## THE GENERALIZED EXPONENTIAL MODEL FOR SAMPLING WEIGHT CALIBRATION FOR EXTREME VALUES, NONRESPONSE, AND POSTSTRATIFICATION

## R. E. Folsom, Jr. and A.C. Singh, Research Triangle Institute R.E. Folsom, RTI, P. O. Box 12194, Research Triangle Park, NC 27709 ref@rti.org

Key Words: Extreme Values, Nonresponse, Poststractification

#### 1. Introduction

Consider a finite population U from which a sample of size n is selected using the design p(s). Denote the data by  $(y_k, x_k, d_k)$ ,  $k \in s$ , where for the  $k^{th}$  unit in the sample,  $y_k$  is the study variable,  $x_k$  is a p-vector of covariates or predictor variables; and d<sub>k</sub> is the design weight. In practice, the d-weights are often adjusted to get the final w-weights in view of the triple concerns of (i) variance inflation of small domain estimates due to extreme values, (ii) bias due to nonresponse (nr), and (iii) bias due to under/over coverage. For the first one, winsorization (i.e., trimming part of the weight beyond the boundary defining extreme values) is often used to adjust extreme values but this may lose its impact after adjustments for nr and coverage; for the second one, weights are adjusted by the inverse response propensity factor (this is typically implemented by calibrating respondent weights to (random) control totals for covariates in the nr model obtained from the full sample of respondents and nonrespondents ) but in the process some weights could become extreme; and for the third one, weights are adjusted by poststratification (ps) (this is typically realized by calibrating weights to nonrandom controls for covariates in the ps model) but in the process some of the final weights could also become extreme. Note that while random controls used in calibration (as in the case of nr and extreme weights resulting from calibration (for nr and ps adjustments) may have the undesirable effect of inflating the variance, this effect could be offset by the anticipated variance reduction due to the correlation between y and x.

There exist methods in the literature which impose bounds on the adjustment factor for ps, see e.g., Deville and Särndal (1992), Rao and Singh (1997) and the review by Singh and Mohl (1996). However, they do not directly restrict the adjusted weight from being too extreme. In this paper we consider the problem of developing a unified approach of weight calibration to address the above three concerns such that there are builtin controls on the adjustment factors to prevent the adjusted weight from being too extreme. For this purpose the logit-type model of Deville and Särndal (1992), denoted by DS in the sequel, is generalized to allow for more general and unit-specific bounds. A review of the DS model is provided in Section 2, and the proposed model is described in Section 3. The asymptotic properties of the proposed calibration estimator are presented in Section 4, and a comparison with alternative methods is given in Section 5. Finally, Section 6 contains numerical results comparing different methods using the 1999 NHSDA data followed by concluding remarks in section 7.

# 2. The Deville-Sarndal Model for Weight Calibration and Statement of the Problem

For ps, in the logit-type model of Deville-Särndal the adjustment factor for unit k is modeled as:

$$a_{k}(\lambda) = \frac{\ell(u-1) + u(1-\ell) \exp(Ax_{k}'\lambda)}{(u-1) + (1-\ell) \exp(Ax_{k}'\lambda)}, \quad (2.1)$$

where  $\ell < 1 < u$ ,  $A = (u-\ell)/(u-1)(1-\ell)$ ;  $\ell, u$  are userspecified bounds, and  $\lambda$  is the column vector of p model parameters corresponding to the p covariates x. The coefficient A in (2.1) is useful to control the behavior of  $a_k(\lambda)$  as the lower or the upper bound approach the center 1. For instance, in the absence of A,  $a_k(\lambda)$  goes to 1 as u goes to 1 regardless of whether  $x'_k\lambda$  is positive or negative which is clearly undesirable. However, in the presence of A, as u approaches 1,  $a_k(\lambda)$  goes to 1 if  $x'_k\lambda$ is positive, and to the lower bound if it is negative. Also note that by construction,  $\ell < a_k < u$ , and as  $\ell - 0$ ,  $u \to \infty$ ,  $a_k(\lambda) - \exp(x'_k\lambda)$  which is the exponential model corresponding to the Raking-Ratio method of poststratification.

The model parameters  $\lambda$  are estimated from

$$\sum_{s} x_k d_k a_k(\lambda) - T_x = 0, \qquad (2.2)$$

where  $T_x$  is the vector of ps controls. The adjusted weights  $w_k$ : =  $d_k a_k$  are close to  $d_k$  in that they minimize  $\Delta(w,d)$  (defined below) subject to (2.2)

$$\frac{1}{A} \sum_{s} d_k \left\{ (a_k - \ell) \log \frac{a_k - \ell}{1 - \ell} + (u - a_k) \log \frac{u - a_k}{u - 1} \right\} \quad (2.3)$$

We wish to generalize the above DS model to allow for

- (i)  $\ell \ge 1$ ; this would be useful for the nonresponse adjustment. This implies that we need to change the center from 1 to c such that  $1 \le \ell \le c \le u$ .
- (ii) nonuniform bounds  $(\ell, u)$  for different subgroups of weights, e.g.,  $(\ell_1, u_1)$  for high extreme values,  $(\ell_2, u_2)$  for nonextreme, and  $(\ell_3, u_3)$  for low extreme values. This would be

useful for providing built-in controls over final adjusted weights for initially identified extreme values.

(iii) a separate weight adjustment for extreme values after nr and ps adjustments. This can be achieved in a manner similar to ps except that the resulting weights meet the tighter bounds on the adjustment factor while continuing to satisfy the ps controls, i.e., preserve the sample distribution of various ps variables.

### 3. The Proposed Model

We propose a generalized exponential model (GEM) with unit-specific bounds  $(\ell_k, u_k), k \in s$ , for the adjustment factor  $a_k(\lambda)$  as follows:

$$a_{k}(\lambda) = \frac{\ell_{k}(u_{k}-c_{k}) + u_{k}(c_{k}-\ell_{k}) \exp(A_{k}x_{k}'\lambda)}{(u_{k}-c_{k}) + (c_{k}-\ell_{k}) \exp(A_{k}x_{k}'\lambda)}, \quad (3.1)$$

where  $c_k$  are prespecified centering constants, such that  $\ell_k < c_k < u_k$  and  $A_k = (u_k - \ell_k)/(u_k - c_k)(c_k - \ell_k)$ . Note that when  $\ell_k - 1$ ,  $c_k - 2$ , and  $u_k - \infty$ , the  $a_k(\lambda)$  approaches the inverse logistic function  $1 + e^{x_k'\lambda}$ .

The  $\lambda$ -parameters are estimated by solving

$$\sum_{s} x_k d_k a_k(\lambda) - \tilde{T}_x = 0, \qquad (3.2)$$

where  $\tilde{T}_x$  denote control totals which could be either nonrandom as is generally the case with ps, or random as is generally the case for nr adjustment.

The final weights  $w_k = d_k a_k$  minimize the distance function  $\Delta(w,d)$  defined as before except that  $(\ell, 1, u)$  is replaced by  $(\ell_k, c_k u_k)$ , and A by A<sub>k</sub>.

Although the proposed model allows for arbitrary unit-specific bounds, in practice, it would generally be sufficient to specify three sets of bounds on the adjustment factors,  $(\ell_1 m_k, u_1 m_k), (\ell_2 m_k, u_2 m_k)$ , and  $(\ell_1, m_k, u_1, m_k)$  for high extreme, nonextreme, and low extreme values identified among the initial weights where  $m_k = b_k / d_k$ ,  $b_k$  is the winsorized value of the design weight  $d_{\nu}$  corresponding to different domains defining extreme values. Clearly,  $m_k = 1$  for nonextreme values. In specifying bounds (l, u)'s, we may first choose them for nonextremes, and then set  $\ell_1 = \ell_2$ ,  $u_3 = u_2$ , and choose  $u_1$  close to  $c_1$ , and  $\ell_3$  close to  $c_3$ . All the three centering constants are typically set to a common value; in the case of ps it is 1, in the case of nr it can be chosen as inverse of the overall response propensity, and in the case of adjustment for extreme weights, it is set to 1 as in the case of ps. It may be noted that allowing  $A_k$  to vary with k might compromise the correlation of the covariate x with y. In practice, it would be sufficient to have only three values of  $A_k$  corresponding to high extreme, nonextreme, and low extreme values, e.g., for high extremes, we can set  $A_k = (u_1 - \ell_1)/(u_1 - c_1)(c_1 - \ell_1)$ ; i.e., remove the factor  $m_k$  from the denominator. Note that the factor  $A_k$  cannot be dropped for reasons mentioned earlier.

Assuming that the solution exists, the model can be fit using Newton-Raphson iterative steps as follows. Let X denote the n x p matrix of auxiliary (or predictor) variables x, and for the vth iteration, let

$$\Gamma_{\varphi v} = \text{diag} (d_k \varphi_k^{(v)}), \varphi_k^{(o)} = 1,$$
  
$$\varphi_k^{(v)} = (u_k - a_k^{(v)}) (a_k^{(v)} - \ell_k) / (u_k - c_k) (c_k - \ell_k)$$

Now the value of the vector  $\lambda$  at iteration v is adjusted as

$$\lambda^{(v)} = \lambda^{(v-1)} + (X' \Gamma_{\varphi, v-1} X)^{-1} (T_x - \hat{T}_x^{(v-1)})$$
  
where  $\lambda^{(0)} = 0$ .

The convergence criterion is based on the Euclidean distance  $||T_x - \hat{T}_x^{(v)}||$ . At each iteration, it is checked whether it is decreasing or not. If not, then half-step length is used in the iteration increment.

## 4. Properties of the GEM Calibration Estimator 4.1 Asymptotic Consistency

Assume the asymptotic setup of Isaki and Fuller (1982). For ps when there is no coverage bias, the weight is adjusted primarily with the goal of variance reduction. In this case, under regularity conditions (see e.g., Deville, and Särndal, 1992) which include the design-consistency of the Horvitz-Thompson estimator, we have

$$\hat{\lambda}_n = O_p(n^{-1/2}) \text{ , and}$$

$$N^{-1} \left[ \sum_s y_k d_k a_k(\hat{\lambda}_n) - T_y \right] \rightarrow 0 \text{ (in design prob.),}$$

i.e., the calibrated estimator is also design-consistent for the population total  $T_y$ . Note that in the model for  $a_k(\lambda)$  as explained in the previous section,  $l_1, u_1$  etc are supposed to be prespecified. The  $m_k$ , however, are sample dependent and hence random. Under our asymptotics, we assume that  $m_k - 1$  in probability uniformly in k so that asymptotically we have only one set of bounds  $(l_2, u_2)$  which are nonrandom as in the DS model. This is only an heuristic argument, and needs rigorous justification. The variance estimators presented in this section do not take into account of the random variability in  $m_k$ . When there is coverage bias, assume a superpopulation model  $\xi_1$  for the multiplicities variable  $(\eta_k, say)$  taking nonnegative integer values, i.e., for each k in U

E<sub> $\xi_1$ </sub> (# times the kth unit in U is enumerated) =  $a_k^{-1}(\lambda)$ , (4.1)

For an explanation of this model, see Singh and Folsom (2000). It follows that for known  $\lambda$ , the calibrated estimator is  $p\xi_1$ -unbiased. Now, under regularity

conditions with respect to the joint  $p\xi_1$ -distribution, the estimator  $\hat{\lambda}_n$  is an asymptotically consistent estimate of  $\lambda$ . Next using Taylor expansion of the estimator  $\hat{T}_y(\hat{\lambda}_n)$  about  $\lambda$ , the poststratified estimator can be shown to be  $p\xi_1$ -consistent.

For nr adjustment, assume that specific to the survey objectives and conditions, one can assign a value of 1 or 0 for the response indicator to each unit in the finite population. Now suppose  $\xi_2$  denotes the superpopulation model for the response indicator ( $\delta_k$ , say), i.e., for each k in U,

$$P_{\varepsilon_2}$$
 (kth unit in U responds) =  $a_k^{-1}(\lambda)$ , (4.2)

Analogous to ps, asymptotic consistency of the nr adjusted estimator follows. To show asymptotic consistency under both nr and ps adjustments, we need to assume two independent superpopulation models  $\xi_1$  and  $\xi_2$  giving rise to adjustment factors  $a_{1k}(\lambda_1)$  and  $a_{2k}(\lambda_2)$ . Now as before, the estimator  $\sum_s y_k d_k a_{1k}(\lambda_1) a_{2k}(\lambda)$  is  $p\xi_1\xi_2$ -unbiased, and asymptotic consistency of the calibration estimator involving estimates  $(\hat{\lambda}_1, \hat{\lambda}_2)$  is established by Taylor expansion about  $(\lambda_1, \lambda_2)$  and the asymptotic consistency of  $(\hat{\lambda}_1, \hat{\lambda}_2)$ .

The extreme value adjustment is part of nr and ps under GEM. If an additional adjustment for extreme values (ev) is used after ps, then as mentioned earlier in Section 2, it is performed by another ps-type GEM such that sample distribution by various ps control variables is preserved but the extreme values are controlled by tight bounds. Thus ev adjustment is analogous to ps (for the case of no coverage bias), and the resulting estimate is design-consistent.

#### 4.2 Asymptotic Variance

For nonresponse, the design-based variance of  $\hat{T}_{v}(\hat{\lambda}_{n})$ below obtained defined is about  $\sum_{U} y_k \, \delta_k \, a_k(\lambda)$ , which is the conditional mean given  $\xi_2$ , and  $\delta_{\nu}$  is the response indicator at the population level. For coverage, the variance is also conditional about  $\sum_{U'} y_k a_k(\lambda)$  (or  $\sum_U y_k \eta_k a_k(\lambda)$ ) given  $\xi_1$  where the population U' exhibits both types of coverage errors (over or under) but U denotes the actual population. Notice that values of the multiplicity factor  $(\eta_{\mu})$  are not needed for unbiased estimation because the target parameter based on U' doesn't involve  $\eta_k$ . The designbased variance gives only the conditional variance. In fact, we need the variance about  $T_{y} := \sum_{U} y_{k}$ , but the second term in the unconditional variance is negligible by comparison. Observe that

$$\hat{T}_{y}(\hat{\lambda}_{n}) := \sum_{s} y_{k} d_{k} a_{k}(\hat{\lambda}_{n}) \approx \hat{T}_{y}(\lambda) + H_{12}(\lambda) (\hat{\lambda}_{n} - \lambda) (4.3)$$
where
$$\hat{T}_{y}(\lambda) := \sum_{s} y_{k} d_{k} a_{k}(\lambda) , H_{12}(\lambda) = \sum_{s} y_{k} (\partial a_{k}(\lambda)/\partial \lambda)' d_{k}$$

Moreover, since  $\hat{\lambda}_n$  solves (3.2), we get from Taylor,

$$0 = \hat{T}_{x}(\hat{\lambda}_{n}) - \tilde{T}_{x} \approx (\hat{T}_{x}(\lambda) - \tilde{T}_{x}) + H_{22}(\lambda)(\hat{\lambda}_{n} - \lambda),$$
  
where  
$$H_{22}(\lambda) = \sum_{s} x_{k} (\partial a_{k}(\lambda)/\partial \lambda)' d_{k}.$$

Therefore,

$$\hat{T}_{y}(\hat{\lambda}_{n}) \approx \hat{T}_{y}(\lambda) - H_{12}(\lambda) H_{22}^{-1}(\lambda) \quad (\hat{T}_{x}(\lambda) - \tilde{T}_{x})$$
$$= \sum_{s} e_{k} d_{k} a_{k}(\lambda) + B(\lambda) \tilde{T}_{x} \quad , \quad (4.4)$$

where  $e_k$  are the residuals  $y_k - B(\lambda)x_k$ , and  $B(\lambda) = H_{12}(\lambda) H_{22}^{-1}(\lambda)$ .

From (4.4), Taylor variance of the calibration estimator can be estimated using standard formulas in sampling theory. Note that in the case of nr adjustment, the vector of control totals  $\tilde{T}_x$  is random since it is derived from the full sample. Therefore, for the nr case the second term in (4.4) leads to an extra contribution to the variance. Also note that  $\lambda$ ,  $H_{12}(\lambda)$ , and  $H_{22}(\lambda)$  are replaced by their consistent estimates  $\hat{\lambda}$ ,  $H_{12}(\hat{\lambda})$ , and  $H_{22}(\hat{\lambda})$  in the variance expression obtained from the right hand side of (4.4). The linearization (4.4) is similar to the one obtained earlier by Folsom (1991) for nr adjustment under a somewhat different model for  $a_k(\lambda)$ .

Now, in the case of ps, if there is coverage bias, we have analogous to the nr bias case,

$$\hat{T}_{y}(\hat{\lambda}_{n}) \approx \sum_{s} e_{k} d_{k} a_{k}(\lambda) + B(\lambda) T_{x}, \qquad (4.5)$$

where the control totals  $\tilde{T}_x$  are now treated as nonrandom, and no longer contribute to the variance.

Observe that in the above linearized approximation to the calibration estimator, presence of the adjustment factor  $a_k(\lambda)$  will tend to increase the variance; however, presence of residuals  $e_k$  will in general tend to decrease the variance, and the net effect is usually a reduction in variance after ps. As in the case of nr, the vector  $\lambda$  and H matrices are replaced by their consistent estimates in the estimated variance from (4.5).

In the absence of coverage bias,  $\lambda=0$  and  $a_k(\lambda)=1$ , we get

$$\hat{T}_{y}(\hat{\lambda}_{n}) \approx \sum_{s} e_{k} d_{k} + B(0) T_{x}, \quad (4.6)$$

where  $\hat{B}(0) = (\sum_s y_k x_k' d_k) (\sum_s x_k x_k' d_k)^{-1}$ , and  $e_k = y_k - B(0)x_k$ . Note that the right hand side of (4.6) is identical to the generalized regression estimator. Thus, for ps when there is no coverage bias, GEM calibration estimator is asymptotically equivalent to the regression estimator. This extends the result of Deville and Särndal (1992) to include GEM. However, in the above linearization, the adjustment factor  $a_k(\lambda)$  is absent unlike the case with coverage bias. Singh and Folsom (2000)

give a simple theoretical justification of why the factor  $a_k(\lambda)$  should be included in variance estimation via the estimating function approach, and obtain an alternative but equivalent sandwich-type variance estimate which is computationally more efficient than the linearization based solution when the covariance matrix for a vector of calibration estimators is required.

So far we considered calibration estimators of totals T<sub>y</sub>. For estimating means or ratios  $R_{yv}$  (: =  $T_y/T_y$ ), the linearized form of the calibration estimator  $\hat{R}_{yv}(\hat{\lambda}_n)$  after subtracting  $R_{yv}$  is given by

$$T_{\nu}^{-1} \left[ \hat{T}_{\nu}(\lambda) - R_{\nu\nu} \hat{T}_{\nu}(\lambda) - B(\lambda) (\hat{T}_{x}(\lambda) - \tilde{T}_{x}) \right]$$
(4.7)

from which the approximate variance estimate can be obtained after substituting consistent estimates of  $\lambda$ ,  $T_{\nu}$ ,  $R_{\nu\nu}$ , and  $B(\lambda)$ .

To estimate variance of the estimator  $\sum_{s} y_k d_k a_{1k}(\hat{\lambda}_1) a_{2k}(\hat{\lambda}_2)$ , adjusted for both nonresponse and poststratification, above type of linearization can be carried through. Alternatively, the estimating function approach of Singh and Folsom (2000) provides a simple sandwich-type estimate for the Taylor variance when successive weight adjustments are performed using GEM.

## 5. Alternative Methods: Review and Comparison

For ps, raking-ratio and regression methods are commonly used. The generalized raking methods such as DS provide bounds on the adjustment factor. As mentioned earlier, raking-ratio (or exponential model) and DS can be obtained as special cases of GEM by choosing uniform bounds  $\ell$  and u suitably. For raking-ratio,  $\ell = 0$ , and u= $\infty$  (which may give rise to extreme values) while for DS, we have  $0 < \ell < 1 < u$ . The regression method does not invoke any bounds, and may give rise to negative weights.

The form of the adjustment factor for the regression method is  $a_k(\lambda) = 1 + x'_k \lambda$ ,  $-\infty < a_k < \infty$ . Despite no range restrictions on  $a_k$ , this method generally works well in practice, and is easy to implement without need to resort to iterative methods. Its use for nonresponse adjustment has also been recently advocated; see Fuller, Loughin, and Baker (1994) for a combined nr and ps adjustment by regression, and Lundström and Särndal (1999) for nr adjustment. Folsom and Witt (1994) proposed a modification of the DS method, termed the scaled constrained exponential model for nr adjustment such that  $a_k(\lambda) \ge 1$ . The basic idea is to multiply the adjustment factor by a constant  $\rho^{-1} \ge 1$  such that  $\rho^{-1}\ell \ge 1$ . By choosing  $\ell = \rho \le 1 \le u$ , we get the desired lower bound as  $\rho^{-1}\ell = 1 < c = \rho^{-1} < \rho^{-1}u$ . They suggest choosing  $\rho$  as the overall response propensity estimated from the sample of respondents and nonrespondents. Folsom (1991), and Singh, Wu, and Boyer (1995) also proposed a modification of the rakingratio method for nr adjustment such that  $a_k(\lambda) \ge 1$ . Here the basic idea is to find  $a_k^*(\lambda)$  (=  $e^{x_k \lambda}$ ) by raking-ratio such that the deficiency control total (defined as the difference between full sample and respondent subsample totals) are met. The final adjustment factor  $a_{i}(\lambda)$  is then defined as  $1 + a_k^*(\lambda)$ . This was termed as the deficiency raking method by Singh, Wu, and Boyer, and their main motivation for proposing this method was to use external control totals for nr adjustment when unit-specific information for the nonrespondents was not available in the context of longitudinal surveys. Another motivation was, of course, to generalize the usual weighting cell adjustment method to more general covariates while ensuring that the adjustment factor was at least 1 as in Folsom (1991).

The extreme values are commonly treated by winsorizing. However, as mentioned in the introduction, this may lose its impact after nr and ps, i.e., the final weights may have extreme values. The proposed method of GEM can be used to directly address this extreme value problem after nr and ps adjustments have been made to reduce biases due to nr and coverage errors. Thus, GEM provides a unified approach for weight adjustments for extreme values, nr, and ps. In addition, by choosing nonuniform bounds on  $a_k(\lambda)$ , GEM allows for the user to exercise control on the extent of adjustment on the initially identified extreme values at each step of weight adjustment.

#### 6. An Illustrative Example

Using the 1999 National Household Survey on Drug Abuse data for the East South Central Census Division (consisting of states, AL, MS, TN, and KY), the three methods RR (raking-ratio or exponential model), DS (in the case of nr, it is modified DS as given by the scaled constrained exponential model), and GEM (generalized exponential model) are compared; see Chen, Penne, and Singh (2000) for more details. For this comparison, we consider weighting (referred to as weight components 12-14 in Chen, Penne, and Singh) for the second phase sample of persons selected for the drug questionnaire after the first phase sample of dwelling units selected for screening questionnaire. For all the three methods, we started with a common set of initial weights. Before respondent person level nr (res.per.nr) and respondent person level ps (res.per.ps) adjustments, a somewhat new step of selected person level ps (sel.per.ps) was introduced to take advantage of the information about selected persons (i.e., both respondents and nonrespondents) in the large first phase sample of households for screening. Here the ps controls for the selected persons are estimated from the first phase sample. This additional step is expected to lead to more stable estimated totals needed for the next step of nonresponse adjustment. Table 1 shows summary statistics for weight adjustment factors and the resulting calibrated weights. It is seen that with the built-in control for extreme weights in GEM, one can reduce the proportion of extreme values in the adjusted weights considerably. Here the extreme value cut-off points are defined as median  $\pm$  3(IQR) where IQR denotes the interquartile range. The cut-off points are specific to the domains defining extreme values. The term "outwinsor" is used to signify the proportion of weight-sum out of the total weight-sum that would be trimmed if weights were winsorized. Also the UWE (unequal weighting effect, i.e., one plus squared coefficient of variation of weights) tends to be smallest for GEM. For domains defined by age groups (12-17, 18-25, 26+), the histograms (not shown here) of adjustment factors are found to be quite similar except for slightly heavier tails for RR. It is interesting to note that for this particular example, the final estimates (not shown here) for recency of use of cigarettes, alcohol, marijuana, and cocaine at the census division level for various age groups turn out to be close to each other despite differences in treatment of extreme values. This is probably due to the fact that the outwinsor proportions are not that high for the alternative methods. The GEM SEs, interestingly, also turn out to be generally similar to the DS ones except being somewhat lower most of the time. Also the RR based estimates (with no bound restrictions on the adjustment factor) turn out to be more or less precise than either DS or GEM. This similarity between estimates is possible for our example because the final UWE for the three methods are similar in magnitude. However, for domains involving high weights under RR (and hence high UWE), we would expect RR based estimates unstable compared to DS and GEM. For a comparison of unadjusted SE, adjusted SE for ps, and adjusted SE for nr and ps, see Vaish, Gordek, and Singh (2000).

## 7. Concluding Remarks

Unlike earlier methods, GEM provides a unified calibration tool for weight adjustments for extreme values, nr and ps. Of special interest is its capability to have a built-in control on extreme values. Under suitable superpopulation modeling and assuming that the bounds on the adjustment factors are prespecified, the resulting calibration estimators were shown to be asymptotically consistent with respect to the  $p\xi$ -distribution for nr and ps, and derivation of the asymptotic Taylor variance estimation formulas analogous to the ones based on residuals for the regression estimator (used for ps) was outlined. In our experience, we find GEM a useful practical alternative to the methods of raking-ratio and Deville-Sarndal while providing comparable results.

## References

Chen, P., Penne, M.A., and Singh, A.C. (2000). Experience with the generalized exponential model of weight calibration for the National Household Survey on Drug Abuse. *ASA Proc. Surv. Res. Meth. Sec.* 

Deville, J.-C., and Särndal, C.E. (1992). Calibration estimation in survey sampling. *JASA*, 87, 376-382.

Folsom, R.E. Jr. (1991). Exponential and logistic weight adjustments for sampling and nonresponse error reduction. *ASA Proc. Soc. Statist. Sec.*, 197-202.

Folsom, R.E., and Witt, M.B. (1994). Testing a new attrition nonresponse method for SIPP. *ASA Proc. Surv. Res. Meth. Sec.*, 428-433.

Isaki, C, T. and. Fuller, W.A.(1982). Survey design under the regression superpopulation model. *JASA*., 77, 89-96.

Fuller, W.A., Loughin, M.M., and Baker, H. D. (1994). Regression weighting in the presence of nonresponse with application to the 1987-88 Nationwide Food Consumption Survey. *Survey Methodology*, 20, 75-85.

Lundström, S. and Särndal, C. -E. (1999). Calibration as a standard method for treatment of nonresponse. *Jour. Off. Statist.*, 15, 305-327.

Rao, J.N.K. and Singh, A.C. (1997). A ridge-shrinkage method for range-restricted weight calibration in survey sampling. *ASA Proc. Surv. Res. Sec.*, 57-65.

Singh, A.C. and Folsom, R.E., Jr. (2000). Bias corrected estimating function approach for variance estimation adjusted for poststrtaification. ASA Proc. Surv. Res. Meth. Sec. (to appear).

Singh, A.C., and Mohl, C.A. (1996). Understanding calibration estimators in survey sampling. *Survey Methodology*, 22, 107-115.

Singh, A.C., Wu, S., and Boyer, R. (1995). Longitudinal survey nonresponse adjustment by weight calibration for estimation of gross flows. *ASA Proc. Surv. Res. Meth Sec.*, 396-401.

Vaish, A.K., Gordek, H., and Singh, A.C. (2000). Variance estimation adjusted for weight calibration via the generalized exponential model with application to the National Household Survey on Drug Abuse. *ASA Proc. Surv. Res. Meth. Sec.* (to appear).

Table 1a - [sel.per.ps]       Bei         UWE       Unwtd         Extreme Values       Unwtd         Wtd       Outwinsor         Weight Distribution       Min         25%       Median         Min       25%         Median       1,         75%       2,         Max       32,         Weight12       Min         25%       Median         75%       2,         Max       32,         Weight12       Min         25%       Median         75%       2,         Max       32,         Weight12       Min         25%       Median         75%       2,         Max       Unwtd         Wtid       Outwinsor         Weight Distribution       Wtid         Weight1-13       Min         25%       Median         Min       25%         Median       1,         75%       2,	aking R           fore         3.05           3.05         0.00%           0.00%         0.00%           131.58         713.36           134.98         714.98           261.61         n/a           n/a         n/a           n/a         n/a           0.84%         2.10%           0.84%         0.69%           0.45         0.45	atio(RR) After 3.32 1.14% 2.40% 0.60% 0.45 678.85 1,114.58 2,787.29 89,355.27 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19% 0.45	Di           Before         3.02           3.00%         0.00%           0.00%         0.00%           131.58         713.36           1,134.98         2,714.98           32,261.61         n/a           n/a         n/a           n/a         n/a           0.95%         2.04%           0.46%         0.46%		GEN Before 3.05 0.00% 0.00% 0.00% 131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a 3.28 0.89% 1.96% 0.40%	After 3.27 1.09% 2.29% 0.35% 39.93 675.77 1,117.42 2,762.72 66,015.70 0.3 0.83 0.96 1.12 3.49
UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution Weight 1-12 Min 25% Median 1, 75% 2, Max 32, Weight12 Min 25% Median 75% Mat Table 1b - [res.per.nr] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution Weight1-13 Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 75% 2, Max 89, Median 75% Max 89, Weight13 Min 25% Median 75% Max 89, Weight13 Min 25% Median 75% Max 89, Weight13 Min 25% Median 75% Max 89, Weight13 Min 25% Median 75% Max 89, Weight13 Min 25% Median 75% Max	3.05 0.00% 0.00% 131.58 713.36 134.98 714.98 261.61 n/a n/a n/a n/a n/a 	3.32 1.14% 2.40% 0.60% 0.45 678.85 1,114.58 2,787.29 89,355.27 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19%	3.02 0.00% 0.00% 131.58 713.36 1,134.98 32,261.61 n/a n/a n/a n/a n/a 0.95% 2.04% 0.46%	3.3 1.11% 2.21% 0.42% 39.93 673.34 1,110.66 2,782.56 65,990.62 0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	3.05 0.00% 0.00% 131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a n/a 	3.27 1.09% 2.29% 0.35% 39.93 675.77 1,117.42 2,762.72 66,015.70 0.3 0.83 0.96 1.12 3.49 3.87 1.14% 2.66%
Extreme Values Unwtd Wtd Outwinsor Weight Distribution Weight1-12 Min 25% Median 1, 75% 2, Max 32, Weight12 Min 25% Median 75% Max Table 1b - [res.per.nr] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution Weight1-13 Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 75% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	0.00% 0.00% 131.58 713.36 134.98 714.98 261.61 n/a n/a n/a n/a n/a 	1.14% 2.40% 0.60% 0.45 678.85 1,114.58 2,787.29 89,355.27 0 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19%	0.00% 0.00% 131.58 713.36 1,134.98 32,261.61 n/a n/a n/a n/a n/a 0.95% 2.04% 0.46%	1.11% 2.21% 0.42% 39.93 673.34 1,110.66 2,782.56 65,990.62 0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	0.00% 0.00% 131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a n/a 	1.09% 2.29% 0.35% 39.93 675.77 1,117.42 2,762.72 66,015.70 0.3 0.83 0.96 1.12 3.49 3.87 1.14% 2.66%
Unwtd Wtd Outwinsor Weight Distribution Weight1-12 Min 25% Median 1, 75% 2, Max 32, Weight12 Min 25% Median 75% Max Table 1b - [res.per.nr] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution Weight1-13 Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 1, 75% 2, Max 89, Median 1, 75% 2, Max 89, Weight13 Min 25% Median 75% Max 89, Median 75% Max 89, Median 75% Max 89, Median 75% Max 75% Max	0.00% 0.00% 131.58 713.36 134.98 714.98 261.61 n/a n/a n/a n/a 	2.40% 0.60% 0.45 678.85 1,114.58 2,787.29 89,355.27 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19%	0.00% 0.00% 131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a 3.29 0.95% 2.04% 0.46%	2.21% 0.42% 39.93 673.34 1,110.66 2,782.56 65,990.62 0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	0.00% 0.00% 131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a n/a	2.29% 0.35% 39.93 675.77 1,117.42 2,762.72 66,015.70 0.3 0.83 0.96 1.12 3.49 3.87 1.14% 2.66%
Wtd Outwinsor Weight Distribution Weight 1-12 Min 25% Median 1, 75% 2, Max 32, Weight12 Min 25% Median 75% Max Table 1b - [res.per.nr] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution Weight1-13 Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 1, 75% 2, Max 89, Median 75% Max 89, Weight13 Min 25% Median 75% Max 89, Median 75% Max 75% Max 75%	0.00% 0.00% 131.58 713.36 134.98 714.98 261.61 n/a n/a n/a n/a 	2.40% 0.60% 0.45 678.85 1,114.58 2,787.29 89,355.27 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19%	0.00% 0.00% 131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a 3.29 0.95% 2.04% 0.46%	2.21% 0.42% 39.93 673.34 1,110.66 2,782.56 65,990.62 0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	0.00% 0.00% 131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a n/a	2.29% 0.35% 39.93 675.77 1,117.42 2,762.72 66,015.70 0.3 0.83 0.96 1.12 3.49 3.87 1.14% 2.66%
OutwinsorWeight DistributionWeight1-12Min25%Median1,75%2,Max32,Weight12Min25%Median75%Max25%Median75%MaxTable 1b - [res.per.nr]UWEExtreme ValuesUnwtdWeight1-13Min25%Median1,75%2,Max89,Weight13Min25%Median75%25%Median75%MaxTable 1c - [res.per.ps]UWEExtreme ValuesUnwtdWtidOutwinsorWeight DistributionWeight Distribution	0.00% 131.58 713.36 134.98 714.98 261.61 n/a n/a n/a n/a 3.38 0.84% 2.10% 0.69% 0.45	0.60% 0.45 678.85 1,114.58 2,787.29 89,355.27 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19%	0.00% 131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a 	0.42% 39.93 673.34 1,110.66 2,782.56 65,990.62 0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	0.00% 131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a n/a 1,28 0.89% 1.96%	0.35% 39.93 675.77 1,117.42 2,762.72 66,015.70 0.3 0.83 0.96 1.12 3.49 3.87 1.14% 2.66%
Weight Distribution         Min           Weight1-12         Min           25%         Median           Median         1,           75%         2,           Max         32,           Weight12         Min           25%         Median           75%         2,           Max         32,           Weight12         Min           25%         Median           75%         Max           Table 1b - [res.per.nr]         Unwtd           WWE         Unwtd           Wtight Distribution         Weight1-13           Weight1-13         Min           25%         Median           Weight13         Min           25%         Median           75%         2,           Max         89,           Weight13         Min           25%         Median           75%         Max           25%         Median           75%         Max           89,         Weight13           Min         25%           Median         75%           Max         75%           Max <td>131.58 713.36 134.98 714.98 261.61 n/a n/a n/a 3.38 0.84% 2.10% 0.69% 0.45</td> <td>0.45 678.85 1,114.58 2,787.29 89,355.27 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19%</td> <td>131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a n/a 3.29 0.95% 2.04% 0.46%</td> <td>39.93 673.34 1,110.66 2,782.56 65,990.62 0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%</td> <td>131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a n/a 3.28 0.89% 1.96%</td> <td>39.93 675.77 1,117.42 2,762.72 66,015.70 0.3 0.83 0.96 1.12 3.49 3.87 1.14% 2.66%</td>	131.58 713.36 134.98 714.98 261.61 n/a n/a n/a 3.38 0.84% 2.10% 0.69% 0.45	0.45 678.85 1,114.58 2,787.29 89,355.27 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19%	131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a n/a 3.29 0.95% 2.04% 0.46%	39.93 673.34 1,110.66 2,782.56 65,990.62 0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	131.58 713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a n/a 3.28 0.89% 1.96%	39.93 675.77 1,117.42 2,762.72 66,015.70 0.3 0.83 0.96 1.12 3.49 3.87 1.14% 2.66%
Weight1-12         Min           25%         Median           Median         1,           75%         2,           Max         32,           Weight12         Min           25%         Median           Weight12         Min           25%         Median           75%         Max           25%         Median           75%         Max           Table 1b - [res.per.nr]         Unwtd           UWE         Unwtd           Extreme Values         Unwtd           Weight1-13         Min           25%         Median           Min         25%           Median         1,           75%         2,           Max         89,           Weight13         Min           25%         Median           75%         Max           25%         Median           75%         Max           25%         Median           75%         Max           75%         Max           75%         Max           75%         Max           75%         Max	713.36 134.98 714.98 261.61 n/a n/a n/a n/a 3.38 0.84% 2.10% 0.69% 0.45	678.85 1,114.58 2,787.29 89,355.27 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19%	713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a n/a 3.29 0.95% 2.04% 0.46%	673.34 1,110.66 2,782.56 65,990.62 0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a 3.28 0.89% 1.96%	675.77 1,117.42 2,762.72 66,015.70 0.3 0.83 0.96 1.12 3.49 3.87 1.14% 2.66%
Min 25% Median 1, 75% 2, Max 32, Weight12 Min 25% Median 75% Max Table 1b - [res.per.nr] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution Weight1-13 Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 7, 75% Max 89, Median 7, 75% Max 89, Median 7, 75% Median 7, 75% Median 7, 75% Median 7, 75% Max 89, Median 7, 75% Max 89, Max 89, Median 7, 75% Max 89, Max 89, Median 7, 75% Max 89, Median 7, 75% Median 7,	713.36 134.98 714.98 261.61 n/a n/a n/a n/a 3.38 0.84% 2.10% 0.69% 0.45	678.85 1,114.58 2,787.29 89,355.27 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19%	713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a n/a 3.29 0.95% 2.04% 0.46%	673.34 1,110.66 2,782.56 65,990.62 0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a 3.28 0.89% 1.96%	675.77 1,117.42 2,762.72 66,015.70 0.3 0.83 0.96 1.12 3.49 3.87 1.14% 2.66%
25% Median 1, 75% 2, Max 32, Weight12 Min 25% Median 75% Max Table 1b - [res.per.nr] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution Weight1-13 Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 75% Max 89, Median 75% Max 9, Median 75% Max 74, Min 25% Median 75% Max 74, Min 25% Max 74, Min 25% Min	713.36 134.98 714.98 261.61 n/a n/a n/a n/a 3.38 0.84% 2.10% 0.69% 0.45	678.85 1,114.58 2,787.29 89,355.27 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19%	713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a n/a 3.29 0.95% 2.04% 0.46%	673.34 1,110.66 2,782.56 65,990.62 0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	713.36 1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a 3.28 0.89% 1.96%	675.77 1,117.42 2,762.72 66,015.70 0.3 0.83 0.96 1.12 3.49 3.87 1.14% 2.66%
Median 1, 75% 2, Max 32, Weight12 Min 25% Median 75% Max 7able 1b - [res.per.nr] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution Weight1-13 Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor	134.98 714.98 261.61 n/a n/a n/a n/a 3.38 0.84% 2.10% 0.69% 0.45	1,114.58 2,787.29 89,355.27 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19% 0.45	1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a 3.29 0.95% 2.04% 0.46%	1,110.66 2,782.56 65,990.62 0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	1,134.98 2,714.98 32,261.61 n/a n/a n/a n/a 3.28 0.89% 1.96%	1,117.42 2,762.72 66,015.70 0.3 0.83 0.96 1.12 3.49 3.87 1.14% 2.66%
Max 32, Weight12 Min 25% Median 75% Max Table 1b - [res.per.nr] UWE Extreme Values Unwtd Wtid Outwinsor Weight Distribution Weight1-13 Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 75% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtid Outwinsor Weight Distribution	261.61 n/a n/a n/a n/a 3.38 0.84% 2.10% 0.69% 0.45	89,355.27 0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19% 0.45	32,261.61 n/a n/a n/a n/a 3.29 0.95% 2.04% 0.46%	65,990.62 0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	32,261.61 n/a n/a n/a n/a 3.28 0.89% 1.96%	66,015.70 0.3 0.83 0.96 1.12 <u>3.49</u> 3.87 1.14% 2.66%
Weight12       Min         25%       Median         75%       Max         Table 1b - [res.per.nr]         UWE         Extreme Values         Unwtd         Win         Outwinsor         Weight Distribution         Weight1-13       Min         25%       Median         Min       25%         Median       1,         75%       2,         Max       89,         Weight13       Min         25%       Median         75%       Max         UWE         Extreme Values         Unwtd         Weight Distribution       Wtd         Outwinsor       Wtd         Weight Distribution       Wtd	n/a n/a n/a 3.38 0.84% 2.10% 0.69%	0 0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19% 0.45	n/a n/a n/a n/a 3.29 0.95% 2.04% 0.46%	0.3 0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	n/a n/a n/a n/a /a 3.28 0.89% 1.96%	0.3 0.83 0.96 1.12 <u>3.49</u> 3.87 1.14% 2.66%
Min 25% Median 75% Max Table 1b - [res.per.nr] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution <i>Weight1-13</i> Min 25% Median 1, 75% 2, Max 89, <i>Weight13</i> Min 25% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor	n/a n/a n/a 3.38 0.84% 2.10% 0.69%	0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19% 0.45	n/a n/a n/a 3.29 0.95% 2.04% 0.46%	0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	n/a n/a n/a 3.28 0.89% 1.96%	0.83 0.96 1.12 3.49 3.87 1.14% 2.66%
25% Median 75% Max Table 1b - [res.per.nr] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution <i>Weight1-13</i> Min 25% Median 1, 75% 2, Max 89, <i>Weight13</i> Min 25% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor	n/a n/a n/a 3.38 0.84% 2.10% 0.69%	0.84 0.96 1.11 5.86 3.91 2.03% 5.23% 1.19% 0.45	n/a n/a n/a 3.29 0.95% 2.04% 0.46%	0.83 0.96 1.12 3.49 3.88 2.11% 5.38%	n/a n/a n/a 3.28 0.89% 1.96%	0.83 0.96 1.12 3.49 3.87 1.14% 2.66%
Median 75% Max Table 1b - [res.per.nr] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution <i>Weight1-13</i> Min 25% Median 1, 75% 2, Max 89, <i>Weight13</i> Min 25% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	n/a n/a 3.38 0.84% 2.10% 0.69%	0.96 1.11 5.86 3.91 2.03% 5.23% 1.19% 0.45	n/a n/a 3.29 0.95% 2.04% 0.46%	0.96 1.12 3.49 3.88 2.11% 5.38%	n/a n/a 3.28 0.89% 1.96%	0.96 1.12 3.49 3.87 1.14% 2.66%
75% Max Table 1b - [res.per.nr] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution Weight1-13 Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 75% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	n/a n/a 3.38 0.84% 2.10% 0.69% 0.45	1.11 5.86 3.91 2.03% 5.23% 1.19% 0.45	n/a n/a 3.29 0.95% 2.04% 0.46%	1.12 3.49 3.88 2.11% 5.38%	n/a n/a 3.28 0.89% 1.96%	1.12 3.49 3.87 1.14% 2.66%
Max         Table 1b - [res.per.nr]         UWE         Extreme Values         Unwtd         Wtd         Outwinsor         Weight Distribution         Weight1-13         Min         25%         Median         75%         Q         Max         89,         Weight13         Min         25%         Median         75%         Max         89,         Weight13         Min         25%         Median         75%         Max         Table 1c - [res.per.ps]         UWE         Extreme Values         Unwtd         Wtd         Outwinsor         Weight Distribution	n/a 3.38 0.84% 2.10% 0.69% 0.45	5.86 3.91 2.03% 5.23% 1.19% 0.45	n/a 3.29 0.95% 2.04% 0.46%	3.49 3.88 2.11% 5.38%	n/a 3.28 0.89% 1.96%	3.49 3.87 1.14% 2.66%
Table 1b - [res.per.nr]         UWE         Extreme Values         Unwtd         Wtd         Outwinsor         Weight Distribution         Weight1-13         Min         25%         Median         75%         Queight13         Min         25%         Median         75%         Max         89,         Weight13         Min         25%         Median         75%         Max         Street         Unwtd         Wtd         Outwinsor         Weight Distribution	3.38 0.84% 2.10% 0.69% 0.45	3.91 2.03% 5.23% 1.19% 0.45	3.29 0.95% 2.04% 0.46%	3.88 2.11% 5.38%	3.28 0.89% 1.96%	3.87 1.14% 2.66%
UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution Weight 1-13 Min 25% Median 1, 75% 2, Max 89, Weight 13 Min 25% Max 89, Weight 13 Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	0.84% 2.10% 0.69% 0.45	2.03% 5.23% 1.19% 0.45	0.95% 2.04% 0.46%	2.11% 5.38%	0.89% 1.96%	1.14% 2.66%
Extreme Values Unwtd Wtd Outwinsor Weight Distribution <i>Weight1-13</i> Min 25% Median 1, 75% 2, Max 89, <i>Weight13</i> Min 25% Max 89, <i>Weight13</i> Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	0.84% 2.10% 0.69% 0.45	2.03% 5.23% 1.19% 0.45	0.95% 2.04% 0.46%	2.11% 5.38%	0.89% 1.96%	1.14% 2.66%
Unwtd Wtd Outwinsor Weight Distribution Weight 1-13 Min 25% Median 1, 75% 2, Max 89, Weight 13 Min 25% Max 89, Weight 13 Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	2.10% 0.69% 0.45	5.23% 1.19% 0.45	2.04% 0.46%	5.38%	1.96%	2.66%
Wtd Outwinsor Weight Distribution Weight 1-13 Min 25% Median 1, 75% 2, Max 89, Weight 13 Min 25% Max 89, Weight 13 Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	2.10% 0.69% 0.45	5.23% 1.19% 0.45	2.04% 0.46%	5.38%	1.96%	2.66%
Weight Distribution Weight 1-13 Min 25% Median 1, 75% 2, Max 89, Weight 13 Min 25% Median 75% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	0.45	0.45		0.92%	0.40%	0.40%
Weight1-13           Min           25%           Median           75%           Max           89,           Weight13           Min           25%           Median           75%           25%           Median           75%           Median           75%           Max           Table 1c - [res.per.ps]           UWE           Extreme Values           Unwtd           Wtd           Outwinsor           Weight Distribution						
Min 25% Median 1, 75% 2, Max 89, Weight13 Min 25% Median 75% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution						
25% Median 1, 75% 2, Max 89, <i>Weight13</i> Min 25% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution						
Median 1, 75% 2, Max 89, Weight13 Min 25% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution			39.93	40.03	39.93	40.02
75% 2, Max 89, Weight13 Min 25% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	664.31	767.5	665.2	764.79	667.11	766.96
Max 89, Weight13 Min 25% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	,078.01	1,326.57	1,080.22	1,317.52	1,079.41	1,327.54
Weight 13         Min         25%         Median         75%         Max    Table 1c - [res.per.ps]          UWE         Extreme Values         Unwtd         Wtd         Outwinsor         Weight Distribution	,462.36	3,136.07 101,325.20	2,482.69 65,990.62	3,108.04 74,851.38	2,487.23 66,015.70	3,161.65
Min 25% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	,333.27	101,525.20	03,990.02	/4,031.30	00,015.70	70,614.65
25% Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	n/a	1	n/a	1	n/a	0.73
Median 75% Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	n/a	1.1	n/a	1.08	n/a	1.07
Max Table 1c - [res.per.ps] UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	n/a	1.18	n/a	1.17	n/a	1.16
Table 1c - [res.per.ps]         UWE         Extreme Values         Unwtd         Wtd         Outwinsor         Weight Distribution	n/a	1.33	n/a	1.33	n/a	1.34
UWE Extreme Values Unwtd Wtd Outwinsor Weight Distribution	n/a	17.63	n/a	3.47	n/a	3.5
Extreme Values Unwtd Wtd Outwinsor Weight Distribution						
Unwtd Wtd Outwinsor Weight Distribution	3.91	3.95	3.88	3.91	3.87	3.87
Wtd Outwinsor Weight Distribution			•			;
Outwinsor Weight Distribution	2.28%	2.20%		2.25%		0.38%
Weight Distribution	5.74%	6.34%		5.69%		0.97%
	1.34%	1.36%	1.05%	1.04%	0.52%	0.09%
Weight1-14						
u <u> </u>	A 40	0.00	40.00	12.01	40.00	12.52
Min	0.45	0.23		12.01	(	13.53
25% Modian	767 5	772.94 1,337.13		762.64 1,332.39		775.42 1,347.02
	767.5	3,138.95		3,145.70		3,093.14
11	,326.57			76,818.01	4	62,606.79
Weight14	,326.57 ,136.07	100 216 20	, ,,001.00	, 0,010.01	, ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	,,,,-,-,-,-,-,-,-,-,-,-,-,-,-,-,
Min	,326.57	100,216.20			n/a	0.3
25%	,326.57 ,136.07 ,325.20		n/a	0.3		0.97
Median	,326.57 ,136.07 ,325.20 n/a	0.05		0.3 0.96		
75%	,326.57 ,136.07 ,325.20 n/a n/a	0.05	n/a	0.3 0.96 1	1	1.01
Max	,326.57 ,136.07 ,325.20 n/a	0.05 0.96 0.99	n/a n/a		n/a	1.01