THE USE OF COMPUTERS IN STUDY OF SOCIAL STRUCTURE: INTERACTION IN A 3-PERSON GROUP.[1]

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The work reported in this paper stems from two sources. The first is a set of propositions made by George Homans in The Human Group, some of which were formalized by Herbert Simon in a paper several years ago. The second is an observation about 3-person groups made by Georg Simmel, and followed up by Theodore Mills and others in recent experiments.

The three-person group as isolate and pair:

Georg Simmel, some years ago, focussed attention on the triad, and made several insightful observations which have been followed up by recent investigators.[2] Simmel's observations tended to suggest that a three-person group breaks up into a pair, between whom association is relatively great, and an isolate, who had considerably less association with either of the pairs. This was only one of a number of configurations which Simmel discussed, but is the one to be examined here. This suggests that a situation of equal interaction between A and B, A and C, and B and C is inherently unstable, and that a stable equilibrium would occur at a less balanced state of affairs, where interaction between A and B was strong, that between A and C was weak, and that between B and C is weak.

This is not at all a trivial proposition, for even if one agrees that a completely balanced state is unlikely, the imbalance might occur in other ways, as indicated in Figure 1.

Theodore Mills has examined Simmel's suggestion in some detail, and found in experimental groups that such a situation did occur; that the three-person group did tend to break up into a highly-interacting pair and a relative isolate.[3]

In quite different investigations, using ad hoc problem-solving groups, Bales has found that in 3-person groups, the rates of interaction between the two highest interacting persons are considerably greater than that of either with the third person (See Table 1.). At the same time, there is much data, both from systematic research and everyday observation, in which 3-person groups do not degenerate in such a fashion.[4] It would be foolish to view the proposition as more than a statement of a tendency, as something which is false if not universally true.

The Homans propositions and Simon model:

The very simplest propositions from Homans were those which Simon undertook to formalize. They concern the mutual reinforcement of social interaction and positive sentiments: as interaction among a set of persons occurs, positive sentiments will develop among them. These positive sentiments, in turn, lead to an increased rate of interaction, over and above that which initially occurred.

Simon formalized these propositions in order to link them together, and thereby construct a system of relations characterizing the group. The formalization consisted of a pair of differential equations which describe the change of positive sentiments (s) with change in interaction rate (r) and vice versa (with parameters a, b, c, g, h, k).[5]

$$\frac{ds}{dt} = a + b\,r - c\,s \qquad (1)$$

$$\frac{dr}{dt} = g + h\,s - k\,r \qquad (2)$$

For appropriate values of the parameters, this will give a system which has a stable equilibrium point, as indicated in Figure 2. The arrows indicate the direction in which interaction and sentiments will move when the group begins at different levels of interaction and sentiments. As Figure 2 indicates, appropriate values of the parameters give a system such that the levels of interaction and sentiment increase, if they begin at a low point, to a stable position (or if they are artifically raised beyond the equilibrium levels, drop to the point where they are mutually sustaining).

It should be emphasized that this is not an empirical analysis, but simply a formalization of propositions about groups, in order to create a system of relations. This system of relations merely shows the joint implications of these two propositions taken together. The empirical adequacy of the propositions is another matter, which will not be discussed here. Though empirical adequacy is important, and certainly the ultimate test of this work, it seems reasonable to withold such tests until after the work has proceeded some distance.

Simon raised the question, as have others, of what might be gained, relative to the above formulation, by carrying out the formalization at the level of individuals and pair relations, rather than the group as a whole. This has some measurement advantages, because sentiments and interaction rates are ordinarily measurements made upon individuals and pairs, not a group. There always arises some aggregation problem if propositions and variables are posited at the group level. Furthermore, by a shift to the individual level, it becomes possible to examine differentiation within the group, which is impossible in the group level formulation. The disadvantages, of course, are the greater complexity of the individual-level formulation, the difficulty of analytic solutions, and the impossibility of graphical representations such as the one above.

This, then, is a second source for the work to be discussed. Together with Simmel's and others' observations on the three-person group, it constitutes the basis for two computer models presented below.

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Taking these two directions of work together, the following question may be raised: accepting as true the isolate-pair tendency in a triad, might it not be explainable in terms of Homans' propositions about the mutual reinforcement of interaction and (positive) sentiments? To be sure, there are personality differences, and differences in compatibility, in any three-person group, and some imbalance certainly results from these. But it would be particularly striking if the imbalance occurred <u>even</u> when there were assumed no initial differences, but only differences which developed through Homans' relations as the group carried out its activities. The conditions under which this might occur is that in which the total interaction between the three was fixed. Thus although there is mutual reinforcement of interaction rate and positive sentiments, there is also a condition of scarcity: only one of the three pairs can be interacting at any one time. This interaction tends to increase the positive sentiments between these two, but because neither of these is interacting with the third, sentiments do not have a chance to build up between him and either of them. This imbalance might then perpetuate itself, resulting in a situation in which the third person was either completely isolated or partially so. The possibility of complete isolation occurring as an initial imbalance became greater and greater seems intuitively easy to conceive, and easy to construct a model for. However, the empirical results seem to correspond to a situation of incomplete imbalance, such that the isolate is not completely disregarded by the others.

Thus with the aim of using the Homans relations to account for the pair-isolate tendency, two formal models were constructed. The concepts and relations in both of these were necessarily at the level of individuals and pairs, in contrast to Simon's model discussed earlier. One is a rather direct analogue of the Simon model, while the other is a stochastic model more directly mirroring the interaction process.

<u>The 3-person analogue of the Simon model, with pairwise interaction, and fixed total interaction:</u>

Simon's concepts are aggregate or group level sentiment and interaction rates. The analogous concepts at the individual level are:

s_{ij} = the (positive) sentiments of individual i for individual j. (There are 6 s_{ij}'s.)

$r_{ij} (=r_{ji})$ = the tendency toward interaction between individuals i and j. (There are 3 r_{ij}'s.)

Note that r_{ij} is here a tendency toward interaction, rather than the rate itself, as in Simon's equations. This is necessary because the tendency must be an increasing function of sentiments, yet the sum of the interaction rates is fixed. The tendencies are translated into interaction rates by dividing up the total interaction proportionally to the sizes of the tendencies r_{ij}.

The system of equations set up is as follows:

(a) For the change in interaction tendency between i and j, as a function of s_{ij} and s_{ji}:

$$\frac{d\, r_{ij}}{dt} = c_1 \left(a_{ij} + k_i\, s_{ij} + k_j\, s_{ji} - r_{ij}\right)$$

(all parameters are > 0)
(3 such equations)

This equation says that without any sentiments, interaction would develop until it equaled a_{ij}, but that sentiments raise this. The parameters k_i show the amount of effect that i's sentiments have on the interaction tendency, and thus are a measure of the power or control i has over the interaction. The parameter c_1 is a coefficient indicating the overall speed of change.

(b) For the change in sentiments of i toward j, as a function of r_{ij} (relative to other r's):

$$\frac{d\, s_{ij}}{dt} = c_2\left(b_{ij} + h_{ij}\, \frac{r_{ij}}{r_{+2} + r_{13} + r_{23}} - s_{ij}\right)$$

(all parameters are > 0)
(6 such equations)

This equation says that some sentiments would develop without interaction, equal to b_{ij}. But the rate of interaction [which is simply $r_{ij} / (r_{12} + r_{13} + r_{23})$] further raises the level of sentiments. The parameter h_{ij} is the amount of effect interaction with j has in increasing i's sentiments toward him - a matter which depends largely on the amount of rewards i gets from the interaction. The parameter c_2 is a coefficient of rate of change.

This system of equations thus consists of three equations for rate of change of r_{12}, r_{13}, and r_{23}, and six equations for the rates of change in sentiments s_{ij} of each toward the other. Given these equations, what can be said about the Simmelian observation about degeneration into an isolate and a dyad? Two conditions can be distinguished:

(a) the condition in which parameters are identical for each person and each pair. In such a circumstance, will there be an equilibrium at balanced interaction (where $r_{12}=r_{13}=r_{23}$)? And if so, will this equilibrium be a stable one? Such a condition provides the most rigorous test, for if there is imbalance when external conditions (represented by the various parameters) are perfectly balanced, then Homan's propositions can under any conditions account for the imbalance which Simmel observed.

(b) the condition in which parameters differ for different persons and pairs. One would naturally expect imbalance in such a case (for example, when the increase of positive sentiments between A and B due to their interaction is especially great). However, two questions are of importance here: does a <u>slight</u> imbalance in the parameters, such as might be found in any group of three, result in a magnified imbalance in the equilibrium interaction? And second, does <u>any</u> type of imbalance in the parameters result in the dyad-isolate configuration, or do some types of imbalance give rise to other patterns, as shown in Figure 1?

These are the questions which are to be investigated with this model. The first condition, of complete balance in the parameters, can be handled by an analytic solution; the second requires numerical analysis, which in this case was carried out on a digital computer. Before examining the results, the second model will be outlined.

A stochastic model of 3-person interaction, with pairwise interaction and total interaction limited:

The model to be described below is not simply the above differential-equation model with a stochastic element introduced. Rather than consisting of a set of simultaneous equations to represent the system, it attempts to mirror more directly the interaction process involved, while capturing the essence of the Homans propositions about interaction and sentiments. The basic processes of the model, as presented in the flow chart of Figure 3, are as follows:

1. Select tentative interaction pair, on the basis of interaction tendencies r_{ij}.

The process is:

a) $p_{12} = \dfrac{r_{12}}{\sum r_{ij}}$

b) If p_{12} > random number, the 12 pair is selected; if not, p_{13} is computed, added to p_{12} and tested against the random number. One pair will definitely be selected, since $\sum p_{ij} = 1$, and the random number lies between zero and 1.

2. Test for rejection of interaction on the basis of interaction tendency r_{ij} relative to pro- (r_{ij}) and anti- (q_{ij}) interaction tendencies.

The process is:

a) If $\dfrac{r_{ij}}{q_{ij} + r_{ij}}$ > random number, then there will be interaction. If not, a new interaction pair will be selected according to (1) above.

3. Interaction between individuals i and j produces (stochastically) a reward or punishment for i, depending on fixed parameters. These rewards and punishments cumulate, to constitute the total rewards and punishments received from interaction with j. The rewards and punishments may be considered directly as positive and negative sentiments toward j, as they will be implicitly below, or they may be considered to affect the positive and negative sentiments. Either way is consistent with the Homans propositions, and either way will give the same formal results.

The process is:

a) $p_{ij} = \dfrac{g_{ij}}{h_{ij} + g_{ij}}$

where g_{ij} = coefficient of reward to i for interaction with j.

where h_{ij} = coefficient of punishment to i for interaction with j.

where p_{ij} = probability of reward.

b) If p_{ij} > random number, reward counter s_{ij} is incremented by 1; if not, punishment counter v_{ij} is incremented.[6]

3.' Same for j as a result of the interaction with i.

4. Modify interaction tendency between i and j on basis of positive sentiments between i and j. This tendency is governed by k_i and k_j, which as in the other model, are i's and j's control over interaction, respectively.

The process is:

$r_{ij} = k_i s_{ij} + k_j s_{ji}$

5. Modify anti-interaction tendency between i and j on basis of negative sentiments between i and j.

The process is:

$q_{ij} = k_i v_{ij} + k_j v_{ji}$

These processes taken together constitute a system of interaction of the following sort: interaction generates either rewards or punishments for the participants. The rewards bias the interaction toward the pair who interacted, in the next selection of interaction; but the punishments act as a "filter" which must be passed before the interaction proceeds.

Results for the two models:

A. The simultaneous-equation model

1. The first question is the equilibrium state when the parameters are identical for all pairs and individuals. It can be easily shown that there is a single equilibrium point, where interaction is exactly balanced. Furthermore, this is a stable equilibrium, as in the original Simon model. If interaction or sentiments are unbalanced to begin with, they will return to the balanced situation, as indicated in (c) of Figure 4.

There is one exception: if there is no externally induced interaction or sentiments (that is, if all a_{ij} and b_{ij} are zero), and the system begins in an unbalanced state, exactly the same imbalance will remain. It will not increase (as

Simmel proposed) nor diminish, but remain exactly as it began.

The first conclusion, then, is that the most difficult test is failed. The Homans propositions, when translated into a deterministic model with no external differences among individuals, do not produce degeneration into a pair and isolate. Quite to the contrary, a completely balanced situation returns when the balance is in some external way upset.

2. When the external parameters are allowed to differ from person to person, the results are quite different. Since the equations cannot be easily solved for the equilibrium state of r's, numerical analysis on the computer was carried out. Two conditions were investigated: a) one in which two persons found greater rewards from their interaction than either did with the third; (h_{13} and h_{31} are greater than other h's) and b) one in which one person had greater control over the interaction than did the other two (k_3 is greater than k_1 or k_2).

The results of these two situations were quite different, as shown in Figure 4. In Figure 4, each point of the triangle represents 100% interaction between two members; the center is completely balanced interaction. In the case where one pair's interaction was mutually more rewarding, the interaction (which began in a balanced state, at the center) moved off toward a new equilibrium, in which r_{13} was high, r_{23} and r_{12} low, shown in (a) of Figure 4. This is, in effect, the isolate-dyad situation. It should be noted, however, that the imbalance appears to be a direct reflection of the imbalance in h's. The values of h_{13} and h_{31} were twice as high as those of the other h's, so that it required a rather large imbalance of the h's to produce even this imbalance in interaction.

In the case where one member has greater control, a very different kind of imbalance occurs, corresponding to (c) in Figure 2. Two pairs (the two in which the strong member takes part) increase their interaction while the third decreases, until a new asymmetric equilibrium is produced, as shown in (b) of Figure 4. This is not at all the kind of asymmetry Simmel discussed, but quite the opposite kind.

On the whole, then, this formalization of Homans' model hardly accounts for the isolate-dyad phenomenon. It does generate various types of disequilibrium, but nothing beyond the imbalances which are introduced as data. Thus the model has little or no explanatory ability with respect to the isolate-dyad phenomenon.

B. The stochastic reward-and-punishment model

The stochastic model gives a somewhat different picture. Only a few runs have been made with the model, so that its general characteristics are not yet known; but these are enough to show some interesting consequences.

The parameters of the model were identical for all persons. The only parameters of the model are these: a) relative power of each person (equal in this case, so that for everyone one unit increase in cumulative reward produces half a unit increment in interaction tendency; and one unit increase in cumulative punishment produces half a unit increment in anti-interaction tendency); and b) the reward-punishment ratio. This ranged from a low of 6:4 to a high of .95:.05 in the runs which have been made.

Making the parameters identical for all is necessary to test the explanatory power of the Homans propositions. If there were imbalance in the parameters, a resulting imbalance in the interaction would be quite expected, as in the deterministic model above.

The initial conditions are important in this model. In all these runs, the assumed existing interaction tendency between each pair was only one unit. Consequently, on the first interaction, the immediate rewards could have as much influence as past experience (one unit to one unit). If the initial "experience" were much greater, a greater tendency toward stability might be expected.

The results of the first run, which used a reward-punishment ratio of 6:4, are shown in detail in Figure 5. There was an immediate and definite drift toward one of the pairs. By chance nine of the first ten interactions were between individuals 1 and 2, and also by chance, both 1 and 2 were rewarded on all nine of these interactions. This initial sequence set into effect a drift which was never overcome, as Figure 5 shows. The drift continues toward the 1,2 pair, though there are interruptions - and cases of rejected interactions. After 100 time periods, the interactions have been far more numerous between 1 and 2 than between any other pair, the rejected interactions have been more numerous, the positive sentiments are far stronger, and greater feelings of hostility have developed. In other words, the 1-3 and the 2-3 relationships simply didn't have a chance to develop. The 1-3 relation was stifled by negative sentiments on 3's part which happened to come about in early interactions. As a consequence, many of the attempted interactions between 1 and 3 simply never came off, thus reducing the likelihood of a relation developing. The 2-3 relation was a different matter - it never got started. There were only 6 attempted interactions altogether, and two of these failed.

The results of this run are not typical, however. In fact, in examining many runs, no typical pattern was observed. Sixty-two runs were made with reward:punishment ratio of 6:4. At the end of 100 trials, some groups were still rather balanced, some were segregated into a dyad and an isolate, and in some, one person dominated the interactions. In other words, the situation sometimes deteriorated into a dyad-isolate combination, sometimes into single-person dominance, and sometimes remained balanced.

A hundred and forty-seven runs were made at a reward:punishment ratio of 8:2, and a hundred at 95:05. There was little change in outcome with these changes, except an apparently slight drift toward imbalance (of both kinds) as the reward:punishment ratio increased.

A quantitative study of the results is not in order until more data are in. These results indicate only that a situation of imbalance frequently does occur, despite the symmetry of the parameters. Surprisingly, the imbalance seems just as frequently to be of the single-person dominance type as of the dyad-isolate type.

Conclusions:

It would be premature to attempt to draw many conclusions from these initial results. Until there are some extensive Monte Carlo runs made, the general characteristics of the model cannot be known. It is clear, however, that in the stochastic model, either a continued balance or a drift toward asymmetry can occur, depending on the initial conditions and upon chance. Yet the asymmetry is not always of the dyad and isolate type, as Simmel and others have written about, but seems equally often to consist of two strong relations and one weak relation, a somewhat surprising result.

It is also clear that under conditions of balanced parameters, the deterministic model does not give asymmetry of any kind. Only with difficulty does it give asymmetry at all, and then an asymmetry which is completely built in by fixing the parameters asymmetrically.

Finally, some points about further development of these models are relevant.

a. This problem has a direct dual, in which the pairs represent individuals and the individuals represent directed relations. (For groups, larger than three in size, the formal duality does not exist, though only a slight variation in the model is necessary for the dual interpretation.) The dual problem is the problem of individual participation in groups. Interaction rate for the pair becomes an individual's participation rate; and the rewards and punishments for the two members in the interaction become rewards and punishments received from the other two members. Though the detailed interpretation will not be discussed here, it is useful to note that the formal model can be appropriate for the dual problem as well as the original one.

b. The model has been set up to investigate specifically a problem of three-person groups; but the same model is appropriate for larger groups where only one pair is interacting at one time. (It is interesting to note that only in a three-person group is this single interaction per unit time dictated by the very numbers themselves. In groups of four or more, simultaneous interactions may occur between two pairs.)

c. The system as it stands is only a bare framework which may be enriched by further processes. For example, Homans and others suggest that the likelihood of an interaction's being rewarding depends upon one's control over it. When power is distributed very unevenly, those with little control over the interaction are more likely to develop hostility. Also, it is known that as sentiments develop in a group, the members extend the relationship into other activities. These are usually activities in which they have common interests, and which consequently have a higher ratio of reward-to-punishment than the original activity.

d. More generally, this model probably will be most important as a piece of other, more inclusive systems. For example, one system now being developed at Johns Hopkins is a worker-management system in which restriction of production by workers is generated by management's reduction of piece rates. But in the model, the normative constraints workers are able to place upon one another depend upon the strength of sentiments which have developed among them. These sentiments are generated over time, according to the configuration of work activities and the stability of personnel. Either of the two models discussed here can be used to generate these sentiments.

FOOTNOTES

1. Work reported in this paper has been facilitated by a grant from the National Science Foundation. Gratitude is also due to the Applied Physics Laboratory of Johns Hopkins, for making its computing facilities (a Univac Scientific, 1103A) available, and Miss Patricia Powers, for aid in running the problems.

2. Georg Simmel, The Sociology of Georg Simmel ed. and trans. by Kurt Wolf (Glencoe: The Free Press, 1950). Work in this area has included that by Fred L. Strodtbeck, "The Family as a Three Person Group" American Sociological Review, 19 (February 1954) pp. 23-29; and Theodore M. Mills, "Power Relations in Three-Person Groups, American Sociological Review, 18 (August, 1953) pp. 351-357.

3. Ibid, and "The Coalition Patterns in Three-Person Groups," American Sociological Review, 19 (December 1954) pp. 657-667.

4. Strodtbeck (loc. cit.) for example, finds that in 3-member family groups this tendency to degenerate into an isolate and a pair is missing.

5. Simon's formulation is more complicated than this, but it reduces to equations like (1) and

(2). Simon also sets up an alternative set of equations, which makes fewer assumptions about quantitative measurement of r and s, and about the form of the relations. Those are not relevant to the present discussion; nor will the present discussion go into questions of measurability of sentiments and interaction, which have been treated in detail by the author elsewhere. (See "Mathematical Models of Small Group Behavior," in Herbert Solomon, ed., Mathematical Thinking in the Measurement of Behavior: Small Groups, Utility, Factor Analysis (Glencoe: The Free Press), to be published Spring, 1960.

6. The random number is rectangular between zero and 1.

TABLE 1.

Participation rates in 3-person groups (data from R. F. Bales - personal communication)

		TO			
		1	2	3	(to group)
FROM	1		1445	660	2044
	2	1455		504	1091
	3	731	530		874

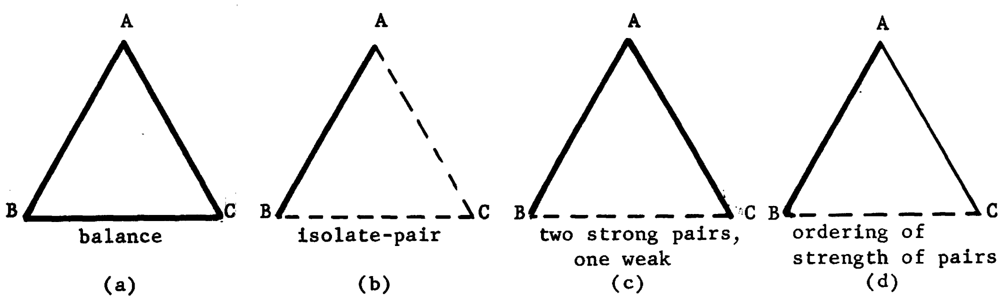

FIGURE 1: Various configurations of interaction in a 3-person group.

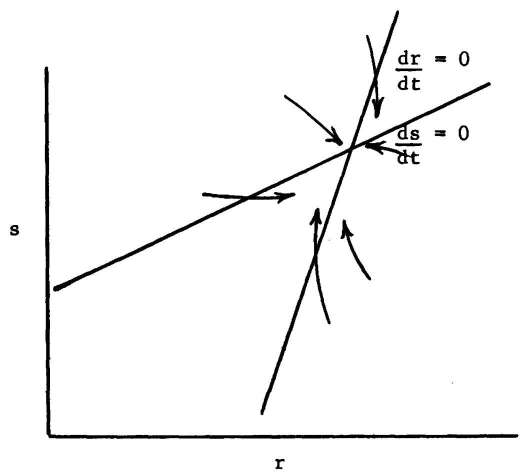

FIGURE 2: The interdependence of sentiments (s) and interaction rate (r) in a group, according to Simon's formalization of Homans' propositions.