Using the Bootstrap to Estimate the Variance from a Single Systematic PPS Sample

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1.0 Introduction

Systematic sampling (either with equal or unequal selection probabilities) is a common sampling scheme in complex sample designs. It is used because of its simplicity of implementation and its potential increase in efficiency, given a good frame ordering, which acts as an additional stratification.

One problem with systematic sampling is that such samples can be viewed as a cluster sample of cluster sample size one. As such, unbiased variance estimation becomes impossible without additional assumptions. One common method for approximating the variance from systematic sampling is to treat the sample as a super-stratified sample. This is accomplished by placing the sample selected within a stratum into the order it was selected and pairing consecutively selected PSUs. Each pair can then be treated as a pseudo-stratum for variance estimation purposes.

There are problems using the pseudo-stratum variance approach. The main problem is that the pseudo-stratum variances still does not reflect the appropriate systematic sampling variance. As such, the variance may only reflect with-replacement sampling. By assumption, the correlation between pseudo-strata is assumed to be zero. At first glance, it seems like these drawbacks would lead to an overestimate of the variance. However, since the correlations can be negative, this need not be the case.

In Kaufman (1998), it is shown that using the pseudo-stratum approach can produce large underestimates of the variance. To reduce this problem, the 1998 paper proposes a consistent bootstrap variance estimation procedure. The advantage of the bootstrap methodology is that it becomes possible to reflect an appropriate systematic sampling variance. The problem with this procedure is that without special adjustments, the bootstrap estimator is biased. To produce an unbiased variance estimator, adjustments are based on estimates from multiple samples. Generally, this is only possible with variables on the frame. Since the required adjustment is dependent on the variable of interest, the proposed procedure can have limited utility.

In this paper, the frame will be randomized in a controlled way, so that some of the affects on efficiency of the frame ordering are maintained, while eliminating the within and between pseudo-stratum correlations. Without the correlations, it becomes possible to estimate the variance in an unbiased fashion, where the expectation is taken across all possible random orderings. With an unbiased variance estimator, the bootstrap variance estimator can be adjusted using only data from a single sample.

The organization of this paper is: 1) define the randomized systematic sampling, 2) define the bootstrap procedure, 3) describe a simulation study to test the bootstrap variance estimator, and 4) present the results and conclusions.

2.0 Systematic Sampling

Systematic probability proportionate to size sampling (PPS) is a common procedure used with complex sample designs. The procedure is described in (Wolter, 1985, pp. 283-286). The idea is to divide the frame into consecutive, exhaustive and disjoint groups of Primary Sampling Units (PSUs), called partition groups, such that the total measures of size in each group are all equal. The total measure of size in a group is called the sampling interval. For this to work, some PSUs must span multiple partition groups. The first sampled PSU is randomly selected from PSUs in the first partition group. All other PSUs are selected systematically, one per partition group, starting from the point of selection of the first PSU.

It is assumed that before sample selection, PSUs with measures of size larger than the sampling interval have been excluded from the sampling. Such units are considered certainty PSUs.

An unbiased estimate for the total of variable X ($\hat{T}_{sy}$) is $\sum_{h=1}^{H}\sum_{i=1}^{n_h} x_i / p_i$, where H is the number of stratum, n_h is the number of sampled PSUs in stratum h, x_i is the value of X for selected PSU i, and p_i is the selection probability for the PSU (i.e., p_i is the measure of size for PSU i divided by the stratum sampling interval).

To simply the development of the randomized systematic sampling procedure $\hat{T}_{sy}$ will be rewritten into an equivalent estimator by treating the PSUs split between partition groups differently. Assume a_h PSUs on the frame split between partition groups. Each of these a_h PSUs will be split into two pseudo-PSUs (j_1 and j_2). For PSU $j \in a_h$ with probability of selection p_j, the first pseudo-PSU selection probability (p_{j_1}) is the part of p_j in the first partition group containing j and the second pseudo-PSU selection probability (p_{j_2}) is $p_j - p_{j_1}$. The partitioning weights (w_{j_1} and w_{j2}) are p_{j_1} / p_j and p_{j12} / p_j, respectively. Without loss of generality, the PPS selection described above can be viewed as selected from this new setup. For a given $j \in a_h$, at most one j_1 or j_2 can be selected. A j_1 or j_2 actually selected will be denoted by $j_\bullet$. And let k_h be the number of $j_\bullet$ selected in stratum h.

Now,

$$\hat{T}_{sy} = \sum_{h=1}^{H} \sum_{i=1}^{n_h} x_i / p_i = \sum_{h=1}^{H} \left(\sum_{\substack{i=1 \\ i \notin a_h}}^{n_h} x_i / p_i + \sum_{\substack{j_\bullet=1 \\ j \in a_h}}^{k_h} (w_{j.} x_j / p_{j.}) \right)$$

3.0 Estimating the $V(\hat{T}_{sy})$ with a Randomized Frame

In this section, an unbiased variance estimator for $\hat{T}_{sy}$ will be derived using only the selected sample. To do this, the sampling frame must be randomized before the sample selection. The variance estimator will then be unbiased across all frame randomizations. The randomization will be done to maintain most of the affects of the implicit stratification induced by the original frame ordering. First, a general expression for $V(\hat{T}_{sy})$ is specified. Next, the frame randomization is specified. Finally, an unbiased variance estimator is derived.

3.1 General Expression for $V(\hat{T}_{sy})$

$V(\hat{T}_{sy})$ can be express as:

$$\sum_{h=1}^{H} \left[\sum_{i=1}^{n_h} V(x_i / p_i) + \sum_{i=1}^{n_h} \sum_{j \neq i}^{n_h} \rho_{hij} \sqrt{V(x_i / p_i) V(x_j / p_j)} \right] \quad (1)$$

where: ρ_{hij} is the weighted correlation between the i^{th} and j^{th} PSUs selected in the systematic selection process.

Of course, without further assumptions, none of the above quantities have unbiased estimates. With the randomized ordering, $\hat{T}_{sy}$ will be denoted as $\hat{T}_{rsy}$.

3.2 New Ordering of the Frame

To simplify the variance estimation, the original frame ordering will be modified. The first step in this process is to define pseudo-strata similar to those described in section 1.0. Within each stratum, place the frame in its original ordering. Next, determine the partition groups as described in the section 2.0. Partition groups are now consecutively paired. Each pair is considered a pseudo-stratum (ps). After, the pseudo-strata are determined, PSUs that are in multiple pseudo-strata must be spilt into two pseudo-PSUs, as described at the end of section 2.0. The final step is to randomize the PSUs and pseudo-PSUs within each pseudo-stratum. It is assumed that n_h is even, so there should be two PSU selections within each pseudo-stratum.

This methodology maintains much of the additional stratification induced by the original ordering. With the original ordering, any contiguous group on the frame would have a selected sample size within one of the expected sample size for that group. With the new ordering, the selected sample size will be within two of the expected sample size.

Another advantage of the new ordering is that across all possible frame randomization the correlation between the i^{th} and j^{th} PSUs selected is zero (i.e., ρ_{hij}=0). (2)

One disadvantage with the new ordering is that if the sum of the covariance terms, from the original ordering, is negative for a variable X then the variance under the new ordering will be less efficient than under the old ordering. The reverse is also true, if the sum of the covariance terms is positive. Of course, with multiple purpose surveys, where many variables are measured, there may be some variables where the sum of the covariance terms is either positive or negative. In this situation, it isn't clear which ordering is overall more efficient. However, variance estimates based on the new ordering should not be negatively biased due to the covariances. A second disadvantage is that it becomes possible to select a pseudo-PSU multiple times. One way of minimizing this impact is to compute the expected number of pseudo-PSUs ($E(ps)$) selected twice and increasing the sample size by this amount: $E(ps) = \sum_h \sum_{j \in a_h} p_{j_1} p_{j_2}$, where a_h refers to the set of PSUs that span multiple pseudo-stratum.

3.3 Estimating $V(\hat{T}_{rsy})$ using the New Ordering

To estimate $V(\hat{T}_{rsy})$, the sampling must be conditioned on three things. The first, denoted by 1, represents the random ordering process described in section 3.2. The second, denoted by 2, represents the PPS systematic sampling process. The third, denoted by N_g, represents the number of PSUs/pseudo-PSUs in partition group g. There are two ways pseudo-PSUs can be formed. The first way is in the formation of the pseudo-strata described in section 3.2, which could generate a pseudo-PSU in each g. For partition group g, assume there are $m_g^{(1)}$ such units (i.e., $m_g^{(1)}$ = 0 or 1). Within a pseudo-stratum, a PSU may still span two partition groups. In this situation, the PSU would be converted into 2 pseudo-PSUs. Within g, assume there are $m_g^{(2)}$ of these units (i.e., $m_g^{(2)}$=0 or 1). The number of PSUs and pseudo-PSUs in a partition group g is: $N_g = N_g^{ns} + m_g^{(1)} + m_g^{(2)}$, where N_g^{ns} is the number of non-splitting PSUs.

Given this:

$$V(\hat{T}_{rsy}) = \underset{N_g}{E}\,\underset{1}{E}\,\underset{2}{V}(\hat{T}_{rsy}) + \underset{N_g}{E}\,\underset{1}{V}\,\underset{2}{E}(\hat{T}_{rsy}) + \underset{N_g}{V}\,\underset{1}{E}\,\underset{2}{E}(\hat{T}_{rsy})$$

$$= \underset{N_g}{E}\,\underset{1}{E}\,\underset{2}{V}(\hat{T}_{rsy}), \text{since } \underset{2}{E}(\hat{T}_{rsy}) = T \text{ (the population total)}$$

$$= \sum_{h}^{H} \sum_{ps \in h} \sum_{g \in ps} \underset{N_g}{E}\,\underset{1}{E}\,\underset{2}{V}(\hat{T}_{rsy,ps,g}), \text{from (1) and (2),}$$

where : $\hat{T}_{rsy,ps,g}$ is the total for partition group g, pseudo-stratum ps

Over all random subdivisions of PSUs and pseudo-PSUs in ps, the probability of any pair of PSUs or pseudo-PSUs being in partition group g is $N_g(N_g - 1)/(N_{ps}(N_{ps} - 1))$, where $N_{ps} = \sum_{g \in ps} N_g$.

By using the argument in (Cochran, 1997, pp. 266-267) for the Rao, Hartley, Cochran estimator and that the number of PSUs/ pseudo-PSUs (N_g) in a group g is a random process, an unbiased estimator for $V(T_{rsy})$ is:

$$\hat{V}(\hat{T}_{rsy}) = \sum_{h}^{H} \sum_{ps \in h} \left(\left(\sum_{g \in ps} N_g^2 - N_{ps} \right) / (N_{ps}(N_{ps} - 1)) \right) \times \left(\sum_{i=1}^{N_{ps}} x_i^2 / p_i - T_{ps}^2 \right)$$

An unbiased sample estimator for $\hat{V}(\hat{T}_{rsy})$ or $V(\hat{T}_{rsy})$, also from Cochran, $v(\hat{T}_{rsy})$, is:

$$v(\hat{T}_{rsy}) = \sum_{h}^{H} \sum_{ps \in h} \left(\left(\sum_{g \in ps} N_g^2 - N_{ps} \right) / \left(N_{ps}^2 - \sum_{g=1}^{2} N_g^2 \right) \right) \times \left(\sum_{g=1}^{2} 1/2(2x_g / p_g - \hat{T}_{ps})^2 \right) \quad (3)$$

where x_g is the variable of interest for the sampled PSU in partition group g and p_g is its selection probability.

The second term of the product in (3) is the balanced half-sample variance estimate (BHR) for the pseudo-stratum. Therefore, any differences between (3) and BHR can be attributed to the first term in (3) (i.e., the scaling term).

The scaling term acts as a finite population correction (FPC). If the N_g 's are all equal in a stratum then this term resembles the simple random sample FPC. However, when the stratum PSUs are skewed in either direction, this term can be greater that 1. In this situation, the BHR estimator should be expected to underestimate the variance.

$v(\hat{T}_{rsy})$ will now be used to produce an unbiased bootstrap variance estimator. When computing N_g , a PSU/pseudo-PSU that spans two partition groups is included in both N_g counts.

4.0 Bootstrap Variance Estimator for $\hat{T}_{rsy}, V^*(\hat{T}_{rsy})$

The bootstrap variance estimator will be generated from a set of bootstrap samples. First, a discussion of the bootstrap sample size used in these samples, n_h^*, will be presented.

n_h^* is chosen so that $E^*(V^*(\hat{T}_{rsy})) = v(\hat{T}_{rsy})$, where E^* represents the expectation with respect to the bootstrap selection. There are two ways to do this:

The first way is to recognize that the sampling scheme proposed here, given a known set of N_g 's, has the same inclusion and joint inclusion probabilities, as well as the same estimator, as the Rao, Hartley, Cochran estimator. Hence, Sitter's (1992) solution to n_h^* can be used. One advantage here is that n_h^* will not be a function of the variable of interest. Therefore, once n_h^* is determined for one variable, across all possible randomizations, it should work for other variables, too. One disadvantage is that Sitter does not provide a closed form solution. Instead, a searching and bracketing process must be used. A possible second disadvantage is that the clustering in the selected sample is ignored.

The second solution is to use a simulation searching process to determine n_h^* that does not ignore the clustering. For this searching process, a number of stratum bootstrap variance estimates are generated, each with a different n_h^*. Each of the bootstrap variance estimates can then be compared to $v(\hat{T}_{rsy})$. A bracketing procedure can now be used to achieve an unbiased variance estimator. The disadvantage here is that this searching process is more involved than the first. However, since the cluster correlations, ρ_{hij} , across all randomizations, are zero, this solution should be reasonably close to the first solution.

In this paper, the second solution will be described and tested in a simulation. The simulation can then be used to verify that this solution, once solved for one variable, works equally well for all other variable.

$\hat{T}_{rsy}$ for stratum h will be denoted by $\hat{T}_h$.

4.1 The Bootstrap Procedure

1. Select a systematic PPS sample (s_h), as described in section 2.0, using the randomization methodology described in section 3.2.
2. Generate a bootstrap frame based on the selected sample s_h . For each selected PSU/pseudo-PSU

j with sampling weight $w_j = 1/p_j$, generate bootstrap-PSUs (bj) by replicating the j^{th} PSU/pseudo-PSU w_j times. Note w_j does not include the partitioning weight. The bj^{th} bootstrap-PSU has the following measure of size (m_{bj}):

$m_{bj} = I_{bj} \cdot 1/w_j$,

$$I_{bj} = \begin{cases} 1, & \text{if } bj \text{ is an integer component of } w_j \\ C_j, & \text{if } bj \text{ is a noninteger component of } w_j \\ & C_j \text{ being the noninteger component} \end{cases}$$

Associate j's pseudo-stratum with each of the bootstrap-PSUs generated from the j^{th} PSU

3. Within each stratum, define a set of bootstrap sample sizes, $n^*_{kh}, k = 1$ to K_h:
   $n^*_{kh} = (n_h - n_{h0})(k-1)/(K_h - 1) + n_{h0}$, where n_{h0} is the lower bound for n^*_{kh}. n_{h0} must be chosen to provide a positively biased variance estimate.
4. Randomize the bootstrap-PSUs within each pseudo-stratum.
5. Choose an n^*_{kh}, say n^*_{0k} to be used to compute the first bootstrap variance.
6. The bootstrap frame, bootstrap frame ordering, measure of size (m_{bj}), and bootstrap sample size (n^*_h) have been specified. Using these quantities select B bootstrap samples using the same procedures used to select the original systematic PPS sample. The one exception to this is that a bootstrap-PSU generated from noncertainy PSUs that become certainty in the bootstrap selection should not be eliminated from the selection process and taken in sample with certainty. The bootstrap weight should properly reflect the bootstrap-PSUs selected multiple times (see 7 below). Before each selection, the bootstrap frame must be re-randomized.
7. For each bootstrap sample, compute a set of bootstrap weights, w^*_j. Compute T^*_{bh} like $\hat{T}_h$, using w^*_j instead of w_j.

   The bootstrap-PSU weight, w^*_j, is: $w^*_j = \sum_{bj \in S^B_j} w^p_{bj}$,

   S^B_j : is the set of all bj's generated from j that are selected in the B^{th} bootstrap sample, and

   $$w^p_{bj} = \begin{cases} I_{bj} \cdot M_{bj} / p_{bj}, & \text{if } bj \text{ is from a PSU} \\ I_{bj} \cdot M_{bj} / p_{bj} \times w_{j.}, & \text{if } bj \text{ is from a pseudo-PSU} \end{cases}$$

   M_{bj} : is the number of times the bj^{th} bootstrap- PSU is selected,

   p_{bj} : is the bootstrap selection probability for the bj^{th} bootstrap-PSU.

   $p_{bj} = m_{bj} / SI_h$, $SI_h = \sum_{bj \in s_h} m_{bj} / n^*_{kh}$.

   $w_{j.}$: is the partitioning weight for the selected bootstrap-PSU.

   When pseudo-PSUs are selected, a bootstrap-PSU weight can be generated by adding up the w^*_j's corresponding to the PSU.
8. The bootstrap variance for $\hat{T}_h$ given n^*_{kh} is:
   $V^*_k(\hat{T}_h | n^*_{kh}) = 1/(B-1)\sum_{b=1}^{B}(T^*_{bh} - \bar{T}^*_h)^2$,
9. Repeat steps 5-8, for each n^*_{kh}, generating $V^*_k(\hat{T}_{ih} | n^*_{kh})$ for $k = 1$ to K_h.
10. Compute $v(\hat{T}_h)$ from sample s_h and compare it to each of the $V^*_k(\hat{T}_{ih} | n^*_{kh})$ for $k = 1$ to K_h. Denote by M_h the stratum bootstrap variance with the smallest negative bias. Denote by P_h the stratum bootstrap variance with the smallest positive bias. Define $q_h = (v_h - M_h)/(P_h - M_h)$. Select a random number between 0 and 1. If it is less than or equal to q_h then use the replicate weights associate with P_h to produce future variances. Otherwise, use the replicate weights associated with M_h. Denote this variance estimator by $V^*(\hat{T}_h)$. This produces unbiased stratum variances because $\underset{q}{E}(V^*(\hat{T}_h)) = q_h P_h + (1-q_h)M_h = v(\hat{T}_h)$, where $\underset{q}{E}$ represents the expectation with respect to the q_h selection. To reduce the instability introduced by the bracketing, P_h and M_h should be determined to be as close to zero bias as possible.

Now, across all randomizations $V^*(\hat{T}_{rsy})$ is unbiased (i.e., $E(V^*(\hat{T}_{rsy})) = \sum_h Ev(\hat{T}_h) = \sum_h V(\hat{T}_h) = V(\hat{T}_{rsy})$)

5.0 Simulation

To demonstrate the advantages of the bootstrap variance estimator, a simulation study is presented comparing BHR and the bootstrap variance estimators. Five hundred simulations are generated using frame variables. In tables 1-6, estimates are computed by each stratification variable (affiliation, region and school level), as well as one of the sort variables (Urbanicity).

5.1 Comparison Statistics

Variance comparisons are based on the relative error of the standard error, relative mean square error of the variance and the 95% coverage rate.

5.2 Sample Design

Following the Schools and Staffing Survey sample design, the list frame component of NCES's Private School Survey (PSS) is stratified by detailed School Association (19 groups) by Census Region (4 levels), and by school level (3 levels). The school sample is selected systematically probability proportionate to size, using square root of the number of teachers, as the measure of size. Before sample selection, the schools are ordered by state, school highest grade, urbanicity, zip code, and school enrollment. One detailed association is simulated.

5.3 BHR Variances

The r^{th} school half-sample replicate is formed using the usual textbook methodology (Wolter, 1985) with 2 PSUs per stratum. When $n_h \geq 2$, PSUs are placed in pseudo-strata (see section 1.0), which are used as strata for estimating variances. This is the BHR without FPC variance. A second BHR variance estimate (BHR with FPC Adjustment) adjusts the first variance estimator by $1 - P_h$, where P_h is the average of the selection probabilities for the selected units within stratum h.

5.4 Number of Replicates and Determining n_h^*

Forty-four and forty-five replicates have been used in the BHR and bootstrap variances, respectively. Total number of students is used to determine n_h^*.

6.0 Results

In terms of relative error, MSE, and coverage rates, tables 1-6 show that the bootstrap variance estimator is better than either of the BHR estimators.

The bootstrap and BHR variance estimates are different only in how they are scaled (see end of section 3.3). Therefore, deficiencies in the BHR estimates are due to the use of an incorrect scaling factor. Each table has examples where BHR produces a large underestimate of the variance. This shows that the correct scaling factor, used in the bootstrap, can be greater than 1 in practice.

The results indicate that the bootstrap performed well for every variable, even though the bootstrap sample size was based on a single variable (number of students). This demonstrates, as argued in the paper, that n_h^* is independent on the variable of interest.

7.0 Conclusion

In the past, the author has proposed using a bootstrap variance estimator when the PSUs are selected with a PPS systematic sampling scheme. With a non-random ordering of the frame, these bootstrap procedures can only be implemented using frame variables. To get around this problem, the PSS systematic selection scheme proposed here introduces a random element to the ordering, while maintaining much of the implicit stratification usually associated with systematic sampling. Now, an unbiased bootstrap variance estimator can be developed for any variable of interest without the dependence of the frame variables.

The simulation study presented in this paper demonstrates that the bootstrap variance estimator is better than the BHR variance estimator, even when a simple FPC adjustment is applied. This is true with respect to relative error, MSE, and coverage rates.

With respect to relative error, the bootstrap performs better than BHR because BHR is not scaled correctly. As such, BHR can provide either an underestimate or overestimate of the variance, depending on the distribution of the PSUs within the pseudo-strata.

When a survey measures one variable or a number of variables all highly correlated with each other, it is likely that a frame ordering exists for an efficient systematic sample using a non-random ordering. In this situation, the standard variance methodologies (e.g. BHR, jackknife, Taylor Series) can safely be used, since these methodologies will likely, but not necessarily, overestimate the variance. In situations where the variance overestimation is unacceptably large (e.g., when the sampling rates are high, the covariance terms are very negative or the sample sizes are small or any combination of these) then the bootstrap procedure in (Kaufman, 1998) may be useful.

When a survey measures a number of unrelated variables, determining an efficient frame ordering for all variables may be impossible. In this situation, using the standard variance methodologies, for the standard systematic sample, can be inappropriate because now variances may have a large underestimation problem. A safer alternative would be using the randomized systematic sampling procedures and bootstrap variance estimator proposed here. With these procedures, all variances can be appropriately estimated. Some estimates may be less efficient than a systematic procedure using a non-random ordering, but there will be no large systematic variance underestimation.

8.0 References

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Table 1 -- % relative error, % relative mean square error and % coverage rates for the Bootstrap and BHR variance estimator for estimating Total Number of Students by Affiliation, Region, Level and Urbanicity

Estimate	Bootstrap			BHR without FPC Adjustment			BHR with FPC Adjustment		
	Rel. Error	Rel. MSE	Cov. Rate	Rel. Error	Rel. MSE	Cov. Rate	Rel. Error	Rel. MSE	Cov. Rate
Other Affil.	-2.3	23.8	92.7	3.2	25.8	92.9	0.2	23.4	92.7
Northeast	-1.9	46.8	91.9	-10.2	30.0	78.6	-12.4	31.9	78.6
Midwest	2.7	46.8	94.6	54.9	149.4	100.0	50.9	137.5	100.0
South	-7.8	28.3	92.3	-10.6	29.2	100.0	-14.0	32.5	99.8
West	-4.2	34.2	93.5	6.4	31.3	92.9	3.9	28.1	92.9
Elementary	0.9	37.0	93.1	2.2	30.1	99.8	0.0	28.7	99.8
Secondary	6.3	43.4	93.8	-5.7	36.6	92.9	-15.5	39.1	85.6
Combined	-2.5	25.2	93.8	5.6	36.8	99.8	2.2	33.0	99.8
Rural	3.0	31.0	96.4	-5.1	29.7	85.9	-7.9	30.3	85.9
Suburban	-2.6	27.5	92.3	17.9	58.2	92.9	14.6	51.3	92.9
Urban	4.9	33.0	95.2	6.1	37.2	93.1	3.1	33.7	93.1

Table 2 -- % relative error, % relative mean square error and % coverage rates for the Bootstrap and BHR variance estimator for estimating the Total Number of Schools by Affiliation, Region, Level and Urbanicity

Estimate	Bootstrap			BHR without FPC Adjustment			BHR with FPC Adjustment		
	Rel. Error	Rel. MSE	Cov. Rate	Rel. Error	Rel. MSE	Cov. Rate	Rel. Error	Rel. MSE	Cov. Rate
Other Affil.	-4.1	25.0	91.7	-11.1	26.8	85.7	-13.5	29.7	85.7
Northeast	-2.3	41.4	91.5	-5.2	40.7	85.7	-7.1	40.7	85.7
Midwest	4.9	44.3	94.6	27.4	71.3	100.0	24.4	64.3	100.0
South	-7.7	31.4	90.9	-1.0	23.1	85.6	-4.8	23.4	85.6
West	-8.3	36.0	90.3	3.9	32.4	93.1	1.4	30.1	93.1
Elementary	-0.8	36.9	91.7	-26.1	52.4	71.5	-27.4	53.8	71.5
Secondary	-3.0	57.8	90.1	23.1	87.5	100.0	13.0	66.8	100.0
Combined	-4.6	26.5	91.7	13.3	45.6	100.0	9.9	39.3	100.0
Rural	-7.2	27.6	91.3	35.8	93.8	100.0	32.5	84.8	100.0
Suburban	2.4	30.1	94.0	-17.1	35.4	85.9	-19.2	38.2	85.9
Urban	-2.2	25.2	93.1	-8.7	29.8	92.9	-11.1	31.6	92.9

Table 3 -- % relative error, % relative mean square error and % coverage rates for the Bootstrap and BHR variance estimator for estimating the Total Number of Teachers by Affiliation, Region, Level and Urbanicity

Estimate	Bootstrap			BHR without FPC Adjustment			BHR with FPC Adjustment		
	Rel. Error	Rel. MSE	Cov. Rate	Rel. Error	Rel. MSE	Cov. Rate	Rel. Error	Rel. MSE	Cov. Rate
Other Affil.	-4.5	24.6	93.1	-15.5	35.3	93.1	-18.0	38.2	93.1
Northeast	-2.7	39.5	91.9	-8.4	37.6	85.9	-10.8	38.3	85.9
Midwest	4.2	38.3	94.8	35.7	94.0	100.0	32.2	84.5	100.0
South	-11.7	31.7	89.7	-12.5	30.3	85.7	-15.9	34.3	85.7
West	-4.7	39.3	91.7	20.0	67.3	93.1	16.9	60.5	93.1
Elementary	-3.2	31.6	93.5	-29.7	53.7	78.4	-31.1	55.3	78.4
Secondary	7.5	45.6	94.8	7.8	42.9	100.0	-2.9	31.9	92.9
Combined	-1.7	27.4	95.0	15.5	52.5	93.1	11.9	45.5	93.1
Rural	5.6	32.2	94.0	4.9	23.5	93.1	1.7	20.4	93.1
Suburban	0.9	27.4	92.7	4.8	26.9	92.9	1.6	23.8	92.9
Urban	9.9	37.0	96.2	49.2	138.9	93.1	44.7	125.5	93.1

Table 4 -- % relative error, % relative mean square error and % coverage rates for the Bootstrap and BHR variance estimator for estimating the Number of Students per School by Affiliation , Region, Level and Urbanicity

Estimate	Bootstrap			BHR without FPC Adjustment			BHR with FPC Adjustment		
	Rel. Error	Rel. MSE	Cov. Rate	Rel. Error	Rel. MSE	Cov. Rate	Rel. Error	Rel. MSE	Cov. Rate
Other Affil.	-4.1	23.3	94.2	-7.3	27.8	85.7	-9.9	29.4	85.7
Northeast	1.2	43.1	93.5	-16.6	37.8	71.7	-18.6	40.1	71.7
Midwest	5.8	43.6	94.6	53.2	147.6	100.0	49.2	135.5	100.0
South	-8.6	28.6	92.3	-13.7	29.6	92.9	-17.0	34.0	92.4
West	-5.2	35.9	92.9	13.7	50.3	85.9	10.9	45.2	85.9
Elementary	-2.9	31.3	91.7	-13.8	40.6	92.7	-15.6	41.6	92.7
Secondary	4.1	43.8	95.2	0.4	40.9	92.9	-9.9	36.6	92.9
Combined	-2.8	25.3	93.5	19.6	63.7	100.0	15.8	55.7	100.0
Rural	2.2	41.2	95.0	-13.3	36.1	85.9	-15.8	38.0	85.9
Suburban	-2.5	30.9	92.5	-30.0	53.1	79.0	-32.0	55.4	79.0
Urban	-3.3	31.2	93.7	-32.2	57.1	71.9	-34.3	59.4	71.9

Table 5 -- % relative error, % relative mean square error and % coverage rates for the Bootstrap and BHR variance estimator for estimating the Number of Teachers per School by Affiliation , Region, Level and Urbanicity

Estimate	Bootstrap			BHR without FPC Adjustment			BHR with FPC Adjustment		
	Rel. Error	Rel. MSE	Cov. Rate	Rel. Error	Rel. MSE	Cov. Rate	Rel. Error	Rel. MSE	Cov. Rate
Other Affil.	-4.3	24.0	92.9	-16.2	34.8	93.1	-18.5	37.7	92.7
Northeast	-2.0	45.7	92.3	-15.4	38.8	85.7	-17.3	40.6	85.7
Midwest	7.9	49.0	94.2	32.7	87.3	100.0	29.3	78.6	100.0
South	-10.3	31.2	91.5	-5.8	24.4	92.7	-9.5	26.8	92.7
West	-5.8	35.9	92.5	18.9	62.7	93.1	15.9	56.1	93.1
Elementary	-2.1	35.3	93.5	-30.4	55.6	71.5	-31.7	57.1	71.5
Secondary	2.3	44.7	94.8	16.3	70.1	100.0	5.1	49.2	100.0
Combined	-3.3	26.1	94.0	11.3	41.9	93.1	7.8	36.2	93.1
Rural	2.1	39.4	93.5	16.8	57.2	100.0	13.5	50.4	100.0
Suburban	0.9	33.2	92.9	-23.5	43.8	79.0	-25.6	46.6	79.0
Urban	-2.0	31.5	94.6	-25.5	48.9	79.0	-27.8	51.5	79.0

Table 6 -- % relative error, % relative mean square error and % coverage rates for the Bootstrap and BHR variance estimator estimating the Student/Teacher Ratio by Affiliation , Region, Level and Urbanicity

Estimate	Bootstrap			BHR without FPC Adjustment			BHR with FPC Adjustment		
	Rel. Error	Rel. MSE	Cov. Rate	Rel. Error	Rel. MSE	Cov. Rate	Rel. Error	Rel. MSE	Cov. Rate
Other Affil.	-0.4	30.8	93.8	14.8	40.7	100.0	11.6	34.5	100.0
Northeast	-5.7	65.7	90.5	-8.8	47.9	85.7	-10.9	47.4	78.6
Midwest	4.4	83.8	94.0	17.2	99.1	92.9	14.4	93.9	92.9
South	-1.1	35.5	92.7	20.8	69.1	92.9	16.2	59.2	92.9
West	-6.2	35.0	91.5	14.7	56.2	93.1	12.1	51.3	93.1
Elementary	5.4	55.2	93.5	-14.4	41.1	79.0	-16.1	42.4	79.0
Secondary	0.0	36.5	90.9	12.2	45.6	92.9	1.5	29.6	85.6
Combined	-2.3	29.8	93.8	-8.1	29.6	92.9	-11.1	31.4	92.9
Rural	0.8	52.4	93.8	11.9	86.8	100.0	8.2	78.8	100.0
Suburban	1.4	40.8	95.0	91.3	296.1	100.0	86.2	276.2	100.0
Urban	0.2	37.5	94.8	-12.7	32.6	85.9	-15.2	35.0	85.9