# Discussion

J.N.K. Rao, Carleton University
J.N.K. Rao, School of Mathematics and Statistics, Carleton University, Ottawa, Canada, K1S 5B6

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In this interesting session on weight calibration, Ralph Folsom and Avi Singh and their colleagues at Research Triangle Institute (RTI) reported their recent work on survey weight calibration for extreme design weights, nonrepsonse and poststratification, using a generalized exponential model (GEM). Taylor linearization method is used to estimate the variance of the calibration estimator of a total, $T_y$, by expressing the estimator as a solution to estimating equations. The proposed methods are applied to data from the National Household Survey on Drug Abuse (NHSDA).

The paper by Folsom and Singh introduces the GEM model. In Section 2, the adjustment factor for k-th sample unit, $a_k(\lambda)$, is modelled as a "logit" model proposed by Deville and Sarandal (1992). This factor satisfies range restrictions (RR), $l < a_k(\lambda) < u$, for specified lower and upper bounds $l$ and $u$. But a solution $\hat{\lambda}_n$ that satisfies the benchmark constraints (BC), $\sum_s x_k d_k a_k(\lambda) - T_x = 0$, may not exist, where $s$ is the sample, $d_k$ is the design weight and $T_x$ is the vector of known population totals of poststratification variables. The logit model is generalized to GEM in Section 3 by introducing unit-specific bounds $(l_k, u_k)$, $k \in s$ which are grouped into three sets to handle extreme design weights. Again a solution $\hat{\lambda}_n$ may not exist. For poststratification no models are involved, but Section 4 introduces superpopulation models for coverage bias and nonresponse, and control totals $\tilde{T}_x$ which are random in the case of nonresponse adjustment are introduced ($\tilde{T}_x = T_x$ for poststratification or coverage bias). Assuming $\hat{\lambda}_n$ exists, the variance of the calibration estimator $\hat{T}_y(\hat{\lambda}_n)$ = $\sum_s y_k d_k a_k(\hat{\lambda}_n)$ under nonresponse or coverage bias is obtained in the paper by Singh and Folsom by expanding $\hat{T}_y(\hat{\lambda}_n)$ and $\hat{T}_x(\hat{\lambda}_n) - T_x = 0$ around the superpopulation parameter $\lambda$, assuming $\hat{\lambda}_n$ converges in probability to $\lambda$. Poststratification is treated as a limiting case with $\lambda = 0$ and $a_k(\lambda) = 1$. In the paper by Vaish, Gordek and Singh, the results on variance estimation are extended to cover both coverage bias and nonresponse adjustment. The paper by Chen, Penne and Singh implemented the proposed method on data from NHSDA.

In a recent paper (Demnati and Rao, 2000), we have developed a new approach to Taylor linearization variance estimation which covers poststratification with general $a_k(\lambda)$ of the form $a_k(\lambda) = F(x_k'\lambda)$ for some $F(\cdot)$; $F(x) = e^{-x}$ gives generalized raking ratio weights. Our method is based on representing Taylor linearization in terms of partial derivatives with respect to design weights $d_k$. It leads to variance estimators with good conditional properties and agrees with a jackknife linearization variance estimator (Yung and Rao, 1996) when the latter is applicable. The method covers general calibration estimators of a total, $T_y$, as well as other estimators defined either explicitly or implicitly as solutions of estimating equations; in particular, estimators of logistic regression parameters with calibration weights. For the general calibration estimator of $T_y$, it is interesting to note that our variance estimator is identical to the Folsom-Singh variance estimator, but we do not assume that $\hat{\lambda}_n$ converges in probability to $\lambda$. Also, our variance estimator is different from the variance estimator proposed by Deville and Sarandal (1992) except in the case of $\partial F(x)/\partial x = F(x)$ which is satisfied for the generalized raking weights case with $F(x) = e^{-x}$.

As noted by Folsom and Singh, the proposed calibration after winsorization of extreme design weights looks appealing, but its asymptotic properties are unknown. The validity of the Taylor variance estimators will depend on the consistency of the solution $\hat{\lambda}_n$.

An alternative to Taylor linearization is to use a resampling method such as the jackknife, BRR or bootstrap (Rao and Wu, 1988) when applicable. The bootstrap or BRR can handle quantiles as well as general parameters such as logist regression parameters; the Taylor methods of Folsom and Singh are not applicable to nonsmooth estimators like quantiles. Also, for the case of calibration after linearization of extreme weights, a resampling method might be more appealing because it applies the same estimation procedure for each pseudo-replicate. The main drawback of a full jackknife (e.g., delete one-cluster jackknife for stratified multistage sampling) is that the computation can become cumbersome when the number of pseudo-replicates is large because the calibration weights have to be obtained for each pseudo-replicate. To simplify the computations, one may use a delete-a-group jackknife variance estimator with fewer pseudo-replicates, but it can be very unstable. On the other hand, the bootstrap variance estimator with the same number of pseudo-replicates is more stable than the delete-a-group jackknife variance estimator (Canti and Davison, 1999). Computation can be further simplified by using only a one-step Newton Raphson method with $\hat{\lambda}_n$ as the starting value to get the jackknife adjustment factors for each pseudo-replicate.

I have already noted that a solution $\hat{\lambda}_n$ that satisfies both RR and BC may not exist, if RR is fairly tight. Rao and Singh (1997) proposed a "ridge-shrinkage" method which is designed to yield a solution by using a built-in tolerance specification procedure to relax BC while satisfying RR and design consistency. It would be worthwhile to explore this method when a solution that satisfies both BC and RR does not exist.

For the case of nonresponse adjustment, Folsom and Singh obtained their variance estimator conditional on population response indicators. But the contribution from the term involving the variance over response indicators under the assumed model may not be negligible if the sampling fraction is not negligible (Shao and Steel, 1999).

All in all, the four papers presented in this section make important contributions to calibration estimation by unifying weight calibration for extreme design weights, nonresponse and coverage bias/poststratification.

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