An Empirical Study of Variance Estimates of Raking Ratio Adjusted Estimators

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Key Words: Raking Ratio Adjustment, Variance Estimation,

1. Introduction

In National Center for Education Statistics (NCES) surveys, ordinary poststratification (or ratio adjustment) and raking ratio adjustment are often used techniques for improving the precision and reducing the bias of estimators. The ratio adjusted post-stratified estimator requires population counts at the cell level. These cell counts, however, are not always available, especially when several auxiliary variables are used. For instance, when age group counts are available from one file and region group counts are available from another file the population marginal counts are known, but the cross-classification cell counts are lacking. This scenario is described as incomplete post-stratification.

Two techniques are often applied to handle incomplete poststratification. The first approach uses regression estimator by introducing multiple poststrata indicator variables (Bethlehem and Keller, 1987). The second approach uses raking ratio adjustment. Raking estimation uses iterative proportional fitting and can be extended to loglinear models for weighting. One disadvantage is that no simple formula for its variance is available (Bethlehem and Keller, 1987).

Deville and Särndal (1992) introduced the calibration estimator, which includes often used estimators such as the ratio estimator, the regression estimator, and the raking ratio estimator as special cases. They proved that any other member of the calibration estimator class is asymptotically equivalent to the regression estimator and, as a consequence, all members of the calibration estimator class share the same asymptotic variance.

Rust (1987) investigated the effect of nonresponse adjustment and ratio weight adjustments on sampling error estimates for two continuous variables from Title IV Quality Control Study. In his study, the differences between the variances estimated via the two approaches are small, which indicates the relationship between the variable of interest and the auxiliary variable was not a strong one. He also noticed, in another study undertaken by Lago et al. (1987), that when variables of interest (weight, height, and level of cholesterol) are highly correlated with the poststratification variables (age and sex), the use of poststratification gave rise to considerable reduction in sampling variance.

Valliant (1993) studied the standard linearization variance estimator, BRR, and the jackknife variance estimator to determine whether they estimate the conditional variance of the poststratified estimator of a finite population total under a super-population model. Yung and Rao (1996) studied the standard linearization variance estimator, jackknife, and the jackknife linearization variance estimators for both the poststratified estimator and the regression estimator. Their simulation study suggests that the jackknife procedure which does not recalculate the regression weights each time a cluster is deleted performs poorly.

In this study, we compare variance estimates which incorporate the raking ratio adjustments and nonresponse adjustment with the variance estimates which ignore these adjustments using the 1993 National Household Education Survey (NHES) School Readiness component data.

2. An Overview of NHES Sample Design and Weighting Procedure

The target population of the NHES:93 survey was children aged 3 through 7, or in second grade or below but at least age 3. The method of sampling used in NHES:93 is a variant of the random digit dialing method, which can be viewed as stratified multistage sampling. 10,888 children sampled from 4577 PSUs completed interview. A clear description of the survey was given by Brick et al. (1994).

The weights for the children in the sample were first adjusted for nonresponse to the extended interview. Six age categories from 3 to 8 and older were used to define the nonresponse adjustment cells. The nonresponse adjustment was the sum of the adjusted base weights for all sampled children in the cell divided by the sum of the adjusted base weights for the respondents in the same cell. The adjustment factors varied from 1.09 to 1.14 across the six cells.

The next stage of weighting was to rake the nonresponse-adjusted person weights to known totals computed from the October 1992 Current Population Survey (CPS). The marginal totals are given in table 1 from Brick et al. (1994). Three dimensions were used in the raking. The first dimension is defined by the cross-classification of home type (owned or not) and Census region. The second dimension is the cross of race/ethnicity and household income. The last dimension is defined by the cross of age and grade.

Sixty jackknife replicate weights were created by Westat, Inc. based on the sampling of clusters of telephone numbers. All 60 replicate weights were created using the same estimation procedures used for the full sample. Also included in the data file are stratum and PSU variables required by software using Taylor series approximation.

Table 1. NHES:93 control totals for School Readiness raking

Control characteristics		Control totals
Home type	**Census region**	
Owned or other	Northeast	2,400,545
Owned or other	Midwest	3,202,557
Owned or other	South	4,116,866
Owned or other	West	2,589,938
Rented	Northeast	1,448,553
Rented	Midwest	1,651,182
Rented	South	2,764,945
Rented	West	1,938,053
Race/ethnicity	**Household income**	
Hispanic	Less than $10,000	818,994
Hispanic	$10,000-$24,999	904,880
Hispanic	$25,000 or more	685,193
Black, non-Hispanic	Less than $10,000	1,360,091
Black, non-Hispanic	$10,000-$24,999	997,013
Black, non-Hispanic	$25,000 or more	792,487
Other	Less than $10,000	1,514,364
Other	$10,000-$24,999	3,610,969
Other	$25,000 or more	9,428,649
Age	**Grade**	
3	All grades	3,905,387
4	All grades	3,806,845
5	All grades	3,832,330
6	All grades	3,763,999
7	All grades	3,809,885
8 and older	Second grade or less	994,193

NOTE: Details do not add to the same total due to rounding.
SOURCE: U.S. Bureau of the Census, *Current Population Survey*, October 1992.

3. Variance estimates comparison

We first used the jackknife replicate weights which incorporated the adjustments to calculate standard errors for two kinds of estimators—total and mean estimators. The variance calculation was performed using WesVar PC; the standard errors calculated by this approach are denoted as ste_T for total estimator, and ste_R for ratio type estimator (this includes estimators of percentage, mean, and the ratio of two variables).

Then we calculated the standard errors for the same estimators but ignored the adjustments. This was implemented in two ways. The first approach was to let WesVar PC generate the jackknife replicate weights and then use these replicate weights to calculate the standard errors with WesVar PC. In this approach, neither nonresponse adjustment nor raking ratio adjustment was performed when a replicate weight was created; therefore these adjustments were not incorporated. The second way was to use the stratum identification variable and PSU identification variable provided with the public use data file to calculate the standard errors with SUDAAN. This approach actually treats the adjusted full sample final weight (FWGT0-Final Raked Weight which incorporates the nonresponse adjustment and the raking ratio adjustment) as a design weight (inverse of inclusion probability). And the variance estimator of the Horvitz-Thompson estimator was used. Also notice that the mean estimator in this study is actually a ratio of two raking ratio adjusted estimators. Although SUDAAN is used here, the underlying variance estimator is actually the variance estimator for the ratio of two Horvitz-Thompson estimators, not a genuine linearized variance estimator for the ratio of two raking ratio adjusted estimators. Therefore the adjustments were also ignored in this approach. The variance estimates calculated from these two approaches (from WesVar PC generated replicate weights and from SUDAAN) are identical. They are denoted by ste_T^* for the standard error of the total estimator and ste_R^* for the standard error for the ratio type estimator.

Table 2 shows standard errors for categorical variables. As we can see, in general, ste_T is much smaller than ste_T^* while ste_R is close to ste_R^* except for the last two variables (which were used as auxiliary variables in the raking ratio adjustment). It seems like the adjustments and the gain in precision cancel out for the ratio type estimator.

For the standard errors of the percentage and mean estimators, when the adjustments are incorporated, the denominator becomes constant C = 20,112,639 for all replicates. Therefore, the standard error equals ste_T/C. When the adjustments are ignored, the denominator varies. But since the numerator is positively correlated with the denominator, the actual standard error is smaller than ste_T^*/C. That is why ste_R/ste_R^* is larger than ste_T/ste_T^*.

Household income is one of the auxiliary variables used for the raking ratio adjustment (table 1) where it has three categories: "Less than $10,000", "$10,000-$24,999", and "$25,000 or more". However in the public use data file, two categories, "Less than $10,000" and "$10,000-$24,999", were collapsed into one category, "Up To $25,000". The marginal totals for all replicates are still the same. Therefore the standard errors are null.

During the raking ratio adjustment, Race/Ethnicity was collapsed into three categories: "Hispanic", "Black, non-Hispanic", "Other". But in the public data file it has the customary four categories: "White / Nonhispanic", "Black / Nonhispanic", "Hispanic", and "All Other Races". Now the marginal totals for category "White/Nonhispanic" and "All Other Races" are not constant anymore, so we observe standard errors for these two categories but no standard error for the other two.

Table 3 shows standard errors for continuous variables. The gain in precision to the total estimator is obvious. Age is an auxiliary variable used for raking ratio adjustment but was treated as a continuous variable here. The two ratios "Number of Bedrooms in Home/Total Number of Household Members" and "Number of Household Members Under 18/Total Number of Household Members" are ratios of two raking ratio adjusted estimators. Incorporating the adjustment results in a reduction of the standard error estimates of about 14 and 7 percent respectively.

Table 4 shows standard errors calculated within the nonresponse adjustment and raking ratio adjustment cells (Home type × Census region × Race/ethnicity × Household income × Age × Grade). Only two cells with comparatively large sample sizes were chosen. Within these cells, the adjustments are the same for all units, so the adjustment factors were canceled out for the ratio type estimator and hence ste_R is about the same as ste_R^*. But still, a gain in precision due to the raking ratio adjustment to the total estimator is present.

Table 2. Standard errors for categorical variables

Categorical Variables	ste_T	ste_T^*	ste_T/ste_T^*	ste_R	ste_R^*	ste_R/ste_R^*
Read story books on own						
Yes	79375	217683	0.3646	0.395	0.507	0.7791
No	79374	230654	0.3441	0.395	0.507	0.7791
Had intensive care when born						
Yes	81658	92717	0.8807	0.406	0.413	0.9831
No	81658	370645	0.2203	0.406	0.413	0.9831
Child's birth order						
Only/Oldest Kid	109700	200995	0.5458	0.545	0.535	1.0187
Later Born	109700	255680	0.4291	0.545	0.535	1.0187
Home choice influenced by school						
Yes	152523	257797	0.5916	0.758	0.788	0.9619
No	152523	252258	0.6046	0.758	0.788	0.9619
Gender						
Female	104303	222735	0.4683	0.519	0.524	0.9905
Male	104303	231969	0.4496	0.519	0.524	0.9905
Number of books child has						
None	23347	25110	0.9298	0.116	0.124	0.9355
1 Or 2 Books	35046	38619	0.9075	0.174	0.191	0.9110
3 To 9 Books	73626	90597	0.8127	0.366	0.422	0.8673
10 To 25 Books	94273	134211	0.7024	0.469	0.465	1.0086
26 To 50 Books	91039	126309	0.7208	0.453	0.469	0.9659
More Than 50	124337	222669	0.5584	0.618	0.667	0.9265
Total household income						
$5,000 Or Less	58528	94562	0.6189	0.291	0.416	0.6995
$5,001 - $10,000	58528	101152	0.5786	0.291	0.434	0.6705
$10,001 - $15,000	58980	79911	0.7381	0.293	0.383	0.7650
$15,001 - $20,000	77404	98786	0.7835	0.385	0.456	0.8443
$20,001 - $25,000	75325	99576	0.7565	0.375	0.455	0.8242
$25,001 - $30,000	69972	80165	0.8729	0.348	0.379	0.9182
$30,001 - $35,000	53173	63908	0.8320	0.264	0.295	0.8949
$35,001 - $40,000	61437	70068	0.8768	0.305	0.319	0.9561
$40,001 - $50,000	81543	96797	0.8424	0.405	0.422	0.9597
$50,001 - $75,000	65695	89348	0.7353	0.327	0.375	0.8720
Over $75,000	76787	87698	0.8756	0.382	0.407	0.9386
Total household income - range						
Up To $25,000	2	255420	0.0000	0	0.804	0.0000
More Than $25,000	0	260352	0.0000	0	0.804	0.0000
Race/Ethnicity						
White/Nonhispanic	52425	319287	0.1642	0.261	0.802	0.3254
Black/Nonhispanic	1	123945	0.0000	0	0.518	0.0000
Hispanic	0	110665	0.0000	0	0.522	0.0000
All Other Races	52425	59301	0.8840	0.261	0.273	0.9560

Table 3. Standard errors for continuous variables

Continuous Variable	ste_T	ste_T^*	ste_T/ste_T^*	ste_R	ste_R^*	ste_R/ste_R^*
Number of bedrooms in home............	231137	1292940	0.1788	0.011	0.014	0.803
Total number of household members....	415720	1953781	0.2128	0.021	0.021	1.024
Number of household member under 18	369884	1161715	0.3184	0.018	0.019	0.952
Number of siblings in household.........	351823	747261	0.4708	0.017	0.018	0.944
Hours of TV from 8am to3pm............	249661	426974	0.5847	0.012	0.014	0.889
Hours of TV after dinner..................	250867	493058	0.5088	0.012	0.012	0.984
Hours of TV Saturday........................	520567	1516009	0.3434	0.026	0.027	0.974
Hours of TV Sunday........................	500809	1201840	0.4167	0.025	0.025	0.988
Age	8698	2125447	0.0041	0.000	0.015	0.000
Total HH bedrooms/Total HH member..				0.003022	0.003515	0.8597
Hhundr18/Hhtotal				0.001987	0.002138	0.9294

Table 4. Standard errors calculated within the nonresponse and raking ratio adjustment cells

		ste_T	ste_T^*	ste_T/ste_T^*	ste_R	ste_R^*	ste_R/ste_R^*
CELL	**Child's birth order**						
1	Only/Oldest Kid	24014.32	26749.49	0.8977	4.637	4.599	1.0083
1	Later Born	22812.91	24063.16	0.9480	4.637	4.599	1.0083
2	Only/Oldest Kid	18091.32	22370.59	0.8087	2.594	2.617	0.9912
2	Later Born	21688.37	24680.15	0.8788	2.594	2.617	0.9912
CELL	**Read story books on own**						
1	Yes	5826.182	6005.819	0.9701	1.408	1.421	0.9909
1	No	27764.57	33962.51	0.8175	1.408	1.421	0.9909
2	Yes	26773.59	36412.67	0.7353	0.866	0.869	0.9965
2	No	5006.48	4970.37	1.0073	0.866	0.869	0.9965
CELL	**Home choice influenced by school**						
1	Yes	20932.59	23983.75	0.8728	4.007	4.003	1.0010
1	No	22133.66	24012.56	0.9218	4.007	4.003	1.0010
2	Yes	19193.92	22665.75	0.8468	2.503	2.523	0.9921
2	No	20255.97	23877.39	0.8483	2.503	2.523	0.9921
CELL	**Total household income - range**						
1	Up To $25,000	15329.57	16214.36	0.9454	3.46	3.49	0.9914
1	More Than $25,000 ..	26211.87	30703.29	0.8537	3.46	3.49	0.9914
2	Up To $25,000	18989.11	20553.96	0.9239	2.924	2.844	1.0281
2	More Than $25,000 ..	24678.83	28751.11	0.8584	2.924	2.844	1.0281
CELL							
1	**Number of household member under 18**	82555.12	92617.02	0.8914	0.104	0.103	1.0097
2	**Number of household member under 18**	70281.32	90818.96	0.7739	0.053	0.053	1.0000
1	**Hours of TV after dinner**	34909.93	40125.09	0.8700	0.056	0.056	1.0000
2	**Hours of TV after dinner**	41062.69	46779.49	0.8778	0.046	0.046	1.0000

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