

# COUNTING PEOPLE WITHOUT A USUAL RESIDENCE IN THE 2000 CENSUS

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## 1. Introduction

In the 2000 Census, the Census Bureau will try to count several components of the population without a usual residence. Service based enumeration is a statistical program used to estimate the number of people who use certain types of services such as shelters, soup kitchens, or mobile food vans.

For the counts used to apportion Congressional seats among the states, due December 31, 2000, each distinct census form captured in these operations will count as one. The Bureau will also deliver adjusted counts for states and smaller geographic regions by April 1, 2001. For these numbers, we will estimate the total number of people using these types of services--those who used them on the day of enumeration, as well as those who used them on other days but were missed on enumeration day. Here, we apply a multiplicity estimator based on the number of times those enumerated used the service facilities during the week prior to enumeration day.

It was shown in Shores, Cantwell, and Kohn (1999) that the multiplicity estimator to be used in the 2000 Census carries a downward bias. In this paper we introduce a very simple method that essentially eliminates the bias in the multiplicity estimator for shelters (alone) or for soup kitchens (alone). When we apply the estimator for both types of services, the task is more complex. We present several options and investigate their statistical properties.

Section 2 provides a review of the procedures used in the enumeration at shelters, the multiplicity estimator to be applied, and some of its important statistical properties. The multiplicity estimator is extended to soup kitchens as well in Section 3. In Section 4, we address the bias in the multiplicity estimator for shelters. Finally, we consider the more complex problem of adjustments for the bias in both components, defining and examining several options in Section 5.

It should be said from the beginning that to perform well, the multiplicity estimator relies on the quality of the responses obtained from those enumerated at the service facilities. If these responses are inaccurate in a fair proportion of cases, the bias or variance of the statistical procedures applied could be increased in a manner difficult to measure.

A longer version of this paper is available from the authors. It contains a discussion of possible duplication between enumeration at services and on "Be Counted Forms," and the Census Bureau's procedures for addressing this problem.

## 2. Multiplicity Estimation for Shelters

On Monday, March 27, 2000, Census Bureau staff visited shelters across the country and enumerated people staying in shelters. The following night, workers visited soup kitchens and mobile food vans and enumerated the patrons there. (For the remainder of the paper, when we mention soup kitchens, we implicitly include mobile food vans as well.)

Early in the morning on March 29, the Bureau also tried to find and count people staying in specific places targeted for enumeration, such as under bridges and in abandoned buildings. Finally, before and after April 1, people across the country had the opportunity to fill out a "Be Counted Form." Placed in convenient locations, these are census forms that one could obtain, fill out, and return to the Bureau. The intent was to give people another chance to be counted in case their residence was missed in the census, or they had no usual residence.

For the adjusted counts, people enumerated under bridges or who returned a Be Counted Form receive a weight of 1, representing only themselves. But for people enumerated at shelters and soup kitchens, we tried to get a better measure of the entire population who use such services. If we had gone out every night for a week or a month, we could have enumerated a larger portion of this population. This approach was not used because it would have introduced certain problems, including measurement error, potential duplication, and

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<sup>1</sup> This paper reports the results of research and analysis undertaken by Census Bureau staff. It has undergone a Census Bureau review more limited in scope than that given to official Census Bureau publications. This report is released to inform interested parties of ongoing research and to encourage discussion of work in progress.

increased cost.

Instead, the census questionnaire used at shelters asked how many times the person used a shelter that night or the prior six nights (March 21 to March 27), the *reference week* for shelter usage. Similarly, respondents at soup kitchens were asked how often they used a soup kitchen from March 22 to March 28, the reference week for the use of soup kitchens. In our estimation, each such person receives a weight greater than or equal to 1, so that they represent (i) themselves, and (ii) others who used these services, but not on the day of enumeration. As will be discussed, it is important to make sure we do not count twice those who are enumerated in any of these programs, or those who are represented in them statistically through the multiplicity estimator.

To see things more clearly, we start by dividing all people without a usual residence into four classes according to their use of shelters, as seen here with the number in each class. (By saying that one uses shelters, we mean around the time of census enumeration, that is, late March through early April, 2000.)

1. Those who used a shelter on March 27, 2000 (enumeration day for shelters),  $N_{1+}$ .
2. Those who did not use a shelter on March 27 (were not enumerated), but used a shelter at least one night in March 21 - 27 (the reference week for shelters),  $N_{2+}$ .
3. Those who sometimes use a shelter, but did not during the reference week,  $N_{3+}$ .
4. Those who never use a shelter,  $N_{4+}$ .

(The reason for including a “+” in the notation will become clear soon.) By merely doing an enumeration of shelters in the area, one cannot measure the fourth class. Therefore, our target is  $N_{SH}$ , defined as  $N_{1+} + N_{2+} + N_{3+}$ , the entire shelter-using population. We note that, while  $N_{SH}$  and  $N_{4+}$  are taken as fixed constants, the components  $N_{1+}$ ,  $N_{2+}$ , and  $N_{3+}$  are observations (realizations) of random variables dependent on shelter usage and other conditions, such as weather, individual financial circumstances, etc. *Over time*,  $N_{SH}$  and  $N_{4+}$  will also change. But to evaluate the multiplicity estimator, they can be taken as fixed.

By making the following assumptions, and asking people who are enumerated on March 27 (those in the first class) how many times they used a shelter during the reference week, a multiplicity estimate can be derived for  $N_{SH}$ :

- (a) The population of shelter users can be divided into

eight mutually exclusive groups  $G_0, G_1, \dots, G_7$ , where  $G_i$  includes all those who used shelters  $i$  times in the reference week.

- (b) For each person in  $G_i$ , the use of a shelter on a specific set of  $i$  days in the reference week is just as likely as the use on any other set of  $i$  days.
- (c) Users in the population visit shelters independently of each other.
- (d) There is no response error. That is, the number of days given as the frequency of shelter use during the reference week is the true number.

Obviously, these assumptions do not hold in reality. For example, consider (b). It is likely that shelters experience heavier usage certain days of the week or different times of the month. Indeed, weather may be an important factor. Assumption (c) ignores the clustering effect of companions and of mothers with children. The most questionable assumption is (d). It is likely that many users will simply not recall how many times they have visited shelters over a week’s time. However, there are few inferences we can make without these or other such assumptions.

It is worth mentioning what these assumptions do *not* imply. To this point, (1) we have *not* assumed that each person in the population behaves the same way with respect to the use of shelters. That is, the probability that a person falls in  $G_i$  can vary from person to person. (2) For any individual, the mechanism for determining whether a visit is made to a shelter need not be independent over the days of the week. In Section 4 we will add these assumptions.

Note that all people in groups  $G_1$  through  $G_7$  belong to one of the first two classes defined above; those in  $G_0$  are in the third class; and people who *never* use shelters are in the fourth class. It helps to keep in mind that the total  $N_{SH} = N_{1+} + N_{2+} + N_{3+}$  is fixed, but each of the three terms is a random variable.

The multiplicity estimator for the shelter component of the population in the 2000 Census is

$$\tilde{N}_{SH} = \sum_{k \in \text{Class 1}} (7/t_k) \quad (1)$$

where the sum is taken over all  $N_{1+}$  people in Class 1 above (those enumerated on March 27), and  $t_k$  is the number of nights person  $k$  used a shelter during the reference week. The fraction  $7/t_k$  is a weight whereby a person enumerated at the shelter represents (i) himself or herself, and (ii) others who used a shelter during the reference week but not on the day of enumeration. The

estimator can be written equivalently as

$$\tilde{N}_{SH} = \sum_{i=1}^7 n_i (7/i) \quad (1a)$$

where  $n_i$  is the number of people (out of all those in  $G_1$ ) who visited shelters  $i$  times in the past week *and* were enumerated on March 27. (Note that  $n_1 + n_2 + \dots + n_7 = N_{1+}$ , above.)

The variance of the multiplicity estimator and an estimator for that variance are derived in Shores, Cantwell, and Kohn (1999). It is also shown there that its expected value is  $E(N_{1+} + N_{2+})$ . Thus, as an estimator of the shelter-using population, the multiplicity estimator,  $\tilde{N}_{SH}$ , is biased downwards by an amount  $E(N_{3+})$ , the expected number of people who sometimes use shelters, but did not use them during the reference week. In Section 4 we address the bias by presenting and examining an adjustment to the multiplicity estimator.

It should be mentioned that some respondents at shelters (and soup kitchens) do not answer the question on shelter (or soup kitchen) usage over the past week. To address this problem, the Census Bureau weights up appropriately within pre-specified cells all those who provide this information. This adjustment for nonresponse is applied before the weighting as in (1) (and as in Sections 4 and 5) and is not discussed further in this paper.

### 3. Extending Multiplicity Estimation to Soup Kitchens

By applying appropriate weights in the multiplicity estimator *for shelters*, the  $N_{1+}$  people in Class 1 (above) represent the  $N_{1+} + N_{2+}$  people in the first two classes. The process works similarly *for enumeration at soup kitchens*, with a slight change to avoid counting people twice. The people who use shelters or soup kitchens can be divided further--similar to the prior section--by their use of these facilities during the appropriate reference weeks. Table 1 at the end of the paper provides the details. (The meaning of the cross hatches will be seen soon.)

Note that the shaded subclass (44) in the table represents those people who never use shelters or soup kitchens. The sizes of the other fifteen subclasses are observations of random variables. Based on enumeration only at these types of service facilities, we cannot estimate  $N_{44}$ . Therefore we now define as our target parameter  $N = N_{++} - N_{44}$ , the number of people who at least sometimes use shelters or soup kitchens.

As was seen in the prior section,  $E(\tilde{N}_{SH}) = E(N_{1+} + N_{2+})$ , which is represented in Table 1 by rows 1 and 2;

the shelter population is underestimated on average by  $E(N_{3+})$ , those people in row 3. People enumerated at soup kitchens are asked a similar usage question: how many times they used a soup kitchen in the reference week (March 22 - 28). We can derive an estimator of the population using soup kitchens similar to (1) :

$$\tilde{N}_{KI} = \sum_{h \in \text{Class 1}} (7/u_h) \quad (2)$$

where the sum is taken over all  $N_{1+}$  people in Class 1 for soup kitchens (those enumerated on March 28), and  $u_h$  is the number of nights person  $h$  used a soup kitchen during the reference week. By means of the weight  $7/u_h$ , person  $h$ , enumerated at a soup kitchen, represents (i) himself, and (ii) others who used a soup kitchen during the reference week but not on the day of enumeration. The sum can be written equivalently as

$$\tilde{N}_{KI} = \sum_{i=1}^7 m_i (7/i) \quad (2a)$$

where  $m_i$  is the number of people who visited a soup kitchen  $i$  times in the past week *and* were enumerated on March 28, with  $m_1 + m_2 + \dots + m_7 = N_{1+}$ .

To estimate  $N$ , the population using these types of services, we could simply add the components for shelters and soup kitchens. But we would then risk counting twice (through direct enumeration, or through statistical representation via weighting) people who used both types of services during the respective reference weeks--subclasses 11, 12, 21, and 22 in Table 1.

To account for this possible overlap, people enumerated at soup kitchens on March 28 were asked whether they had used a shelter at least once during the prior seven days. If so, they were excluded from the sum in (2) or (2a). The multiplicity estimator for soup kitchens, then, is  $\tilde{N}'_{KI}$ , the sum in (2) or (2a), *but only* over those people enumerated at soup kitchens *who did not use a shelter* during the reference week. The service based enumeration estimator, to be used by the Census Bureau for the 2000 Census, becomes

$$\tilde{N}_{SH} + \tilde{N}'_{KI} \quad (3)$$

As was shown in Shores et al. (1999),  $E(\tilde{N}_{SH} + \tilde{N}'_{KI}) = E(N_{1+} + N_{2+} + N_{31} + N_{32} + N_{41} + N_{42})$ . As an estimate of  $N$ , this leaves a downward bias of  $E(N_{33} + N_{34} + N_{43})$ , those people who sometimes use either shelters or soup kitchens or both, but used neither during the respective reference weeks. What is covered by the shelter component (////) and the soup kitchen component (\\\\\\) is depicted in Table 1. The unshaded subclasses represent the bias in the service based enumeration estimator.

#### 4. Adjusting for the Bias in the Multiplicity Estimator: The Shelter Component

At this point we present a simple adjustment to the multiplicity estimator to account for its bias. To simplify matters, we start by ignoring soup kitchens, that is, we first consider shelters only. Our target is then  $N_{SH} = N_{1+} + N_{2+} + N_{3+}$ . As  $\tilde{N}_{SH}$  covers  $N_{1+}$  and  $N_{2+}$  without bias, the goal is to include  $N_{3+}$  efficiently in an estimator. Up to this point, no modeling assumptions were made beyond those described in Section 2. Because our enumeration never captures anyone in Class 3 for shelters, without additional information or assumptions we cannot say anything further about  $N_{3+}$ . We now introduce an additional “Bernoulli model” assumption:

- (e) On any day in the reference week, each person is assumed to use a shelter with probability  $p_{SH}$ , with behavior independent from day to day and over facilities.

(Although this modeling assumption is restrictive, our research (not included here) demonstrates that, under certain circumstances, this model can approximate the more realistic situation where each person  $k$  behaves according to his own Bernoulli model with probability  $p_k$ . In fact, we have shown that--again, under reasonable conditions--the latter model can be approximated quite well by a simple model with only two parameters, which can be estimated by the data. Derivations and results will be documented in another paper.)

As assumption (e) represents a special case of the earlier derivations, the results shown before under general conditions still hold. In particular, the expected value of  $\tilde{N}_{SH}$  in (1) and (1a) is equal to  $E(N_{1+} + N_{2+})$ , that is, the estimator is biased downwards by  $E(N_{3+})$ .

Before proceeding, let us consider how important this bias could be. Under the Bernoulli model in (e), this bias equals  $N_{SH}(1 - p_{SH})^7$ . If the probability of using a shelter on each day,  $p_{SH}$ , is .20, by using the multiplicity estimator without adjusting for the bias we expect to miss about 21.0% of  $N_{SH}$ , the target parameter. If  $p_{SH} = .10$ , that bias is 47.8% of  $N_{SH}$ . One should note that other biases can enter the multiplicity estimator without detection. For example, it is possible that some people living in permanent living quarters may be erroneously enumerated at shelters or soup kitchens and included in either component of the multiplicity estimator.

To adjust for the bias in the multiplicity estimator, one can use the observed data, that is,  $N_{1+}$  and the individual responses  $t_1, t_2, \dots$ , to make inferences about the unknown totals. Then, we can estimate  $N_{SH} = N_{1+} + N_{2+} + N_{3+}$  directly, or estimate  $N_{3+}$  first, and add this to  $\tilde{N}_{SH}$ , an estimate of  $E(N_{1+} + N_{2+})$ .

One method uses the fact that, under the Bernoulli model, the expected number of people enumerated is the total number of people who sometimes use shelters,  $N_{SH}$ , times the probability of their using a shelter on any day. That is,  $E(N_{1+}) = N_{SH} \times p_{SH}$ . From this, we can estimate the total shelter population as

$$\tilde{N}_{SH}^{ADJ,1} = N_{1+} / p_{SH} \quad (4)$$

where  $p_{SH}$  is replaced by an appropriate estimator.

A second approach is to use the observation that  $\tilde{N}_{SH}$  is an unbiased estimator of  $E(N_{1+} + N_{2+})$ . A person is a member of  $N_{1+}$  or  $N_{2+}$  if he or she visited a shelter at least once during the reference week. Under the Bernoulli model, it follows that  $E(\tilde{N}_{SH}) = E(N_{1+} + N_{2+}) = N_{SH} \times \Pr(t_k > 0) = N_{SH} (1 - (1 - p_{SH})^7)$ . Thus, a second estimator follows as

$$\tilde{N}_{SH}^{ADJ,2} = \tilde{N}_{SH} / (1 - (1 - p_{SH})^7) \quad (5)$$

where, again, an estimate of  $p_{SH}$  is substituted.

How should we estimate  $p_{SH}$ ? For each person in  $N_{SH}$  there are seven independent trials with probability  $p_{SH}$  of success (using a shelter), summarized in  $t_k$ , the number of times person  $k$  used a shelter in the reference week. However, we observe  $t_k$  only for the  $N_{1+}$  people enumerated on March 27. The seventh trial is necessarily a success for this class, so it yields no information about  $p_{SH}$ . But the sequence  $t_1 - 1, t_2 - 1, \dots$  provides a set of independent binomial observations based on six trials and probability of success  $p_{SH}$ . Consequently, conditional on  $N_{1+}$ , the sum of the  $t_k - 1$  over the people enumerated at shelters follows a binomial distribution with parameters  $6(N_{1+})$  and  $p_{SH}$ . An estimator for  $p_{SH}$  is then

$$\sum_{\text{Class 1}} (t_k - 1) / (6 N_{1+}) \quad (6)$$

where the sum is taken over the  $N_{1+}$  people in Class 1 for shelters. It is easily shown that the estimator is unconditionally unbiased for  $p_{SH}$  with a variance of  $(1/6) p_{SH} (1 - p_{SH}) E(1 / N_{1+})$ , where the expectation is taken under the Bernoulli model. If  $N_{SH} \times p_{SH}$  is relatively large, the variance becomes approximately  $(1 - p_{SH}) / (6 N_{SH})$ .

This estimator for  $p_{SH}$  can be inserted into (4) and (5). But which estimator is preferred to adjust  $\tilde{N}_{SH}$ ? We would suggest  $\tilde{N}_{SH}^{ADJ,2}$ . Although we have not yet compared the performance of the two through simulations, the following reasoning points us toward  $\tilde{N}_{SH}^{ADJ,2}$ .  $\tilde{N}_{SH}^{ADJ,1} = N_{1+} / p_{SH}$  tries to project directly from the number of people in Class 1,  $N_{1+}$ , to the total shelter population,  $N_{SH}$ , through their relationship under

the Bernoulli model. Although this is likely an efficient estimator when the model is true, it may be riskier under deviations from the model.

The second,  $\tilde{N}_{SH}^{ADJ,2}$ , starts by estimating  $N_{1+} + N_{2+}$  via the multiplicity estimator, and then adjusts upwards to account for the final component,  $N_{3+}$ . Whether or not the Bernoulli model accurately describes the stochastic mechanism for using shelters, the multiplicity estimator is unbiased for  $E(N_{1+} + N_{2+})$ . It appears then that  $\tilde{N}_{SH}^{ADJ,2}$  may be more robust to model misspecification than is  $\tilde{N}_{SH}^{ADJ,1}$ .

In what follows, we will take  $\tilde{N}_{SH}^{ADJ,2}$  as the multiplicity estimator for the shelter population, and suppress the superscript 2. In that case, the estimator for  $N_{3+}$  is  $\tilde{N}_{SH}^{ADJ}(1 - p_{SH})^7$ .

### 5. Adjusting for the Bias in the Multiplicity Estimator: Shelters and Soup Kitchens

To extend the bias adjustment to address both components of service usage, let us first expand the assumption introduced in Section 4:

- (e') On any day in the reference week, each person is assumed to use a shelter (soup kitchen) with probability  $p_{SH}$  ( $p_{KI}$ ), with behavior independent from day to day and over facilities.

In Section 4 we noted that the bias of the unadjusted multiplicity estimator for shelters can be considerable when estimating  $N_{SH}$ . We saw in Section 3 that the unadjusted estimator in (3),  $\tilde{N}_{SH} + \tilde{N}_{KI}'$ , is biased downward by *at least*  $E(N_{34} + N_{43})$ , which, under the Bernoulli model in (e'), equals  $N_{SH,ONLY}(1 - p_{SH})^7 + N_{KI,ONLY}(1 - p_{KI})^7$ , where  $N_{SH,ONLY}$  ( $N_{KI,ONLY}$ ) is defined as the class of people who sometimes use shelters (soup kitchens) but never use soup kitchens (shelters). (The size of the other component of the bias,  $E(N_{33})$ , depends on the interaction between one's use of shelters and soup kitchens.) Similar to what we saw in Section 4, if  $p_{SH}$  is .20, the bias of  $\tilde{N}_{SH} + \tilde{N}_{KI}'$  is at least 21.0% of  $N_{SH,ONLY}$ . The same statement can be made about  $N_{KI,ONLY}$  if  $p_{KI} = .20$ .

To construct an adjustment, the probability of using a shelter,  $p_{SH}$ , is estimated as in (6), while the probability of using a soup kitchen,  $p_{KI}$ , is estimated analogously:

$$\sum (u_h - 1) / (6 N_{+1}) \quad (7)$$

where the sum is over the  $N_{+1}$  people enumerated at soup kitchens. Adjusting for the bias in shelters and in soup kitchens leads to (5') and (8), respectively:

$$\tilde{N}_{SH}^{ADJ} = \tilde{N}_{SH} / (1 - (1 - p_{SH})^7) \quad (5')$$

$$\tilde{N}_{KI}'^{ADJ} = \tilde{N}_{KI}' / (1 - (1 - p_{KI})^7) \quad (8)$$

The problem arises if we apply both bias adjustments together: several subclasses are covered twice. That is, some people represented by the bias adjustment for shelters are also covered at soup kitchens through direct enumeration, representation in  $\tilde{N}_{KI}'$ , or the bias adjustment for soup kitchens. We consider several competing estimators for  $N = N_{++} - N_{44}$ :

$$\tilde{N}_{SH} + \tilde{N}_{KI}' \quad (3)$$

$$\tilde{N}_{SH}^{ADJ} + \tilde{N}_{KI}'^{ADJ} \quad (9)$$

$$\tilde{N}_{SH}^{ADJ} + \tilde{N}_{KI}' \quad (10)$$

$$\tilde{N}_{SH} + \tilde{N}_{KI}'^{ADJ} \quad (11)$$

The first estimator applies no adjustment for the bias in either component, while that in (9) adjusts both terms. The estimators in (10) and (11) adjust one component but not the other.

The estimator in (9) leads to an overestimate (on average) of  $N$  by the expected number of people in subclasses 31, 32, and 33 of Table 1. Under the Bernoulli model defined in (e'), this bias can be expressed as approximately  $N_{SH \cap KI}(1 - p_{SH})^7$ , where  $N_{SH \cap KI}$  is the number of people who sometimes use shelters *and* sometimes use soup kitchens (the upper left  $3 \times 3$  block in Table 1).

The estimator in (10) tends to over- or underestimate  $N$  by the amount  $E(N_{31} + N_{32} - N_{43})$ , that is, two subclasses are covered twice, but another is left out completely. Finally, the estimator in (11) leads to an average underestimate of approximately  $E(N_{34})$ . Under the model this tends to be around  $N_{SH,ONLY}(1 - p_{SH})^7$ .

Obviously, if there are no soup kitchens in the geographic area under consideration,  $N_{SH \cap KI}$  and  $N_{KI,ONLY}$  would be 0, and the appropriate procedure is to use the shelter estimator with the adjustment for its bias,  $\tilde{N}_{SH}^{ADJ}$ , as in (5'). Under this scenario, there is no overlap among the people covered, and the estimator is approximately unbiased. Analogously, if there are no shelters in the area, one would apply  $\tilde{N}_{KI}'^{ADJ}$ , the same as  $\tilde{N}_{KI}^{ADJ}$  in this context.

It would appear that none of the three estimators that adjust completely or partially for the bias in the basic multiplicity estimator is better than the others under all circumstances. Without specific knowledge about the relative sizes of  $N_{SH \cap KI}$ ,  $N_{SH,ONLY}$ , and  $N_{KI,ONLY}$ , and about  $p_{SH}$  and  $p_{KI}$ , it is difficult to advocate one estimator in all situations. The following are some observations about the statistical properties of the four estimators listed above in (3), (9), (10), and (11).

1. Among the four, we know the direction of the bias in (3), (9), and (11). Without more information, we don't know if the estimator in (10) tends to over- or underestimate N.
2. Whatever the parameters, the downward bias in (11) is less than (or, at worst, equal to) that in (3).
3. From our discussions with experts on the locations and operations of shelters and soup kitchens, we believe that  $N_{SH,ONLY}$  is smaller than  $N_{SH,KI}$  in many geographic areas. If so, the absolute bias using (11) is smaller than that using (9).
4. Some might argue the following: because we are inflating the measure of the service-using population above what was actually counted, one would prefer a conservative adjustment--one that underestimates the true target. Under this argument, (11) is preferred over (9).
5. From (5') one can see that  $\tilde{N}_{SH}^{ADJ}$  is simply  $\tilde{N}_{SH}$  divided by  $1 - (1 - p_{SH})^7$ , a random variable taking values between 0 and 1, where  $p_{SH}$  is estimated as in (6). This indicates that the variance of the estimator in (9) is larger than that in (11). A similar argument implies that the variance is larger in (9) than in (10). The variance is smallest in (3).

Several options have been presented for adjusting the multiplicity estimator to address its bias. There may well be better alternatives. However, the estimator in (11) eliminates a large part of the bias of the unadjusted multiplicity estimator,  $\tilde{N}_{SH} + \tilde{N}_{KI}'$ , moving it (on average) *closer to the target parameter without overestimating it*.

For the 2000 Census, the Bureau of the Census has decided *not* to adjust the multiplicity estimator in the service based enumeration to address its bias. Rather, it will apply the estimator in (3). This decision is based on several factors, including the need for further research on the bias in the multiplicity estimator.

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#### References

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**Table 1: Components of the Population Covered by the Multiplicity Estimator**

Shelters ////// : covered by $\tilde{N}_{SH}$		Soup Kitchens // : covered by $\tilde{N}_{KI}'$				Total
		1 Used on 3/28/00	2 Used in RW <sup>1</sup> , but not on 3/28	3 Sometimes, but not in RW	4 Never use soup kitchens	
1	Used on 3/27/00	$N_{11}$	$N_{12}$	$N_{13}$	$N_{14}$	$N_{1+}$
2	Used in RW <sup>1</sup> , but not on 3/27	$N_{21}$	$N_{22}$	$N_{23}$	$N_{24}$	$N_{2+}$
3	Sometimes use, but not in RW	$N_{31}$	$N_{32}$	$N_{33}$	$N_{34}$	$N_{3+}$
4	Never use shelters	$N_{41}$	$N_{42}$	$N_{43}$	$N_{44}$	$N_{4+}$
<b>Total</b>		$N_{+1}$	$N_{+2}$	$N_{+3}$	$N_{+4}$	$N_{++}$

<sup>1</sup> The reference week (RW) in the table includes the day of enumeration and the prior six days: March 21 - 27 for shelters, and March 22 - 28 for soup kitchens.