and π_{i1} in Stasny's model. In Stasny's model parameters like μ_1 and τ_1 are assumed fixed but unknown, and these hyperparameters are estimated using the maximum likelihood method. We provide a full Bayesian analysis. Our expansion model as described here allows different degrees of nonignorability in different areas, states or domains.

Since the number of visits among the nonrespondents is unknown, we denote it by the latent variable z_i, and hence, the number of households with no visits among them is $n_i - r_i - z_i$. We note that the distributions of the z_i can be obtained from the distributions of r_i and y_i through multinomial sampling.

The augmented likelihood function can be represented by a four-cell multinomial probability mass function of y_i, $r_i - y_i$, z_i and $n_i - r_i - z_i$ with probabilities $\gamma_i \pi_i p_i$, $\pi_i(1 - p)$, $(1 - \gamma_i \pi_i p_i)$, and

$(1-\pi_i)(1-p_i)$, respectively. Then it is easy to show that the augmented likelihood function is proportional to $f(\mathbf{y}, \mathbf{r}, \mathbf{z} \mid \mathbf{p}, \boldsymbol{\gamma}, \boldsymbol{\pi}) = \prod_{i=1}^{\ell} f(y_i, r_i, z_i \mid p_i, \gamma_i, \pi_i)$.

Using Bayes' theorem, it is convenient to make the transformation $\phi_i = \gamma_i \pi_i$ with other parameters π_i, p_i, and z_i not transformed, the joint posterior density of all the parameters can be written down.

Inference about p_i, δ_i and γ_i can be obtained by using this posterior density function. Because the posterior density is complex, we use Markov chain Monte Carlo (MCMC) methods.

4. ANALYSIS OF NHIS DATA

We discuss the goodness of fit of the expansion model, and inference on the NHIS data using this model.

4.1 Assessment of the Model

We assess our model by using a Bayesian cross validation analysis to obtain deleted residuals. These residuals are studied in the usual way. We obtain the predictive distribution of $y_i/r_i \mid \mathbf{y}_{(i)}, \mathbf{r}_{(i)}$ where $\mathbf{y}_{(i)}$ and $\mathbf{r}_{(i)}$ are the vectors of all y_i and all r_i, respectively with the i^{th} state deleted. We assess how y_i/r_i differs from its predicted value under the expansion model. To assess the model fit, we computed $DRES_i = (y_i/r_i - \hat{E}(y_i/r_i \mid \mathbf{y}_{(i)}, \mathbf{r}_{(i)}))/(\widehat{Std}(y_i/r_i \mid \mathbf{y}_{(i)}, \mathbf{r}_{(i)}))$, $i = 1, \ldots, \ell$. Then we plotted $DRES_i$ versus $E(y_i/r_i \mid y_{(i)}, r_{(i)})$.

Except for Maryland and Oregon, all the points are between -2.0 and 2.0. For Maryland DRESS is 2.31, and for Oregon DRES is -2.24. It is interesting that Maryland has an observed value of .33 and a predicted value of .27 while Oregon, a state among those with the smallest observed rates, has an observed value of .27 and a predicted value of .33, thereby incurring large residuals. The observed response rate for Maryland is .91, among the smallest observed response rates, versus .98 for Oregon, the largest observed response rate. While the number of points above the zero is 31 out of 51, the difference is not statistically significant for a standard large sample approximation. Also, a simple linear regression of DRES on PRED has adjusted $R^2 = 6.6\%$, and the Pearson correlation is negative. Also, deletion of Maryland and Oregon reduces the adjusted R^2 to 0.0%. Thus, there is a very weak relationship between DRES and PRED.

We also investigated whether the residuals are related to the sample size within states. We cross-classified the DRES (> 0 and ≤ 0) and sample size a ($> a$ and $\leq a$). Using Fisher's exact two-sided test on SAS, the P-values at $a = 100, 200$, and 500 are .271, .204, and .778, respectively. Thus, the sign of DRES is not associated with the sample size.

4.2 Posterior Inference for Expansion Model

Next, we apply our methodology to the NHIS data. Then, we perturb a key assumption in our model to investigate how sensitive inference is to this assumption.

In columns 2, 3, and 4 of the first part in Table 2 we present 95% credible intervals for the p_i, δ_i, and γ_i for the original expansion model.

Table 2: 95% Credible interval for p, δ and γ by Models

St	p	δ	γ
Expansion			
CO	(.30, .36)	(.87, .92)	(.70, .84)
DE	(.31, .38)	(.86, .94)	(.68, .94)
DC	(.30, .38)	(.84, .93)	(.61, .90)
FL	(.33, .37)	(.90, .92)	(.81, .87)
LA	(.32, .37)	(.89, .93)	(.79, .90)
MD	(.33, .39)	(.89, .93)	(.78, .89)
NE	(.31, .38)	(.89, .96)	(.77, .97)
NY	(.33, .36)	(.90, .92)	(.80, .86)
SC	(.32, .38)	(.89, .94)	(.79, .92)
WV	(.31, .38)	(.88, .94)	(.73, .92)
Alternative			
CO	(.28, .33)	(.87, .91)	(.90, 1.06)
DE	(.29, .36)	(.87, .94)	(.91, 1.11)
DC	(.29, .35)	(.85, .94)	(.89, 1.09)
FL	(.29, .33)	(.89, .92)	(.98, 1.08)
LA	(.30, .35)	(.89, .93)	(.92, 1.07)
MD	(.30, .35)	(.88, .92)	(.92, 1.12)
NE	(.29, .35)	(.89, .95)	(.93, 1.11)
NY	(.29, .32)	(.89, .91)	(1.00,1.10)
SC	(.29, .35)	(.89, .93)	(.93, 1.11)
WV	(.29, .35)	(.87, .94)	(.91, 1.11)

NOTE: see Table 1 for p and δ.

Some of the intervals do not contain the observed values of the p_i. For example, Colorado, Florida and New York do not contain the observed proportions. This implies that the ratio method may provide unreasonable estimates for the true proportions for these states. As for the δ_i, there are some variations among the states where for the 95% credible intervals the upper ends are reasonably close, but the lower ends differ. But, in general, the response rates are similar. These intervals are useful because they provide information about the sample size that might be required for a future survey. For example, for the District of Columbia the 95% credible interval for δ_i is $(0.612, 0.903)$. This means that if a future survey requires 1,000 respondents from the District of Columbia, the interviewer might need to visit 1107-1634 households, with a much sharper statement for many of the states. It is clear that many of the intervals for the γ_i do not contain 1, and so for these states the nonresponse mechanism should be considered nonignorable, and therefore the ratio estimator should not be used.

We also cross-classified the DRES (> 0 and ≤ 0) and nonignorability (95% credible interval for γ_i is to the left of 1 versus it contains 1). Again using Fisher's exact two-sided test on SAS, the P-value is 0.554. Thus, the goodness of fit is not associated with ignorability.

5. ALTERNATIVE MODEL AND SIMULATION

First, we assess the assumption on the γ_i in (4) by introducing an alternative (but less favored) model. Finally, we describe a simulation study on the expansion model to investigate how well it can reproduce the true parameter values. In this simulation study we also compare inference for the original expansion model with the alternative expansion model, an ignorable model and a nonignorable model (see Stasny (1991) for these latter two models).

5.1 An Alternative Expansion Model

Next, we show how to assess the effect of the assumption $\gamma_i \mid \nu, \pi_i \overset{iid}{\sim} \text{Gamma}(\nu, \nu),\ 0 < \gamma_i < 1/\pi_i$ *and* $0 < \pi_i < 1$ in our model. We retain the same conditions for r_{ij} and p_i. We obtain a new prior density for (π_i, γ_i). For γ_i we take $\gamma_i \mid \pi_i \overset{ind}{\sim} U(2 - \pi_i^{-1}, \pi_i^{-1}),\ 2 - \pi_i^{-1} < \gamma_i < \pi_i^{-1}$ and for π_i we take $\pi_i \mid \mu_2, \tau_2 \overset{iid}{\sim}$ Beta $(\mu_2\tau_2, (1 - \mu_2)\tau_2)$.

Note that we have eliminated the parameter ν from the original model. Thus, the joint prior density for $\pi_i, \gamma_i \mid \mu_2, \tau_2$ is $p(\pi_i, \gamma_i \mid \mu_2, \tau_2)$. Then, we take the hyperparameters $\mu_1, \mu_2 \overset{iid}{\sim} \text{Beta}(1, 1)$, and $p(\tau_1) = 1/(\tau_1 + 1)^2, \tau_1 \geq 0$ and independently $p(\tau_2) = 1/(\tau_2 + 1)^2, \tau_2 \geq 0$.

Using Bayes' theorem, again it is convenient to make the transformation $\phi_i = \gamma_i\pi_i$ with other parameters π_i, p_i, and z_i not transformed, the joint posterior density of all the parameters $(\mathbf{z}, \mathbf{p}, \boldsymbol{\pi}, \boldsymbol{\phi}, \mu_1, \tau_1, \mu_2, \tau_2)$ for given data $(\mathbf{y}, \mathbf{r})$ can be obtained apart from a constant of proportionality.

A strategy for the computations is similar to that used for the original model. The bottom three columns in Table 2 contain the corresponding quantities for the alternative expansion model. First, inference for the p_i and the δ_i are similar with much more similarity for the δ_i. For the p_i, the intervals based on the alternative model overlap on the left of the intervals based on the original model. For the γ_i, the intervals based on the alternative model overlap on the right of the intervals based on the original model. We note that $\mid \gamma_i - 1 \mid < \pi_i^{-1} - 1$, so that the credible interval for γ_i always contains 1. In fact, the alternative model actually pins down the posterior densities of the γ_i very close to 1. In other words, the alternative model should not be used to assess ignorabilty.

Thus, while the two models differ considerably with respect to the γ_i, inference about the p_i and the δ_i is very similar.

5.2 Simulation Study

We perform a simulation study to assess how well the expansion model can reproduce the true parameters, and we compare the expansion model, alternative expansion model, an ignorable model and a nonignorable model. We do so by using the reduced set of ten states in Table 1. We also compare 95% credible intervals of the p_i for the expansion, alternative, ignorable and nonignorable models at two values of the γ_i.

We have compared four models: the expansion model, alternative expansion, an ignorable model and a nonignorable model (see Stasny 1991) in the simulation study. However, for the ignorable and the nonignorable models, unlike Stasny (1991) who used a Bayes empirical Bayes analysis, we provide a full Bayesian analysis of these latter two models as well as using Markov chain Monte Carlo methods. We consider two values of the γ_i. Because the true γ_i are close to one, we pick all the $\gamma_i = 1$ for a choice of ignorability and to enforce nonignorability, we choose $\gamma_i = .75$. At each of these two values, we obtained 100 data sets as described above. Then, we fit the four models to these data sets. We obtain 95% credible intervals for the p_i, the parameters of central interest.

In Table 3 we present these intervals for $\gamma_i = 1$ and $\gamma_i = .75$. At $\gamma_i = 1$, the intervals are all very similar across the models, although the intervals for the nonignorable model are wider. At $\gamma_i = .75$, the intervals for expansion model and the alternative model are very similar. As seen in the bottom half of Table 3, the intervals for the ignorable model are to the left of those for the expansion and the alternative models with almost no overlaps, while there are substantial overlaps with those of the nonignorable model. Also, the intervals for the nonignorable model are much wider than those for the expansion and the alternative models. Here the ignorable model is clearly inappropriate, and it is unfair to compare it with the other three nonignorable models.

The first half of Table 3 shows that the three nonignorable models can produce inference similar to the ignorable model when there is ignorability. The bottom half of Table 3 shows that while the three nonignorable models can produce comparable inference under nonignorabilty, the ignorable model can not cope with this situation.

Table 3: 95% Credible interval for p by model

St.	p	Expan.	Alter.	Igno	Nonig.
$\gamma_i = 1$					
CO	.34	.31-.37	.30-.36	.31-.36	.31-.39
DE	.34	.29-.37	.29-.37	.30-.37	.27-.42
DC	.34	.29-.37	.29-.37	.29-.37	.28-.42
FL	.34	.31-.35	.32-.35	.32-.35	.33-.37
LA	.34	.31-.37	.30-.36	.30-.36	.31-.38
MD	.32	.30-.36	.30-.35	.30-.35	.30-.37
NE	.32	.29-.36	.29-.36	.29-.36	.27-.40
NY	.31	.30-.33	.29-.32	.30-.33	.30-.34
SC	.33	.29-.35	.30-.36	.30-.36	.30-.38
WV	.33	.29-.37	.29-.36	.30-.36	.28-.39
$\gamma_i = .75$					
CO	.34	.31-.37	.29-.35	.25-.30	.26-.36
DE	.34	.29-.37	.28-.35	.24-.30	.22-.37
DC	.34	.29-.37	.28-.35	.24-.30	.23-.38
FL	.34	.32-.36	.30-.33	.26-.29	.27-.35
LA	.34	.31-.37	.29-.35	.24-.30	.26-.35
MD	.32	.30-.36	.29-.34	.24-.29	.25-.34
NE	.32	.29-.37	.28-.35	.24-.30	.23-.36
NY	.31	.30-.33	.27-.30	.24-.27	.25-.32
SC	.33	.30-.36	.29-.35	.24-.29	.25-.35
WV	.33	.30-.37	.28-.35	.24-.30	.24-.36

NOTE: Expansion, Alternative, Ignorable, Nonignorable. The end points of intervals are averages over the 100 simulated 95% credible intervals.

6. CONCLUSION

We have presented a Bayesian method to estimate the proportion of doctor visits and the probability that a household responds in the NHIS. In doing so we have been able to incorporate a degree of uncertainty about the ignorability of the nonresponse mechanism. We proposed a model and assessed one of its critical assumptions.

Our method is potentially useful to incorporate uncertainty about ignorability of the nonresponse mechanism for many surveys. We have shown that it is possible to decide for which states the nonresponse mechanism can be treated as ignorable. For these states it is possible to use the ratio method for nonresponse adjustment. For the other states one must be reluctant to use the ratio method. In either case our method provides adjusted estimates for p_i and δ_i based on the extent of nonignorability.

We studied the sensitivity of the original expansion model to its specifications. Perturbation of the distribution assumption on the γ_i in an alternative model leads to similar inference about p_i and δ_i as in the original model. When the nonresponse mechanism is ignorable, there is similarity in inference about p_i for the two expansion models, an ignorable model and a nonignorable model. When the nonresponse mechanism is nonignorable, there is agreement in inference about p_i for the two expansion models and a nonignorable model, but no agreement with the ignorable model. The nonignorable model provides estimates of p_i with larger variability for both situations.

There are two possible extensions to our work. Even though the segments are small, many of them do not have nonrespondents. Thus, it may be useful to include the segments as an additional stage in the hierarchical model. Another important extension is to polychotomous (more than two cells) data that are so prominent in many complex surveys. While we are currently investigating nonignorable nonresponse regression models elsewhere, the issue of covariate is a distinct problem.

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