MODELING 2-BEDROOM MEDIAN RENT OF OCCUPIED HOUSING UNITS USING AHS-MS DATA

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KEY WORDS: Variance component models, Bias, Variance

I. Introduction

The Department of Housing and Urban Development (HUD) is required by law to establish Fair Market Rents (FMR) for rental units at small area level. HUD defines FMR of an area as the 45th percentile of rent. HUD estimates FMRs on an annual basis for 339 metropolitan areas and 2,416 non-metropolitan counties in the United States. The universe in estimating the FMR consists of rental units that satisfy certain criteria.

In the current methodology, the 45th percentile rent for two-bedroom units is used in estimating subnational differentials. There are two different methods used to estimate FMR. One method is based on survey data - American Housing Survey- Metropolitan Sample (AHS-MS), and the other method is based on Decennial Census estimates. The AHS-MS surveys cover 44 largest metropolitan areas. The AHS-MS are conducted on a four-year cycle, 11 metropolitan areas each year are in the AHS-MS survey. Outside of these areas, FMRs are based on the Decennial Census of Housing. The survey based or the census based estimates are updated by the Consumer Price Index (CPI). For all areas, FMRs are rebenchmarked every 10 years when the results of the decennial Census become available. For the 44 largest metropolitan areas with AHS data, the FMRs are rebenchmarked every four years on the basis of these surveys. Estimates for SMSAs/MSAs of other housing sizes included in the AHS - Metropolitan Sample (MS) are produced by proportionally adjusting the sample estimate for two - bedroom units for the SMSA/MSA by the national ratios for units of other sizes. The current methodology disregards any sample data (AHS-MS) for units of other sizes, except to the extent that these units contribute to the estimation of the overall national ratios. Fay (1988) proposed using a multivariate variance components model to estimate median rent for unit size of bedroom sizes from 0 to 4.

The Census Bureau employed a model (see Fay (1986) and Fay, Nelson and Litow (1993)) combining sample estimates from the Current Population Survey (CPS) with other information to produce estimates of median income for 4-person families by state for the Department of Health and Human Services. In this paper, we did some empirical study using variance component models similar to CPS application to do the alternative estimation research of the FMR problem. (see Shapiro (1993)).

We first used the published AHS-MS data of median rents for different bedroom sizes (1 to 3) for an empirical study because of the availability of the published data (Note that AHS-MS median rent is not FMR). When the fair market rent of different bedroom sizes and its variance estimates were available for MSAs in AHS-MS (1986-1990) (See Kim, J. (1996)), we then conducted an empirical study using the fair market rent data to seek improvement in the sample estimate of median FMR for the MSAs.

The study of median rent for rental housing units is given in sections II and III. In section II, models for estimating 2-bedroom median rent are presented; they include univariate and multivariate variance component models. In section III, empirical examples of estimation of 1990 median rent of 2-bedroom rental housing units of 11 MSAs (in the 1990 AHS-MS survey) using AHS-MS data for the single year (1990), 4 years (1987-1990) and 7 years (1984-1990) are presented, respectively.

The empirical study of median rent for rental housing units that satisfy the HUD criteria (FMR) is given in section IV; a univariate variance component model for estimating 1990 2-bedroom Fair Market Rent of 11 MSAs (in the 1990 AHS-MS survey) is given by using 1 year's, 4 years' and 5 years' worth of AHS-MS data. We conclude this report with a summary in section V.

II. Models for Estimating 2-Bedroom Median Rent by MSA

A. Univariate Variance Component Model (Isaki, et. al 1991)

Let $\mathbf{Y}$ denote the (n x 1) vector of direct sample estimates of 2-bedroom median rent for n MSAs from AHS-MS survey.

Let $\mathbf{y}$ be an n x 1 vector of the true 2-bedroom median rents.
Let $\mathbf{X}$ be an n x k auxiliary data matrix. Let

$$\mathbf{Y} = \mathbf{y} + \mathbf{e} \quad \text{where } e \text{ is } N(0, \mathbf{V}), \tag{2.1}$$

$$\mathbf{y} = \mathbf{X}\beta + v \quad \text{where } v \text{ is } N(0, \sigma^2 I), \tag{2.2}$$

e and v are independent, and β and σ^2 are unknown, $\mathbf{V}$ is assumed known but estimated.

Based on models (2.1), (2.2), the estimated best linear unbiased predictor of $\mathbf{y}$ or empirical Bayes estimate of $\mathbf{y}$ (or the smoothed estimator) is

$$\hat{y} = \mathbf{X}\hat{\beta} + \hat{\sigma}^2 \mathbf{I}\hat{\Sigma}^{-1}(\mathbf{Y} - \mathbf{X}\hat{\beta}) \quad \text{with} \tag{2.3}$$

$$\hat{\beta} = (\mathbf{X}'\hat{\Sigma}^{-1}\mathbf{X})^{-1}\mathbf{X}'\hat{\Sigma}^{-1}\mathbf{Y}, \tag{2.4}$$

$\hat{\Sigma} = \hat{\sigma}^2 \mathbf{I} + \mathbf{V}$ and $\hat{\sigma}^2$ is the maximum likelihood estimator of σ^2 obtained via iteration.

An estimator of the mean square error of $\hat{y}$ was given by Fuller (1989).

B. Multivariate Variance Component Model

The univariate model can be extended to a multivariate model when more than one characteristic of sample estimates are available. (See Fay (1988)).

Let Y be a series of sample estimates of K related characteristics $Y_{k(i)}$, k = 1,...,K, in MSA i, and
$Y = (Y_{1(1)}, Y_{2(1)}, \ldots Y_{k(1)}, Y_{1(2)} \ldots Y_{k(2)}, \ldots, Y_{1(n)} \ldots Y_{k(n)})'$,
a (nk x 1) sample vector where n is the sample size.

Let X be a design matrix of size nk x nk. Let p be the number of auxiliary variables for each of k characteristics.
$x_{k(i)} = (x_{1k(i)},...,x_{pk(i)})$, a 1 x p vector
y is the true unknown median rent vector of n k x1

$$Y = y + e \qquad e \text{ is distributed } N(0,\mathbf{D}) \tag{3.1}$$

$$y = X\beta + \upsilon \qquad \upsilon \text{ is distributed } N(0,\mathbf{A}) \tag{3.2}$$

e and υ are independent.

D is the sample covariance matrix of Y.
$\mathbf{D} = \text{diag}(D_i)$, i = 1,...,n.
D_i is a k x k diagonal matrix. (3.3)

The model covariance **A** is a block diagonal matrix of nk x nk.
$\mathbf{A} = \text{diag}(A_i)$, i = 1,...,n, (3.4)
and we assume A_i, a k x k matrix, is equal to a common A^*. If **A** and **D** are known, the best linear unbiased estimator of y is given by Harville (1976).

$$\hat{y} = \mathbf{X}\hat{\beta} + \mathbf{A}\,(\mathbf{D} + \mathbf{A})^{-1}(\mathbf{Y} - \mathbf{X}\hat{\beta}) \tag{3.5}$$

and

$$\hat{\beta} = (\mathbf{X}'\,(\mathbf{A} + \mathbf{D})^{-1}\,\mathbf{X})^{-1}\,\mathbf{X}'(\mathbf{A} + \mathbf{D})^{-1}\,\mathbf{Y}. \tag{3.6}$$

III. Estimation of Two-bedroom Median Rent of Housing Units by MSA Using AHS-MS Data

The data used in this section are the published data of median rent of one to three bedroom housing units from AHS-MS survey (using a single year or multiple years' survey data), 1980 and 1990 census data. The annual changes of the Consumer Price Index (CPI) from 1980 to 1990 for a selected MSAs and regions are also used.

Our objective is to estimate 1990 two-bedroom median rent of units of MSAs in the 1990 AHS-MS sample. Several sources of estimates of 1990 median rent of the two-bedroom size are considered in this section:

(1) The direct sample estimate from the AHS-MS survey.
(2) The 1980 census estimates adjusted for 1990 by using consumer price indices (CPI).
(3) The regression estimates derived from the univariate or multivariate variance component models.
(4) The smoothed estimates $\hat{Y}$ from (2.3) or (3.5) derived from the univariate or multivariate variance component models.

To compare the different estimates, the corresponding 1990 census figures are used as a measure of truth.
Let c_i denote the 1990 census estimate for the i^{th} MSA (i = 1,...,n). For any estimate $m = (m_1,...,m_n)'$, we compute

$$\text{Average Relative Bias} = n^{-1} \sum |c_i - m_i| / c_i$$
$$\text{Average Squared Relative Bias} = n^{-1} \sum |c_i - m_i|^2 / c_i^2 \tag{3.7}$$
$$\text{Average Absolute Bias} = n^{-1} \sum |c_i - m_i|$$
$$\text{Average Squared Deviation} = n^{-1} \sum (c_i - m_i)^2$$

We used single year (1990 AHS-MS) 4 years' (1987-1990) and 7 years' (1984-1990) worth of AHS-MS data to estimate 1990 2-bedroom median rent for 11 MSAs in the 1990 AHS-MS surveys and compared the different estimates using the above 4 criteria.

III. A. Using One Year's (1990 AHS-MS) Data

III.A. 1. Using Univariate Variance Component Model

We used 1990 AHS-MS 2-bedroom median rent (excluding no cash rent units) as dependent variable Y in (2.1).

The X variables are
$X_{0i} = 1$, i = 1,...,11,
X_{1i}: the updated 1980 census gross median rent of a 2-bedroom of the i^{th} MSA using the annual percent change of Consumer Price Index (CPI) from 1980 to 1990,
X_{2i}: the 1980 census gross median rent of a 2-bedroom of the i^{th} MSA.

There are 11 metropolitan areas in the 1990 AHS-MS, and each MSA sample estimator is independent of every other estimator. The V defined in (2.1) is a diagonal matrix. The element on the diagonal is the sample estimate of the variance of the estimated 2-bedroom median gross rent from the 1990 AHS-MS. The variance of an estimated median rent is calculated by $(\text{Var}(p)/d^2)$. Var(p) is the variance of the estimated percentage that the number of the occupied rental housing units is less than or equal to the median. We used the table of the standard error of the estimated percentage provided in the Appendix of the Current Housing Report of the 1990 AHS-MS publication. d^2 is the square of the density which is derived from the interval where the median is located. The median rent in AHS-MS publication is obtained by interpolation of the categorical data - the number of housing units in the rent interval groups. The $\hat{\sigma}^2$ is obtained via maximum likelihood.

The fitted regression model for 2-bedroom median rent is

$$X\hat{\beta} = -113.2702 + 0.8614\,X_1 + 0.8115\,X_2 \tag{3.8}$$
$$(86.45) \qquad (0.219) \qquad (0.565)$$
$$\hat{\sigma}^2 = 853.96 \quad R^2 = 0.93$$

The standard error of the regression coefficient is in the parenthesis under the corresponding regression coefficent.

The smoothing results showed that the smoothed estimate $(\hat{y})$ is the best estimate in terms of the smallest variance, the regression estimate, y, has the largest variance. However, the smoothed estimate $(\hat{y})$ is very close to the direct sample estimate Y because the model variance is much bigger than the sampling variance (derived from the generalized variance). The decrease in variance of the smoothed estimates over the direct sample estimate for 11 MSAs when the sampling variance was derived from the generalized variance is 2%.

Since the sampling variance derived from the generalized variance is underestimated; we then used an estimated sampling variance from a replication method (Jackknife variance estimation via VPLX of Fay's computer software). The estimated variance using VPLX is double or triple of the sample variance derived from the generalized variances. The fitted regression model for the 2-bedroom median rent for 11 MSA's in 1990 AHS-MS using sampling variance via VPLX is:

$$X\hat{\beta} = -113.0847 + 0.8622\,X_1 + 0.8089\,X_2 \tag{3.9}$$
$$(87.99) \qquad (0.222) \qquad (0.564)$$
$$\hat{\sigma}^2 = 809.01. \qquad R^2 = 0.93$$

The decrease in variance of the smoothed estimate over the direct sample estimate for 11 MSAs when the sampling variance is estimated via VPLX is 5%.

III. A. 2. Using Multivariate Variance Component Models

III. A.2. a. Estimation of the Model Errors Using One Year's Data

The median rent of k-bedroom units (k = 1, 2, 3) based on 1990 census data of 11 MSAs (in the 1990 AHS-MS) was fitted to the auxiliary variables X_{1k} and X_{2k}. The estimated model variance is obtained from the difference between the resulting predicted and the 1990 census values. The estimated model error covariance matrix A^* for 4-variate (Y_1, Y_2, Y_3, Y_c) median rent for different bedroom sizes are

$$\begin{array}{llll} a^*_{11} = 615.01 & a^*_{12} = 405.58 & a^*_{13} = 1021.31 & a^*_{1c} = 742.57 \\ & a^*_{22} = 868.20 & a^*_{23} = 1550.67 & a^*_{2c} = 866.03 \\ & & a^*_{33} = 3421.11 & a^*_{3c} = 1975.36 \\ & & & a^*_{cc} = 1222.47 \end{array}$$

where $Y_c = 0.6\ Y_1 + 0.4\ Y_3$, the weighted combination of 1-bedroom and 3-bedroom median rent. The weights 0.6 and 0.4 are approximately proportional to the respective sample sizes of 1 and 3 bedroom median rent from the AHS national sample.

III. A.2. b. The Bivariate Model (Y_2 Y_c)

Let Y_{2i} be the median rent of 2-bedroom units in MSA i from the 1990 AHS-MS, and $Y_{ci} = 0.6Y_{1i} + 0.4Y_{3i}$ be the weighted median rent of 1-bedroom and a 3-bedroom units from the 1990 AHS-MS.

Let $\mathbf{Y} = (Y_{21}\ Y_{c1},...,Y_{2,11},Y_{c,11})'$, a 22 x 1 column vector.
Let $\mathbf{X}_{2i} = (x_{2i1}\ x_{2i2}\ x_{2i3})$ denote a row vector with 3 predictor variables for a 2-bedroom unit median rent in the MSA i, and $\mathbf{X}_{ci} = (x_{ci1}\ x_{ci2}\ x_{ci3})$ the predictors for the weighted combined variable, $Y_{c,i}$, and let

$$X = \begin{pmatrix} X_{21} & 0 \\ 0 & X_{c1} \\ X_{22} & 0 \\ 0 & X_{c2} \\ \ldots & \ldots \end{pmatrix}$$

be a 22 x 6 matrix containing all predictors.
The estimated model error covariance matrix $\mathbf{A}_*$ derived using 11 MSAs for 2-variate (Y_2, Y_c) is

$$A^* = \begin{pmatrix} 868.20 & 866.03 \\ 866.03 & 1222.47 \end{pmatrix}$$

The estimated sampling variance of Y using the published generalized variance formula is used in the diagonal of D. The same predictors are used as before.

The regression predictor and the smoothed estimate of y are calculated and compared with 1990 census values using the 4 measurement criteria.

Similar procedures are carried out for other bivariate and 3 variate models using 1 year's and 4 years' (1987-1990 worth of data.

The results of the comparison using 4 criteria for different models of the smoothed estimates for estimating 1990 2-bedroom median rent for the 11 MSAs are tabulated in Table 1.

B. Using 7 Years' Worth of (1984-1990) AHS-MS Survey Data

B.1. Using a Univariate Variance Component Model

We used median rent of 2-bedroom units over 7 years' worth of (1984-1990) AHS-MS data to estimate 1990 2-bedroom median rents. Since AHS-MS are conducted on a four-year cycle, three of the 7 years' worth of data are recurrent samples. Hence there is some correlation of median rent that are 4 years apart. The estimated correlation of different bedroom sizes based on AHS national survey samples are as follows:

- Correlation between 1985 and 1989 median rent

1-bedroom	2-bedroom	3-bedroom	4+-bedroom
0.485	0.311	0.558	0.386

- Correlation between 1987 and 1991 median rent

1-bedroom	2-bedroom	3-bedroom	4+bedroom
0.494	0.432	0.560	0.226

The fitted regression model of 2-bedroom median rent using 7 years' worth of data and for **V** being a diagonal matrix **D** with sample variances on the diagonal is

$$\begin{array}{l} X\hat{\beta} = 12.7931 + 1.0095\ X_1 + 0.0878\ X_2 \\ \quad (39.88) \quad (0.096) \quad (0.240) \\ \hat{\sigma}^2 = 1629.39 \quad R^2 = 0.84 \end{array}$$

We also fitted regression models using the same independent variables X_1 and X_2 for **V** under different assumptions:

1. **V** is a **V1** where the diagonal elements are the sample variance of 2-bedroom median rent, the covariance of 2-bedroom median rent 4 years apart are calculated using the correlation of 2-bedroom median rent, $r_2 = 0.311$.

2. **V** is a **V2** where the diagonal elements are the same as D but the covariance of 4 years apart is calculated using correlation, $r_2 = 0.432$.

3. **V** is a **V3** where the diagonal elements are the same as **D** but the covariance of 4 years apart is calculated by regions correlations. ($r_{ne} = 0.455$, $r_{nc} = 0.315$, $r_s = 0.485$, $r_w = 0.581$)

4. **V** is a **V4** where the diagonal elements are the same as **D** but the covariance of 4 years apart is calculated by regions correlations. ($r_{ne} = 0.287$, $r_{nc} = 0.475$, $r_s = 0.548$, $r_w = 0.671$)

The smoothed estimate and its variance for estimating 1990 2-bedroom median rent of 11 MSAs are calculated. The performance of the smoothed estimates are compared with 1990 census figures according to 4 criteria and is tabulated in Table 1.

IV. Estimation of Two-bedroom Fair Market Rent by MSA using AHS-MS Data

To model the sample estimate of 2-bedroom Fair Market Rent, a special effort was made to calculate the direct sample estimate of 2-bedroom Fair Market Rent and its variance from the AHS-MS files. The Fair Market Rent (FMR) for a given bedroom size is the median rent of the occupied rental units that satisfy certain criteria as defined by HUD. Kim (1996) provided the direct sample estimate and its variance of the median rent for bedroom sizes 1-3 for rental housing units that satisfy HUD criterions via VPLX (Jackknife variance estimation) using 1986-1990 AHS-MS data files. In the following, the univariate variance component model is used to estimate 1990 2-bedroom fair market rent for MSAs in the 1990 AHS-MS survey using data of 1, 4 and 5 years duration.

IV.A. Using One Year's (1990 AHS-MS) Data

The 2-bedroom median rent for the 11 MSAs that satisfy HUD criteria is used as a dependent variable Y in (2.1). The X variables are the same as defined before.

The V defined in (2.1) is a diagonal matrix **D**. The sample variance estimate of Y's (via VPLX) is the diagonal element of **D** which we assumed known. The $\hat{\sigma}^2$ is obtained via maximum likelihood.

The fitted 2-bedroom median rent of the occupied rental housing units satisfying the HUD criteria (FMR) yielded the regression model below

$$X\hat{\beta}\ (FMR) = -67.2594 + 0.8085\ X_1 + 0.8033\ X_2 \qquad (4.1)$$
$$(95.852)\quad (0.241)\quad (0.615)$$
$$\hat{\sigma}^2 = 919.35 \quad R^2 = 0.91$$

The smoothing results of FMR are given in Table 2 which include the direct sample estimate (Y), and the smoothed estimate ($\hat{y}$) along with their variance estimate.

From Table 2., the smoothed estimate $\hat{y}$ of fair market rent (FMR) for 2-bedroom is better than the sample estimate, the increase in efficiency is from 1 to 16 percent over the direct sample estimate (or in an average of 7 percent).

VI.B. Using 4 Years' (1987-1990) worth of AHS-MS Data

The 44 MSA sample estimates of 2-bedroom median rent that satisfy HUD criteria (FMR) for MSAs in 1987-1990 AHS-MS surveys are used to estimate the 1990 2-bedroom fair market rent (FMR).

The estimated variance of the sample estimate Y via VPLX (Jackknife variance estimate) is used as the diagonal element of the sample covariance matrix D.

The fitted regression model is

$$X\hat{\beta}\ (FMR) = 16.8728 + 1.0944\ X_1 + 0.0438\ X_2 \qquad (4.2)$$
$$(62.54)\quad (0.149)\quad (0.379)$$
$$\hat{\sigma}^2 = 2058.01 \quad R^2 = 0.82$$

The smoothed estimate $\hat{y}$ along with their variance for 1990 2-bedroom fair market rent of MSAs in the 1990 AHS-MS survey are tabulated in Table 2. The increase in efficiency of the smoothed estimate over the direct sample estimate is from 5 to 15 percent (or an average of 7 percent).

VI.C. Using 5 Years' (1986-1990) worth of AHS-MS Data

The 55 sample estimates and their variance of 2-bedroom median rent that satisfy HUD criteria- the fair market rent (FMR), for MSAs in 1986-1990 AHS-MS surveys were obtained using VPLX. We smoothed these sample estimates to estimate the 1990 2-bedroom fair market rent for 11 MSAs using different covariance assumptions.

1) **V** = **D**, the sample variance of **Y** is used in the diagonal of **V**.

The fitted regression model is

$$X\hat{\beta}\ (FMR) = 24.199 + 1.103\ X_1 + 0.002\ X_2 \qquad (4c.1)$$
$$(51.98)\quad (0.127)\quad (0.316)$$
$$\hat{\sigma}^2 = 1874.91 \quad R^2 = 0.83$$

2) $\mathbf{V} = \mathbf{V}_1$, the sample variance of Y is in the diagonal of V_1 and the sample covariance of 2-bedroom fair market rent for the same MSAs in 1990 and 1986 is estimated by using the sample correlation estimate $r_2 = 0.311$.

The smoothed results in Table 2 showed that the variance of the smoothed estimate is a little better for the 5 years' data than the 4 years' data case (an average of 8 percent instead of 7 percent over the variance of the direct sample estimate). By comparing the smoothed results using a sample covariance **V** versus a diagonal case **D** and for 5 years' worth of data, the increase in efficiency of the smoothed estimate over the direct sample estimate is not much, an average of 9 percent instead of 8 percent. Notice that in the 5 years' worth of data case (55 MSAs), only 11 MSAs of 1986 and 1990 AHS-MS are correlated.

V. Summary

In this paper, we presented the results of an empirical study of the use of univariate and multivariate variance component models to estimate 1990 2-bedroom median rent for MSAs (in the 1990 AHS-MS surveys) using AHS-MS published data. We also presented the results of an empirical study of the use of univariate variance component models to estimate 1990 2-bedroom fair market rent for MSAs using the individual survey responses to compute sample estimates and their variance via VPLX.

We presented the results of estimating 1990 2-bedroom median rent of 11 MSAs by using a single year's worth of data (1990 AHS-MS), 4 years' worth of AHS-MS (1987-1990) data and 7 years' worth of AHS-MS (1984-1990) data. For the 7 years' worth of data we incorporated the covariance between estimates of 4 years apart. The auxiliary information used in the regression was the adjusted 1980 census figures (using the consumer price index) and the 1980 census data. Smoothed estimates derived from the variance component models (the weighted average of the sample estimate and the regression estimate) were used. The results of the smoothed estimates for 1990 2-bedroom median rent for 11 MSAs are compared with the 1990 census figures using 4 criteria (average relative bias, average squared relative bias, average absolute bias, average squared deviation).

The models used are the variance component models of univariate (Y_2- 2-bedroom median rent), bivariate models (Y_2 Y_1), (Y_2 Y_3), (Y_2 $0.6Y_1 + 0.4Y_3$) and 3-variate model (Y_1 Y_2 Y_3), where Y_i is the median rent of bedroom size i, i = 1, 2, 3. In all the models considered, the smoothed estimate from the bivariate model (Y_2 $0.6Y_1 + 0.4Y_3$) is the best, although the gain is not much in comparison with other models.

For 2-bedroom median rent, among all of the estimates considered, the smoothed estimate was best, the regression estimate was worst. However, the gain in the smoothed estimate is not much over the sample estimate. Since the model error is large in comparison with the sampling error of the direct sample estimate derived from the generalized variance in all the models considered; the smoothed estimate is very close to the sample estimate.

For the smoothed estimate, there is a little gain in using 4 years' worth of data than 1 year's data . This is because all 4 years of data are independent samples. There is no correlation among the 4 years' worth of data. For 7 years' worth of data (1984-1990) there is a recurrence of MSAs in the sample. By assuming stationary covariance of data 4 years apart, the sample correlation of 4 years apart estimated from 1985 and 1989 AHS-N, and 1987 and 1991 AHS-N were used to estimate covariances and incorporated in model estimation. However, there was little gain in the smoothed estimate of 2-bedroom median rent using sample covariance data as compared to the model which assumed independent samples over 7 years.

For 2-bedroom median rent all comparisons are based on the sample variance of the direct sample estimate of the median rent derived from the generalized variance. We found that the sample variance derived from the generalized variance is ⅓ to ½ of the sample variance derived from the VPLX. When the estimated sample variance of the median rent for 2-bedroom size is calculated via VPLX (Jackknifing) for the 1990 AHS-MS data, the average efficiency of the smoothed estimate of a 2-bedroom median rent over the sample estimate is 5 percent using VPLX variance instead of 2 percent by using generalized variance.

For fair market rent estimation, we used a univariate variance component model to estimate the 1990 2-bedroom fair market rent (for rental units that satisfy HUD's criteria) of MSAs in 1990 AHS-MS surveys using 1 year (1990 AHS-MS), 4 years (1987-1990 AHS-MS) and 5 years' worth (1986-1990 AHS-MS) data. All the direct sample estimate of fair market rent and their variances for 2-bedroom size were obtained using VPLX for 5 years (1986-1990) AHS-MS data.

The average efficiency of smoothed estimate of a 2-bedroom fair market rent over the sample estimate is 7 percent by using 1 year or 4 years of data; 8 percent by using 5 years data assuming independent covariance and 9 percent by using 5 years data with sample covariance.

It seems that using 4 years uncorrelated data (1987-1990) does not help much in the estimation of 1990 2-bedroom fair market rent. The regression estimate is worst of all 3 estimates for each MSA.

Since we don't have 1990 census fair market rent of different bedroom sizes, we didn't compare the smoothed estimate of the 1990 2-bedroom fair market rent with the 1990 census estimate of fair market rent.

In the current fair market rent estimation, for the non-MSA areas or the MSA's not in the current year sample, the Consumer Price Index (CPI) data are used for rents and utilities to adjust census or previous years' sample estimates. The HUD regional rent change factors developed from Random Digit Dialing telephone surveys are also used to adjust the base-year fair market rents. One alternative estimate of the fair market rent for MSAs not in the sample is to use the regression model derived from the sample to predict the fair market rent of MSAs not in the sample. However the regression estimate of the 2-bedroom fair market rent derived from the sample in this study is the worst of all the estimates considered. Further study is needed to see whether some socio-economic variables can be incorporated to improve the regression estimate.

This paper reports the general results of research undertaken by Census Bureau staff. The views expressed are attributable to the author (s) and do not necessarily reflect those of the Census Bureau.

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Table 1. Comparison of the Smoothed Estimates for the Different Models Using 1-year (1990) or 4 years (1987-1990) or 7 Years (1984-1990) Data to Estimate 1990 2-bedroom Median Rent of MSAs in 1990 AHS-MS

	Avg Rel Bias	Avg Sq Rel Bias	Avg Abs Bias	Avg Sq Dev
1990 sample estimate	0.0542	0.0041	26.05	1006.86
Adjusted 1980 Census	0.0547	0.0061	28.12	1662.03
Univariate (Y_2) model				
1 year-D	0.0540	0.0040	25.89	975.65
1 year-D (VPLX)	0.0534	0.0040	25.52	951.91
4 years-D	0.0536	0.0039	25.60	949.73
4 years-D (+yr effect)	0.0540	0.0040	25.76	958.01
7 years-D	0.05351	0.00391	25.56	944.49
7 years-V1 (r = 0.311)	0.05333	0.00387	25.45	932.90
7 years-V2 (r = 0.432)	0.05327	0.00386	25.41	928.58
7 years-V3 (region r's)	0.05317	0.00385	25.36	924.77
7 years-V4 (region r's)	0.05311	0.00384	25.33	922.20
Bivariate (Y_2 Y_1) model				
1 year	0.0535	0.0040	25.56	956.03
4 years	0.0530	0.0038	25.23	923.36
Bivariate (Y_2 Y_3) model				
1 year	0.0535	0.0039	25.54	944.98
4 years	0.0540	0.0040	25.94	998.29
Bivariate (Y_2 Y_c) model				
1 year	0.0528	0.0039	25.16	928.09
4 years	0.0534	0.0039	25.58	970.39
3-variate (Y_1 Y_2 Y_3) model				
1 year	0.0534	0.0039	25.55	944.31
4 years	0.0533	0.0039	25.46	954.18

Table 2. The Different Estimates of 1990 2-bedroom Median Rent That Satisfying HUD Criteria (FMR) for MSAs in 1990 AHS-MS Using AHS-MS Data (1 year (1990), 4 years (1987-1990) and 5 years (1986-1990)

			1 Year		4 Years		5 Years (V=D)		5 Years (V = V_1)	
	Y	V(Y) VPLX	$\hat{y}$	$V(\hat{y})$	$\hat{y}$	$V(\hat{y})$	$\hat{y}$	$V(\hat{y})$	$\hat{y}$	$V(\hat{y})$
1. Anaheim-Santa Ana, CA	830.34	498.47	825.03	471.38	829.16	425.66	828.68	416.31	828.29	416.55
2. Cincinnati, OH	441.10	221.43	441.60	192.40	442.24	201.95	442.39	199.83	442.47	198.72
3. Denver, CO	492.15	130.34	494.81	125.59	491.96	123.60	491.84	122.68	492.15	122.07
4. Kansas City, MO	449.24	144.74	452.59	133.37	451.14	136.17	451.31	135.21	452.35	134.71
5. Miami-Ft Lauderdale, FL	608.41	328.74	594.61	306.87	598.44	292.37	597.37	287.44	595.51	287.10
6. New Orleans, LA	432.02	136.24	430.85	125.42	431.63	128.60	431.62	127.71	430.86	126.86
7. Pittsburgh, PA	430.54	262.66	430.93	221.95	431.32	235.53	431.43	232.62	432.05	231.18
8. Portland, OR	456.72	120.70	462.60	119.63	458.80	114.83	458.93	114.09	459.58	113.94
9. Riverside-San Banardino-Ontario, CA	593.46	156.85	587.80	147.96	591.47	146.94	591.28	145.88	591.50	144.42
10. Rochester, NY	528.40	154.54	534.74	153.52	533.40	146.71	533.90	145.62	534.44	144.72
11. San Antonio,TX	434.68	127.85	429.60	124.92	432.90	121.48	432.79	120.65	431.30	120.27