SOME DESIGN CONSIDERATIONS FOR STRATIFIED SAMPLING

Richard L. Greenstreet and Ragan B. Madden, VPI&SU

Introduction

Consider a finite population of N units where with each unit is associated a continuous variable of interest, y, which is assumed to be a realized value of a random variable, Y. Let the total of the y's be the quantity of interest. Assume that the y's are unknown a priori, but prior knowledge of two qualitative variables, for example, race and sex, is available on each unit. We will consider two models which express a stochastic dependence of the y's upon the known, qualitative information.

The first model assumes that

$$Y_{ijk} = \gamma_{ij} + \varepsilon_{ijk} \quad (1)$$

where the Y_{ijk} are uncorrelated random variables with expectation γ_{ij} and variance σ^2_{ij}. The second model is additive, with

$$Y_{ijk} = \tau_i + \beta_j + \varepsilon_{ijk}\,. \quad (2)$$

For both models (1) and (2), i,j = 1,2, and represent the possible levels for the two prior, qualitative variables; k = 1,..., N_{ij}, where N_{ij} is the number of units in the population at level (i,j), and $\sum_{i=1}^{2}\sum_{j=1}^{2} N_{ij}=N$. The quantity which must be estimated is $T = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{N_{ij}} y_{ijk}$.

The Estimators

It is not difficult to show for any sample of size n with $n_{ij}\neq 0$ units from each level (i,j), that the minimum variance, linear, unbiased (MVLU) estimator of T under model (2) is

$$\hat{T}_a = \sum_s y_{ijk} + \sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij}\,\bar{y}_{ij}(s), \text{ where}$$

$\sum_s$ indicates the sum of sample units, $\bar{y}_{ij}(s)$ is a sample average at level (i,j), and

$$a_{ij} = M_{ij} + (-1)^{i+j}\, B/\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\sigma^2_{ij}}{n_{ij}}, \text{ where}$$

$$M_{ij} = N_{ij} - n_{ij} \qquad i,j = 1,2, \text{ and}$$

$$B = \left(\frac{\sigma^2_{12}M_{12}}{n_{12}} + \frac{\sigma^2_{21}M_{21}}{n_{21}} - \frac{\sigma^2_{11}M_{11}}{n_{11}} - \frac{\sigma^2_{22}M_{22}}{n_{22}}\right).$$

Further, it is easy to show that under model (2)

$$E(\hat{T}_a-T)^2=VAR(\hat{T}_a-T)=\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\sigma^2_{ij}N_{ij}M_{ij}}{n_{ij}} - B^2/\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\sigma^2_{ij}}{n_{ij}}\,. \quad (3)$$

Under model (1) the MVLU estimator of T is

$$\hat{T}_g = \sum_s y_{ijk} + \sum_{i=1}^{2}\sum_{j=1}^{2} M_{ij}\bar{y}_{ij}(s), \text{ and}$$

under either (1) or (2)

$$E(\hat{T}_g-T)^2 = VAR(\hat{T}_g-T) = \sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\sigma^2_{ij}N_{ij}M_{ij}}{n_{ij}}\,. \quad (4)$$

Objective of Paper and Some Analytical Results

This paper reports results of a computer implemented study designed to determine the relative merits of $\hat{T}_a$ and $\hat{T}_g$ as estimators for the total.

To begin, observe that $E(\hat{T}_a-T)^2\leq E(\hat{T}_g-T)^2$ for any sample of size n, $n_{ij}\neq 0$, provided model (2) holds. If (2) fails and (1) holds then

$$E(\hat{T}_a-T) = (\gamma_{11}+\gamma_{22}-\gamma_{12}-\gamma_{21})B/\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\sigma^2_{ij}}{n_{ij}}\,. \quad (5)$$

Consequently, under model (1)

$$E(\hat{T}_a-T)^2 = VAR(\hat{T}_a-T)+[E(\hat{T}_a-T)]^2$$

and one may easily show that $E(\hat{T}_a-T)^2\leq E(\hat{T}_g-T)^2$ if and only if

$$|\gamma_{11}+\gamma_{22}-\gamma_{12}-\gamma_{21}| \leq \left(\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\sigma^2_{ij}}{n_{ij}}\right)^{1/2}, \quad (6)$$

where $\hat{T}_a$ and $\hat{T}_g$ are based on the same sample. (We must point out that (6) applies only if $B\neq 0$. If $B=0$, then, $E(\hat{T}_a-T)^2=E(\hat{T}_g-T)^2$ when both $\hat{T}_a$ and $\hat{T}_g$ are based on the same sample.)

An application of either $\hat{T}_a$ or $\hat{T}_g$ requires the determination of n_{11},n_{12},n_{21}, and n_{22}. The sample which minimizes (4) is given by

$$n_{ij} = n\,\sigma_{ij}N_{ij}/\sum_{i=1}^{2}\sum_{j=1}^{2}\sigma_{ij}N_{ij}\,. \quad (7)$$

The allocation which minimizes (3) must be obtained by direct enumeration of all possible samples of size n, except when $\sigma^2_{ij}=\sigma^2$, in which case,

$$n_{ij} = n\,N_{ij}/N \quad (8)$$

minimizes either (3) or (4).

If we denote the sample which minimizes (3) by $n^{(a)}_{ij}$ and that which minimizes (4) by $n^{(g)}_{ij}$, then, under model (2) and for these two samples, $VAR(\hat{T}_a-T)^2$ is equal to expression (3) with n_{ij} replaced by $n^{(a)}_{ij}$, and, also, $VAR(\hat{T}_g-T)^2$ is equal to (4) with n_{ij} replaced by $n^{(g)}_{ij}$. Clearly, $VAR(\hat{T}_g-T)^2-VAR(\hat{T}_a-T)^2=D\geq 0$. Now, if model (2) fails and (1) is appropriate we may easily show that $E(\hat{T}_a-T)^2 \leq E(\hat{T}_g-T)^2 \Longleftrightarrow$

$$|\gamma_{11}+\gamma_{22}-\gamma_{12}-\gamma_{21}|\le[(\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\sigma_{ij}^2}{n_{ij}(a)})^2\,D/B^2]^{1/2}, \quad (9)$$

where it is assumed that $B\neq 0$. If $B=0$, then $E(\hat{T}_g-T)^2-E(\hat{T}_a-T)^2=D\geq 0$.

Determining the samples which minimize (3) or (4) requires knowledge of σ_{ij}^2 and N_{ij}, $i,j=1,2$. In most applications the N_{ij} are known, but the σ_{ij}^2 must be estimated, either from a past, similar, known population or a pilot study of the population currently under study.

It should be noted that $\hat{T}_a$ is functionally dependent upon the assumed σ_{ij}^2, whereas $\hat{T}_g$ is not. We should expect, then, $\hat{T}_a$ to be more sensitive to incorrect estimates of the σ_{ij}^2 than $\hat{T}_g$. We will examine this issue more carefully in a later section.

Results of Study (Comparing $\hat{T}_a$ with $\hat{T}_g$ Based on the Optimal Allocations for Each)

Let us, at this point, consider the results tabulated in tables I, II, and III. These tables contain comparative results for samples of size $n=20$ drawn from populations with $N_{11}=N_{12}=N_{21}=N_{22}=25$; $N_{11}=N_{22}=10$, $N_{12}=N_{21}=40$ or $N_{11}=N_{12}=10$, $N_{21}=N_{22}=40$, respectively. For each choice of the N_{ij} computations are made for a number of reasonable, relative values of the σ_{ij}^2. (The cases $n=8,16$ were also studied, but will not be reported in this paper since the patterns observed were qualitatively similar to the $n=20$ case.)

Note in tables I and II that using the optimal allocation under (2) compared with allocation (7) yields a ratio $VAR(\hat{T}_a-T)/VAR(\hat{T}_g-T)$ which is significantly less than one for certain choices of the σ_{ij}^2. For example, in table I observe for $\sigma_{11}^2=1$, $\sigma_{12}^2=4$, $\sigma_{21}^2=2$, $\sigma_{11}^2=1$ that $VAR(\hat{T}_a-T)=518.86$ based on the optimal allocation $n_{11}=9$, $n_{12}=1$, $n_{21}=1$, $n_{22}=9$. The allocation which minimizes $VAR(\hat{T}_g-T)$ is $n_{11}=4$, $n_{12}=7$, $n_{21}=5$, $n_{22}=4$ and gives $VAR(\hat{T}_g-T)=719.64$. The ratio $VAR(\hat{T}_a-T)/VAR(\hat{T}_g-T)$ is .72.

Notice in table III, however, that $VAR(\hat{T}_a-T)/VAR(\hat{T}_g-T)$ is near unity for all choices of the σ_{ij}^2. No benefit is derived from using $\hat{T}_a$ and its optimal allocation instead of $\hat{T}_g$ with its optimal sample for this particular configuration of the N_{ij}.

Thus, under model (2) $\hat{T}_a$ is preferred to $\hat{T}_g$ for some choices of the N_{ij} and σ_{ij}^2. If $\hat{T}_a$ is used along with its optimal allocation, then, column (5) of tables I, II, and III gives the right hand side of condition (9). This indicates if model (2) fails the amount of interaction which can be tolerated and still have $E(\hat{T}_a-T)^2\leq E(\hat{T}_g-T)^2$, where $\hat{T}_a$ and $\hat{T}_g$ are each used with their own optimal allocations.

($\hat{T}_a$ compared with $\hat{T}_g$, both based on allocation (7).)

Because allocation (7) is simple to obtain, one might use it with $\hat{T}_a$. This is not recommended since $VAR(\hat{T}_a-T)$ and $VAR(\hat{T}_g-T)$ both based on (7) yield a ratio which is near unity for all studied configurations of σ_{ij}^2 and N_{ij}. (See column (10) of tables I, II, III.) Thus, using (7) with $\hat{T}_a$ gives no significant advantage over (7) with $\hat{T}_g$ if model (2) holds, and would be inefficient if model (1) holds and if condition (6) is violated.

($\hat{T}_a$ compared with $\hat{T}_g$, both based on allocations independent of the σ_{ij}^2)

Because the allocations which minimize (3) and (4) require estimates for the σ_{ij}^2, we are interested in the performance of $\hat{T}_a$ and $\hat{T}_g$ for equal allocation and allocation (8), that is, proportional allocation. Neither of these two allocations require knowledge of the σ_{ij}^2.

Note in tables V and VI that equal allocation for $\hat{T}_a$ gives values for $VAR(\hat{T}_a-T)$ which are comparable or uniformly larger than $VAR(\hat{T}_a-T)$ for proportional allocation. Also, note in table V that equal allocation severely increases $VAR(\hat{T}_g-T)$. We will conclude that equal allocation is never recommended. (Of course, equal and proportional allocation are equivalent for equal N_{ij}.)

To assess proportional allocation observe columns (15) and (16) of table IV and columns (18) and (19) of tables V and VI. Columns (16) and (19) contain the ratio of $VAR(\hat{T}_a-T)$ based on the optimal allocation under (2) to $VAR(\hat{T}_a-T)$ based on proportional allocation. Columns (15) and (18) contain the ratio of $VAR(\hat{T}_g-T)$ based on allocation (7) to $VAR(\hat{T}_g-T)$ based on proportional allocation. Notice that the above ratio for $\hat{T}_g$ is very close to unity for unequal N_{ij}, however, for equal N_{ij} this ratio is often around ten percent less than one. The same ratio for $\hat{T}_a$ is near unity for $N_{11}=N_{12}=10$, $N_{21}=N_{22}=40$, (see table VI), but is often much less than one for the other two configurations of N_{ij} studied, particularly when the N_{ij} are equal.

One may conclude, then, that proportional allocation is definitely reasonable to use with $\hat{T}_g$, particularly for unequal N_{ij}.

As for $\hat{T}_a$, proportional allocation does not eliminate the need to estimate the σ_{ij}^2 since $\hat{T}_a$

depends on the σ_{ij}^2. If one assumes $\sigma_{ij}^2 = \sigma^2$, then, with proportional allocation, $\hat{T}_a = \hat{T}_g$.

If, in a particular application, a proportional allocation has been performed, then, data would exist to estimate the σ_{ij}^2 and the information in tables IV, V, and VI would provide a basis for comparing the performance of $\hat{T}_a$ in this situation to its performance with optimal allocation.

Robustness Considerations

As pointed out earlier, the allocations which minimize (3) or (4) require estimates for the σ_{ij}^2. In addition, $\hat{T}_a$ is functionally dependent upon the σ_{ij}^2. It is of interest, then, to study the relative behavior of $\hat{T}_a$ and $\hat{T}_g$ when the true σ_{ij}^2 are different than the estimates used to determine the optimal allocations.

What was done, then, was to compute for each of the σ_{ij}^2 configurations listed in tables I, II, and III and for the corresponding optimal allocations under (1) and (2), $VAR(\hat{T}_a - T)$ and $VAR(\hat{T}_g - T)$ under alternate configurations of the σ_{ij}^2. The alternate σ_{ij}^2 configurations for any assumed configuration would be the other 26 or 15 configurations listed in the tables. The averages of $VAR(\hat{T}_a - T)$ and $VAR(\hat{T}_g - T)$ across the alternate configurations were computed. The ratio of the average $VAR(\hat{T}_a - T)$ to the average $VAR(\hat{T}_g - T)$ is used as an index of relative performance under alternate σ_{ij}^2 configurations.

This ratio is in almost all cases greater than one for the unequal N_{ij} studied. However, for equal N_{ij} this ratio is less than one in nearly all cases.

We may conclude that $\hat{T}_a$ and its optimal allocation is (less, more) reliable for alternative σ_{ij}^2 than $\hat{T}_g$ with its optimal allocation when the N_{ij} are (unequal, equal).

It should be noted that using proportional allocation with $\hat{T}_a$ rather than the allocation which is optimal for the assumed set of σ_{ij}^2 is not a good strategy for increasing reliability under alternate σ_{ij}^2 configurations. The averages of $VAR(\hat{T}_a - T)$ across alternate σ_{ij}^2 configurations for proportional allocation and optimal additive allocation are nearly the same, for all cases studied. (A similar statement is true for $\hat{T}_g$.)

Conclusion and Summary

If reliable estimates of the σ_{ij}^2 are available and $|\gamma_{11}+\gamma_{22}-\gamma_{12}-\gamma_{21}|=0$, then, $\hat{T}_a$ with its optimal allocation is preferred to $\hat{T}_g$ with allocation (7), particularly for populations having nearly equal N_{ij}. If knowledge of the σ_{ij}^2 is unreliable, but the interaction is known to be zero, $\hat{T}_a$ is, again, recommended over $\hat{T}_g$ for equal N_{ij}.

If the absolute interaction violates condition (9), $\hat{T}_a$ is inferior to $\hat{T}_g$, where each is used with its optimal allocation.

The use of proportional allocation as a defense against errors in the σ_{ij}^2 is not recommended.

Proportional allocation does perform reasonably well, though, compared to the optimal allocations for either $\hat{T}_a$ or $\hat{T}_g$, and if one is presented with a study which used proportional allocation, the extra expense of obtaining optimal allocations would usually be unjustified.

Comments on Tables

Most of the entires in tables I - VI are explained in the table headings or in the text. A few additional remarks are in order, though.

The σ_{ij}^2 combinations for which computations are presented are listed in column (1) of tables I, II, and III. Table IV is really a continuation of table I and the σ_{ij}^2 configuration for the ith line of table IV corresponds to that given in the ith line of table I. Similarly, tables V and VI are, respectively, continuations of tables II and III.

Columns (3) and (7) of tables I, II, and III list the number of samples which give the same minimum value for either $VAR(\hat{T}_a - T)$ or $VAR(\hat{T}_g - T)$. The criteria by which the listed samples are selected is described in the headings of columns (2) and (6). In a few cases more than one allocation minimizes either $VAR(\hat{T}_a - T)$ or $VAR(\hat{T}_g - T)$ and meets the appropriate criterion.

Bibliography

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TABLE I

$N_{11}=N_{12}=N_{21}=N_{22}=25 \quad (n=20)$

(1)				(2) Opt Additive Allocation With Largest Value for Column (5)				(3) Number of Samples	(4)	(5) R-H Side of Expression (9) in Text	(6) Opt Int Allocation With Smallest Additive Variance				(7) Number of Samples	(8)	(9)	(10)	(11)
σ_{11}^2	σ_{12}^2	σ_{21}^2	σ_{22}^2	n_{11}	n_{12}	n_{21}	n_{22}		$VAR(\hat{T}_a-T)$		n_{11}	n_{12}	n_{21}	n_{22}		$VAR(\hat{T}_g-T)$	$VAR(\hat{T}_a-T)$	(9)/(8)	(4)/(8)
1	1	1	1	5	5	5	5	9	400.0	.00	5	5	5	5	1	400.00	400.00	1.00	1.00
1	2	1	1	9	1	1	9	1	435.04	.35	5	6	4	5	3	489.58	476.52	.97	.89
1	4	1	1	9	1	1	9	1	492.43	.47	4	8	4	4	1	606.25	597.80	.99	.81
1	1	2	1	9	1	1	9	1	435.04	.35	5	4	6	5	3	489.58	476.53	.97	.89
1	2	2	1	9	1	1	9	1	465.84	.49	4	6	6	4	1	579.17	565.14	.99	.80
1	4	2	1	9	1	1	9	1	518.86	.63	4	7	5	4	1	719.64	687.45	.94	.72
1	1	4	1	9	1	1	9	1	492.43	.48	4	4	8	4	1	606.25	597.80	.99	.81
1	2	4	1	9	1	1	9	1	518.86	.63	4	5	7	4	1	719.64	678.45	.94	.72
1	4	4	1	9	1	1	9	1	569.94	.77	3	7	7	3	1	880.95	861.68	.98	.65
1	1	1	2	1	9	9	1	1	435.03	.35	4	5	5	6	3	489.58	476.52	.97	.88
1	2	1	2	2	6	9	3	3	579.17	.00	4	6	4	6	1	579.17	579.17	1.00	1.00
1	4	1	2	7	2	1	10	1	647.52	.44	4	7	4	5	1	719.64	716.09	1.00	.90
1	1	2	2	2	9	6	3	3	579.17	.00	4	4	6	6	1	579.17	579.17	1.00	1.00
1	2	2	2	7	1	1	11	2	618.01	.40	4	5	5	6	3	689.58	675.48	.98	.90
1	4	2	2	7	1	1	11	2	678.20	.58	3	7	5	5	1	840.48	835.36	.99	.81
1	1	4	2	7	1	2	10	1	647.52	.44	4	4	7	5	1	719.64	716.09	.99	.90
1	2	4	2	7	1	1	11	2	678.20	.58	3	5	7	5	1	840.48	835.36	.99	.81
1	4	4	2	7	1	1	11	2	732.53	.75	3	6	6	5	1	1016.67	968.28	.95	.72
1	1	1	4	1	9	9	1	1	492.43	.48	4	4	4	8	1	606.25	597.80	.99	.81
1	2	1	4	1	10	7	2	1	647.52	.44	4	5	4	7	1	719.64	716.09	1.00	.90
1	4	1	4	4	6	3	7	6	876.61	.76	3	7	3	7	1	880.95	880.95	1.00	1.00
1	1	2	4	1	7	10	2	1	647.52	.44	4	4	5	7	1	719.64	716.09	1.00	.90
1	2	2	4	2	7	7	4	3	831.11	.28	3	5	5	7	1	840.48	838.94	1.00	.99
1	4	2	4	6	1	1	12	1	921.00	.47	3	6	5	6	1	1016.67	1016.45	1.00	.91
1	1	4	4	4	3	6	7	6	876.61	.76	3	3	7	7	1	880.95	880.95	1.00	1.00
1	2	4	4	6	1	1,	12	1	921.00	.47	3	5	6	6	1	1016.67	1016.45	1.00	.91
1	4	4	4	6	1	1,	12	1	982.76	.70	3	6	5	6	3	1216.67	1186.22	.97	.81

TABLE II

$N_{11}=N_{22}=10,\ N_{12}=N_{21}=40 \quad (n=20)$

(1)				(2) Opt Additive Allocation With Largest Value for Column (5)				(3)	(4)	(5)	(6) Opt Int Allocation With Smallest Additive Variance				(7)	(8)	(9)	(10)	(11)
σ^2_{11}	σ^2_{12}	σ^2_{21}	σ^2_{22}	n_{11}	n_{12}	n_{21}	n_{22}	Number of Samples	$VAR(\hat{T}_a-T)$	R-H Side of Expression (9) in Text	n_{11}	n_{12}	n_{21}	n_{12}	Number of Samples	$VAR(\hat{T}_g-T)$	$VAR(\hat{T}_a-T)$	(9)/(8)	(4)/(8)
1	1	1	1	2	8	8	2	9	400.00	.00	2	8	8	2	1	400.00	400.00	1.00	1.00
2	1	1	1	1	9	9	1	1	420.03	.62	3	8	7	2	2	435.24	435.24	1.00	.97
1	2	1	1	9	1	1	9	1	450.03	.27	2	9	7	2	1	544.13	534.62	.98	.83
2	2	1	1	3	9	6	2	3	579.17	.89	2	9	7	2	1	584.13	584.04	1.00	.99
1	1	2	1	9	1	1	9	1	450.03	.27	2	7	9	2	1	544.13	534.61	.98	.83
2	1	2	1	3	6	9	2	3	579.17	.89	2	7	9	2	1	584.13	584.04	1.00	.99
1	2	2	1	9	1	1	9	1	494.84	.41	2	8	8	2	1	720.00	677.33	.94	.69
2	2	2	1	10	1	1	8	1	633.70	.31	2	8	8	2	1	760.00	752.00	.99	.83
1	1	1	2	1	9	9	1	1	420.03	.62	2	8	7	3	2	435.24	435.24	1.00	.97
2	1	1	2	1	9	9	1	1	434.84	.87	3	7	7	3	1	470.47	470.47	1.00	.92
1	2	1	2	2	9	6	3	3	579.17	.89	2	9	7	2	1	584.13	584.04	1.00	.99
2	2	1	2	1	10	8	1	1	603.70	.75	2	9	7	2	1	624.13	615.95	.99	.97
1	1	2	2	2	6	9	3	3	579.17	.89	2	7	9	2	1	584.13	584.04	1.00	.99
2	1	2	2	1	8	10	1	1	603.01	.75	2	7	9	2	1	624.13	615.95	.99	.97
1	2	2	2	8	1	1	10	1	633.70	.31	2	8	8	2	1	760.00	752.00	.99	.83
2	2	2	2	2	8	8	2	9	800.00	.00	2	8	8	2	1	800.00	800.00	1.00	1.00

TABLE III

$N_{11}=N_{12}=10,\ N_{21}=N_{22}=40 \quad (n=20)$

1	1	1	1	2	2	8	8	3	400.00	.00	2	2	8	8	1	400.00	400.00	1.00	1.00
2	1	1	1	1	3	9	7	1	418.82	.62	3	2	8	7	2	435.24	433.91	1.00	.96
1	2	1	1	3	1	7	9	1	418.82	.62	2	3	7	8	2	435.24	433.91	1.00	.96
2	2	1	1	3	2	7	8	4	467.67	.92	3	3	7	7	1	470.48	470.48	1.00	1.00
1	1	2	1	3	1	8	8	1	526.73	.67	2	2	9	7	1	544.13	540.62	.99	.97
2	1	2	1	2	2	10	6	2	579.46	1.10	2	2	9	7	1	584.13	582.34	1.00	.99
1	2	2	2	3	1	8	8	1	540.52	.91	2	2	9	7	1	584.13	563.69	.97	.93
2	2	2	1	4	1	8	7	2	603.77	.73	2	2	9	7	1	624.13	622.13	1.00	.97
1	1	1	2	1	3	8	8	1	526.73	.67	2	2	7	9	1	544.13	540.66	.99	.97
2	1	1	2	1	3	8	8	1	540.52	.91	2	2	7	9	1	584.13	563.69	.97	.93
1	2	1	2	2	2	6	10	2	579.95	1.10	2	2	7	9	1	584.13	582.34	1.00	.99
2	2	1	2	1	4	7	8	2	602.99	.73	2	2	7	9	1	624.13	622.13	1.00	.97
1	1	2	2	2	1	8	9	2	706.62	1.18	2	2	8	8	1	720.00	720.00	1.00	.98
2	1	2	2	1	2	10	7	1	735.41	.85	2	2	8	8	1	760.00	752.00	.99	.97
1	2	2	2	2	1	7	10	1	735.41	.85	2	2	8	8	1	760.00	752.00	.99	.97
2	2	2	2	2	2	8	8	3	800.00	.00	2	2	8	8	1	800.00	800.00	1.00	1.00

TABLE IV

$N_{11}=N_{12}=N_{21}=N_{22}=25$ (n=20)

Equal and Proportional Allocation (They are the same in this case.)

(12) $VAR(\hat{T}_a-T)$	(13) $VAR(\hat{T}_g-T)$	(14) (12)/(13)	(15) (8)/(13)	(16) (4)/(12)
400.00	400.00	1.00	1.00	1.00
484.00	500.00	.97	.98	.90
597.17	700.00	.85	.87	.82
484.00	500.00	.96	.98	.90
546.67	600.00	.91	.97	.85
640.00	800.00	.80	.90	.81
597.14	700.00	.85	.87	.82
640.00	800.00	.80	.90	.81
712.00	1000.00	.71	.88	.93
484.00	500.00	.97	.98	.90
600.00	600.00	1.00	.97	.97
760.00	800.00	.95	.90	.85
600.00	600.00	1.00	.97	.97
688.57	700.00	.98	.99	.90
820.00	900.00	.91	.93	.83
760.00	800.00	.95	.90	.85
820.00	900.00	.91	.93	.83
918.18	1100.00	.83	.92	.80
597.14	700.00	.85	.87	.82
760.00	800.00	.95	.90	.85
1000.00	1000.00	1.00	.88	.88
760.00	800.00	.95	.90	.85
891.11	900.00	.99	.93	.93
1092.73	1100.00	.99	.92	.84
1000.00	1000.00	1.00	.88	.88
1092.73	1100.00	.99	.92	.84
1244.62	1300.00	.95	.94	.79

TABLE V $N_{11}=N_{22}=10$, $N_{12}=N_{21}=40$ (n=20)

Equal Allocation (5, 5, 5, 5)			Proportional Allocation (2, 8, 8, 2)				
(12) $VAR(\hat{T}_a-T)$	(13) $VAR(\hat{T}_g-T)$	(14) (12)/(13)	(15) $VAR(\hat{T}_a-T)$	(16) $VAR(\hat{T}_g-T)$	(17) (15)/(16)	(18) (8)/(16)	(19) (4)/(15)
400.00	580.00	.69	400.00	400.00	1.00	1.00	1.00
469.00	590.00	.79	430.86	440.00	.98	.99	.98
499.00	860.00	.58	548.36	560.00	.98	.97	.82
600.00	870.00	.69	600.00	600.00	1.00	.97	.97
499.00	860.00	.58	548.36	560.00	.98	.97	.82
600.00	870.00	.69	600.00	600.00	1.00	.97	.97
576.67	1140.00	.51	677.33	720.00	.94	1.00	.73
703.57	1150.00	.61	752.00	760.00	.99	1.00	.84
469.00	590.00	.79	430.86	440.00	.98	.99	.97
516.67	600.00	.86	451.56	480.00	.94	.98	.96
600.00	870.00	.69	600.00	600.00	1.00	.97	.97
673.57	880.00	.77	633.26	640.00	.99	.98	.95
600.00	870.00	.69	600.00	600.00	1.00	.97	.97
673.57	880.00	.77	633.26	640.00	.99	.98	.95
703.56	1150.00	.61	752.00	760.00	.99	1.00	.84
800.00	1160.00	.69	800.00	800.00	1.00	1.00	1.00

TABLE VI $N_{11}=N_{12}=10$, $N_{21}=N_{22}=40$ (n=20)

Equal Allocation (5, 5, 5, 5)			Proportional Allocation (2, 8, 8, 2)				
(12) $VAR(\hat{T}_a-T)$	(13) $VAR(\hat{T}_g-T)$	(14) (12)/(13)	(15) $VAR(\hat{T}_a-T)$	(16) $VAR(\hat{T}_g-T)$	(17) (15)/(16)	(18) (8)/(16)	(19) (4)/(15)
580.00	580.00	1.00	400.00	400.00	1.00	1.00	1.00
589.00	590.00	1.00	430.86	440.00	.98	.99	.97
589.00	590.00	1.00	430.86	440.00	.98	.99	.97
600.00	600.00	1.00	480.00	480.00	1.00	.98	.97
811.00	860.00	.94	548.36	560.00	.98	.97	.96
840.00	870.00	.97	600.00	600.00	1.00	.97	.97
816.67	870.00	.94	565.87	600.00	.94	.97	.96
845.00	880.00	.96	633.26	640.00	.99	.98	.95
811.00	860.00	.94	548.36	560.00	.98	.97	.96
816.67	870.00	.94	565.87	600.00	.94	.97	.96
840.00	870.00	.97	600.00	600.00	1.00	.97	.97
845.00	880.00	.96	633.26	640.00	.99	.98	.95
1140.00	1140.00	1.00	720.00	720.00	1.00	1.00	.98
1149.29	1150.00	1.00	752.00	760.00	.99	1.00	.98
1149.29	1150.00	1.00	752.00	760.00	.99	1.00	.98
1160.00	1160.00	1.00	800.00	800.00	1.00	1.00	1.00