CALIBRATION ESTIMATION IN THE 1991 AND 1996 CANADIAN CENSUSES

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1. 1986/1991 Census Estimation Techniques

In the Canadian Census, basic person and dwelling information is gathered on a 100% basis. This will be called 2A information after the 2A Census short form. For a 1 in 5 systematic sample of private households, additional questions are asked. This will be called 2B information after the 2B Census long form. In the 1991 Canadian Census, the 1 in 5 systematic sample of private households was selected from each of 40,072 enumeration areas (EAs) out of a total of 45,995 Eas. Sampled EAs contained on average 249 households (all references to households in this report indicate private households). The other 5,923 EAs (which were remote EAs, Indian reserves or EAs containing exclusively collective dwellings) were sampled 100%. The 1 in 5 sample (as will be shown later) is subject to small but significant biases.

A weight for each sampled household is calculated. This single weight is used to produce all published household and person 2B characteristic estimates. Published 100% counts of 2A information (which includes institutional residents) should agree closely with (and hopefully not be less than!) published estimates of 2A information (which do not include institutional residents) based on the 20% sample. Published estimates of 2A information appear in cross-tabulations of 2A information by 2B information. Large differences between the 100% counts and the 20% estimates cause concern to users of Census data. The Census estimation methodology is designed, therefore, to reduce or eliminate such population/estimate differences for small geographical areas. At the same time, the variances of the Census estimators are also reduced. The estimation methodology was designed to perform well for the thousands of published estimates generated with a minimum of manual intervention during the processing of Census data. Characteristics for which consistency is required between the sample estimate and the population count will be called constraints on the weights.

Sampled EAs are grouped into weighting areas (WAs). The weighting system is run separately in each WA. The 5,736 WAs formed in 1991 contained, on average, 7 sampled EAs that are geographically contiguous. WA boundaries must respect the boundaries of census divisions (CDs), and as far as possible, of census subdivisions (CSDs or municipalities), census tracts (CTs), and federal electoral districts (FEDs).

In the 1986 Census, raking ratio (RR) estimation generated sample weights that ensured agreement between certain sample estimates and known population counts at the weighting area (WA) level. Although RR estimators generally have smaller variances than estimators based on weights equal to the inverse of the probability of selection (see, for example, Brackstone and Rao 1979), certain problems have been documented with this weighting procedure. Small differences remained between some sample estimates and population counts because the RR iterative solution (as proposed by Deming and Stephan 1940) had not completely converged after 40 cycles. Also, while there was close or exact agreement between the sample estimates and population counts at the WA level, this was frequently not achieved at the EA level. In fact, usually the agreement with RR estimates was no better than for estimates calculated using weights which equalled the inverse of the probability of selection. Finally, because different weights were used to produce household and person estimates, this caused discrepancies between these estimates in certain cases. For the 1991 Canadian Census (and also the 1991 Brazilian Census), two step calibration or regression estimation was used. Two recent papers on this subject are Zieschang (1990) and Deville and Särndal (1992). Earlier papers include Stephan (1942), Friedlander (1961), Cassel, Särndal and Wretman (1976), Huang and Fuller (1978), Isaki and Fuller (1982), Wright (1983), and Särndal and Hidiroglou (1989). Renewed interest in calibration estimation was generated among practitioners by Luery (1986) and Bethlehem and Keller (1987). Examples of papers applying the calibration estimator are Alexander (1987), Copeland, Peitzmeier and Hoy (1987) and Lemaitre and Dufour (1987).

A primary objective of the 1991 estimation methodology was to generate household weights such that differences between known population counts and the corresponding estimates were reduced for small areas (EAs) while at the same time eliminating or reducing these population/estimate differences for larger areas (WAs). To achieve this objective, two adjustments were made to the initial household weights which equal the inverse of the probability of selection. The first adjustment was calculated separately for each EA. The constraints were partitioned into two groups. Weighting adjustment factors were calculated for each group of constraints using regression estimation. The two resulting sets of weighting adjustment factors were then combined to form an average weighting adjustment factor. When applied to the initial weights, the resulting average adjusted weights reduced but did not eliminate population/estimate differences at the EA level. Regression estimation was then applied a second time at the WA level using all the constraints. The weighting adjustment factors which resulted were applied to the average adjusted weights. The final adjusted weights eliminated the population/estimate differences at the WA level. The Census weights were adjusted in two steps because this made it possible to achieve reasonable consistency between sample estimates and population counts at the EA level. At the same time, the variance of the two step regression estimator was significantly lower than that of the 1986 Census estimator at the EA level and somewhat lower at the WA level. This was important because EAs are the basic building blocks for tabulations of larger geographical areas.

Methods were developed for discarding constraints which were linearly dependent (LD, i.e. redundant), nearly linearly dependent (NLD, i.e. they caused a large condition number for the regression weight matrix being inverted where the condition number of the matrix is the ratio of its largest eigenvalue to its smallest eigenvalue) or small (i.e. they applied to 20 or less households). In addition, constraints, if necessary, were discarded to ensure that all the adjusted weights were within the desired range [1,25]. Any weights outside this range were called outlier weights. If weights less than 1 or a large condition number occurred when all the constraints were used, a forward selection procedure was utilized starting with the two constraints which applied to the largest number of households. Constraints were then added sequentially in descending order of size. If weights less than 1 or a large increase in the condition number resulted from adding a constraint, it was discarded. Additional information on this estimation methodology is provided in Bankier, Rathwell and Majkowski (1992). Constraints were also dropped in 1986 but for somewhat different reasons.

This approach contrasts with that of Deville and Särndal (1992) who select distance measures such that the optimum weighting adjustment factors fall within a certain range. If the set of solutions that satisfy the constraints (regardless of distance measure) does not include a solution where all the adjustment factors fall within the desired range or if the iterative solution used does not converge, then some of the constraints must be dropped to find a solution.

Averages were calculated of the number of WA level constraints that were discarded per WA in 1991. Of the 62 WA level constraints applied in 1991 (see Appendix A), there were, on average, 7.1 WA level constraints discarded for smallness, 6.3 discarded for being LD, 8.0 discarded for being NLD and 6.9 discarded for causing outlier weights. This resulted in an average of 28.3 WA level constraints being discarded per WA. This left 33.7 constraints retained on average per WA that hence had a zero discrepancy. For each of the WAs, there were also constraints representing counts at the EA level. These constraints were the number of households at the EA level (HHEACT) and the number of persons at the EA level (PPEACT). On average per WA, HHEACT was discarded for 1.3 EAs and PPEACT was discarded for 2.1 EAs. Different constraints were eliminated in each WA. Because of this, some population/estimate differences remained for most constraints at higher geographical levels. Table 4 provides 1986 and 1991 population/estimate differences for some constraints at the Canada level.

A study was done comparing the absolute differences between sample estimates and population counts for 62 characteristics (49 of which were constraints in 1991) in 1991 and 1986 for various geographical levels. The results of the study are summarized in Table 1. The table contains the percentage of characteristics that had an R value within a certain range where

$$R=\left(\frac{\sum_{i=1}^{N_{91}}|\hat{X}_i^{91}-X_i^{91}|}{\sum_{i=1}^{N_{91}}X_i^{91}}\bigg/\frac{\sum_{i=1}^{N_{86}}|\hat{X}_i^{86}-X_i^{86}|}{\sum_{i=1}^{N_{86}}X_i^{86}}\right)*100$$

R is the ratio of 1991 to 1986 differences where X_i^{91} and X_i^{86} are respectively the 1991 and 1986 population counts for a characteristic. The sample estimate in 1991 based on regression weights is $\hat{X}_i^{91}$, while the sample estimate in 1986 based on RR weights is $\hat{X}_i^{86}$. R values were calculated for each of six geographic levels (EA, WA, CSD, CD, Province, and Canada). The sum of the absolute values of the population/estimate differences were calculated where N_{91} equals the number of areas for the particular geographical level in 1991 and N_{86} equals the number of areas for the particular

Table 1. Percentage of the Characteristics with R Values Falling in Certain Ranges, 1991 vs 1986

R	EA	WA	CSD	CD	Prov	Can
< 95	87	58	81	47	31	29
95-105	11	11	8	18	14	10
> 105	2	31	11	35	55	61

geographical level in 1986. An R value in the range of 95 to 105 means that the 1991 estimation system and 1986 estimation system performed equally as well. An R value less than 95 means that the 1991 system performed better than the 1986 system for that characteristic at that particular geographical level, while an R value greater than 105 means that it did worse.

Table 1 shows, for the 62 characteristics, that 87% of them had an R value less than 95% at the EA level. Only 2% (or one characteristic) had an R value greater than 105% at the EA level. The 1991 estimation system was effective at reducing the population/estimate differences at the EA level compared to the 1986 estimation system. The effectiveness of the 1991 estimation system, however, decreases as the geographical areas become larger. At the provincial and Canada levels, the percentage of characteristics having an R value greater than 105% is over 50%.

There are consistently smaller population/estimate differences for small areas in 1991 because the 1991 estimator has a smaller sampling variance than the 1986 estimator because of the two step approach. The consistently larger population/ estimate differences for large areas may be the result of the 1991 estimator controlling less well on the bias present in the sample. In addition, the use of a two step regression estimator in 1991 may have introduced more bias into the estimation procedure than the one step RR estimator did in 1986. It should be noted, however, that the 1991 differences in percentage terms at the higher geographical levels are still very small.

Small relative biases of the same sign at the WA level can become very significant at the Canada level. This can be easily demonstrated under some simplifying assumptions. Assume that $B(\hat{Y}_k)=b$ and $V(\hat{Y}_k)=\sigma^2$ for all K = 5,736 WAs where $\hat{Y}_k$ represents, for the kth WA, the sample estimator for the characteristic of interest. Also, define the relative bias at the WA level as $r=b/\sigma$. The variance at the Canada level would be $K\sigma^2$ while the bias is Kb for the Canada level estimator $\hat{Y}$. The square root of mean square error (MSE) at the Canada level is

$$\sqrt{MSE(\hat{Y})} = \sqrt{K^2b^2+K\sigma^2} = \sigma\sqrt{K}\sqrt{Kr^2+1}$$

Thus, a relative bias of r = 0.1 at the WA level can result in an increase by a factor of $\sqrt{Kr^2+1}=\sqrt{5,736*0.01+1}=7.6$ in the square root of the MSE of $\hat{Y}$ at the Canada level. Thus the impact of small biases at the WA level on the MSE of the estimator at the Canada level can be very significant.

2. 1996 Census Estimation

In the 1996 Census, the sampled EAs were formed into 5,932 WAs with an average population of 1,795 households. A WA contained, on average, 8 sampled EAs. Because of budget constraints, the 1991 weighting system was used in 1996 with very few changes to the software. The weighting system parameters (such as those which defined the constraints), however, could be modified. The objective was to keep a higher proportion of the constraints in 1996. Our work was made easier by the elimination of many of the 100% characteristics in the 1996 Census for budgetary reasons. The weighting system was run twice in 1996 because of problems noted with the first run.

In the first run, a total of 27 constraints (see Appendix A) were used at the WA level including 5 year age ranges, marital status, sex and household size. The constraint Marital Status Separated was not used because it was linearly dependent on the other marital status constraints. Similarly, the constraints Age 60 - 64 and Household Size 6+ were not used because they were linearly dependent on other constraints used. The constraint Household Size 1 was not used because it was dropped frequently during testing for NLD. To help retain constraints, the weights in 1996 were allowed to be in the range (0, 25]. This resulted in 0.17% and 0.49% of the households having weights in the ranges (0, 0.5) and [0.5, 1) respectively.

Of the 27 WA level constraints applied in the first run of the weighting system in 1996, there was on average, 0.06 WA level constraints discarded for smallness, 1.01 discarded for being LD, 0.51 discarded for being NLD and 0.41 discarded for causing outlier weights. This resulted in an average of 1.99 WA level constraints being discarded per WA. The constraints AGE4, AGE9, AGE14, TOTPERS and TPERGE15, however, were linearly dependent. (The constraint short form names are defined in Appendix A.) Thus, in each WA, one of the age constraints AGE4, AGE9, and AGE14 was discarded for LD without any impact on population/estimate differences. Thus, of the remaining 26 WA level constraints, only 0.99 of them was discarded on average. For the constraints representing counts at the EA level, HHEACT was discarded for 0.77 EAs and PPEACT was discarded for 1.64 EAs, on average, per WA.

The second run of the weighting system used an additional constraint, Common-Law Status = Yes. The first run did not use this constraint because problems with the common-law responses in 1991 precluded them being used for the testing of the weighting system. Also, there was not time to study the quality of 1996 common-law responses before the first run of the weighting system.

The other change made for the second run was to modify the initial weights. In the first run, the initial household weight equalled the EA household population count divided by the EA household sample count. In the second run, these initial weights were calculated at the EA level and then adjusted so that the estimated number of households of size 1, 2, 3, 4, 5 and 6+ agreed with the corresponding population counts at the WA level. These will be called poststratified initial weights. For a small number of households, the poststratified initial weights were larger than 20. The poststratified initial weights for these households were set to 20 to allow some "room" for adjustment by the two step regression process (where weights of size 25 or less were allowed).

Finally, the additional constraints Age 60-64 and Household Size 1 were used. Age 60-64 was used in an attempt to reduce the size of the population/estimate difference for that characteristic which was quite large in the first run because Age 60-64 was always dropped. Household Size 1 was used because it made the programming of the poststratified weights easier.

Of the 30 WA level constraints applied in the second run of the weighting system in 1996, there was on average, 0.06 WA level constraints discarded for smallness, 2.19 discarded for being LD, 1.58 discarded for being NLD and 0.39 discarded for causing outlier weights. This resulted in an average of 4.22 WA level constraints being discarded per WA. In each WA, however, one of the age constraints for persons under the age of 15 and one of the age constraints for persons 15 or over were discarded for LD without any impact on population/estimate differences. Thus, of the remaining 28 WA level constraints, 2.22 of them were discarded on average.

The increase in the number of constraints dropped over the first run is primarily the result of the use of the constraint Household Size 1 in the second run. This resulted in more household size constraints being discarded for being NLD. At the EA level, HHEACT was discarded for 0.78 EAs and PPEACT was discarded for 1.68 EAs, on average, per WA. In addition, 0.17% and 0.48% of the households had weights in the ranges (0, 0.5) and [0.5, 1) respectively.

A study was done, based on the second run, comparing the absolute differences between sample estimates and population counts for 33 characteristics (29 of which were constraints in 1996) in 1996 and 1986 for various geographical levels. The results of the study are summarized in Table 2.

Table 2. Percentage of the Characteristics with R Values Falling in Certain Ranges, 1996 vs 1986

R	EA	WA	CSD	CD	Prov	Can
< 95	94	64	94	64	52	49
95-105	0	15	0	15	21	18
> 105	6	21	6	21	27	33

The 1996 estimation system was effective at reducing the population/estimate differences at all geographical levels compared to the 1986 estimation system. This is a significant improvement over the results for the 1991 Census as displayed in Table 1.

Table 3 displays the population/estimate differences for the various constraints using the initial weights in the first production run and the poststratified initial weights in the second production run. With the first run, there are some large population/estimate differences. For example, there are overestimates of 52,402, 91,338 and 46,666 for the number of children (Age < 15), married persons and persons living in four person households respectively. There is also an underestimate of 77,834 in the number of persons living in either 1 or 6 person households. The rightmost column of Table 3 gives the ratio of the first run difference to the standard error of the estimator. The standard error was calculated under the simplifying assumption that a simple random sample of persons (or households for household constraints) was selected at the Canada level. The actual sample design is a stratified (by EA) sample of households selected under systematic sampling. Thus these standard errors should be viewed as rough approximations. With many of the ratios greater than 4, however, this suggests that the population/estimate differences are larger than can be explained by sampling variability. These results indicate that the Census sample is biased upwards in terms of married couples with children. It is hypothesised that this bias may be partially caused by some Census Representatives preferring to give 2B long forms to households that are easy to enumerate. It may also be the result of 2B long forms being converted to 2A short forms during processing if there is total non-response to the questions asked on a sample basis.

The population/estimate differences based on the two step regression weights for the various constraints used are provided in Table 4 for the two runs of the 1996 weighting system along with the corresponding population/estimate differences from the 1991 and 1986 Censuses. The first run was very successful at eliminating some of the large population/estimate differences that were present using the initial weights. For example, after the two step regression adjustments were applied, the population/estimate difference for married persons was reduced from 91,338 to 10. It was less successful at eliminating population/estimate differences related to household size because these constraints (especially HHSIZE5) were frequently discarded for being NLD and to a lessor extent for causing outlier weights. For example, the population/estimate difference for the number of persons living in 5 person households increased from 15,850 in Table 3 with the initial weights to 57,775 in Table 4 with the regression weights. Also, somewhat surprisingly, the characteristic Common-Law Status = Yes, which was not used as a constraint in the first run, had it population/estimate difference change from an underestimate of 1,404 to an overestimate of 46,646 when Tables 3 and 4 are compared.

For these reasons, it was decided to run the Census weighting system a second time with the constraint Common-Law Status = Yes added and with poststratified initial weights used. Table 3 shows that the second run poststratified initial weights reduced the population/estimate differences for age, marital status and (as would be expected) household size. Some discrepancies remained for household size because some poststrata had zero sample sizes at the WA level. Also, the large households grouped together in the HHSIZEG6 constraint are probably sampled at progressively lower rates as the household size increases. Males and persons age 15 or more, however, have larger population/estimate differences under poststratification. This has to be studied further. Overall, however, the benefits to poststratification are substantial with the sum of the absolute values of the population/estimate differences under the first run being 25% larger than the corresponding sum under the second run in Table 3.

Table 4 shows with the two step regression weights that the sum of the absolute values of the population/estimate differences are 95% and 167% larger for the first run of 1996 and for 1991 respectively compared to the corresponding sum under the second run in 1996. The sum of the absolute values of the population/estimate differences for the 5 year age ranges, however, is 30% lower under the first run of 1996 compared to the second run of 1996. Also, the number of persons aged 75 and over is underestimated by 2,377 in the first run while it is underestimated by 9,207 in the second run. The sum of the absolute differences for 5 year age ranges, however, is 170% larger for 1991 than the second run of 1996. The population/estimate difference for Common-Law Status Yes was reduced from 46,646 under the first run to 2,415 under the second run. Thus overall, we have done considerably better in terms of population/estimate differences with the second run of 1996.

3. Further Research for the 2001 Census

Regression estimation was used in 1991 and 1996, because its methodology is well known and well accepted. In addition, regression estimation has a non-iterative solution so there are no problems with lack of convergence. For the 2001 Canadian Census, however, some possible enhancements (as outlined below) will be studied.

Under calibration estimation (including regression), the initial weights are adjusted by as little as possible such that the constraints are satisfied. One reason this is done is that it is assumed that an estimator based on the initial weights is unbiased. Thus, if small changes are made to the initial weights under calibration estimation, the calibration estimator should be nearly unbiased. If, however, the sample picked is biased, then the estimator based on the initial weights will be biased and the calibration estimator is likely to be biased also. It is desirable, therefore, to modify the initial weights to adjust for major sources of bias, before applying the 30 constraints at the EA and WA level. This was why the initial weights were poststratified on household size at the WA level in 1996. Doing this, however, was only partially successful at removing bias. We will experiment with using calibration estimation to adjust the initial weights at the WA level for major sources of bias identified at the provincial level. Besides household size, the total number of households, persons and persons aged < 15 could be used as constraints on the adjusted initial weights.

Possible causes of the bias in the sample will also be studied. Initially, data analysis will be done to study the patterns of bias in the sample. Does it vary by province? Is it uniformly distributed or clustered in a minority of the EAs? How much does using weighting to adjust for non-response to the sample questions (by converting 2B forms to 2A forms) increase the bias? Would it be better to use imputation to deal with this non-response? Can the field procedures be modified to reduce the biases?

The methodology to discard constraints because they are small, nearly linearly dependent or cause weights outside the range 0 to 25 could be reassessed. Some modifications to the methods might allow fewer constraints to be discarded.

The use of estimators based on alternative distance measures such as the raking ratio estimator or the bounded logit estimator could be considered. Estimators of this type have been discussed by Deville and Särndal (1992), Darroch and Ratcliff (1972), Huang and Fuller (1978) and Rao (1992a). The raking ratio estimator always generates non-negative weights while the bounded logit estimator can generate weights within any specified range. It has been found, based on limited testing with 12 Census WAs using the 1991 Census constraints, that the iterative process used to generate these weights does not always converge. This may sometimes be the result of there being no solution which satisfies the constraints and has weights in the required range. Dropping some constraints (usually fewer than with the regression estimator) results in the iterative process converging. Given the small number of constraints dropped in the 1996 Census for causing outlier weights, however, the benefits of switching to one of these other estimators may be minimal.

Bardsley and Chambers (1984) and Dustan and Chambers (1986) as well as Rao (1992b, 1993) have suggested using ridge regression. Ridge regression allows the constraints to be approximately satisfied (some with greater precision than others) but at the same time lowers the condition number of the matrix being inverted to determine the ridge regression weights. Lowering the matrix condition number tends to reduce the number of negative weights generated. Ridge regression has been applied to 12 WAs. It was found, unless the constraints were significantly relaxed, that negative weights were generated and many constraints had to be dropped. When the constraints were significantly relaxed, few constraints were dropped but the population/estimate differences when summed over 12 WAs were larger than those achieved with the logit estimator. Rao (1992b) also suggested using quadratic programming to relax the constraints on the regression weights. Alternatively, the method applied at the EA level in the 1991/1996 Censuses to relax the constraints could be extended to the WA level as well.

The Generalized Estimation System (GES) at Statistics Canada (Hidiroglou, Carpenter and Estevao 1996) allows the use of a regression estimator with bounds on the weights. It also allows the constraints on the weights to be relaxed if there is no solution. Non-linear programming techniques are used to guarantee that the distance measure is minimized subject to the constraints applied and the weight range required. The GES may be tried with Census data.

Another method to reduce the size of provincial population/estimate differences would be to do a third step weighting adjustment at the provincial level to reduce or eliminate such differences. The matrix to be inverted at the provincial level equals the sum of the corresponding WA level matrices. It could therefore be accumulated as the second step adjustment factors are calculated. Having inverted the provincial level matrix, another pass would be made through the Census data to calculate adjusted weights for individual households. Any households with adjusted weights outside the range 0 to 25 would not have the adjustment done. This means that population/estimate differences would be reduced but not necessarily eliminated at the provincial level. The size of the differences would be a function of how many households had out of bounds weights generated by the third step adjustment. It is assumed that the third step weighting adjustment would be done using a regression estimator because this technique would be difficult to implement with the iterative solution required for the raking or bounded logit estimator. The effectiveness of this approach would have to be checked by applying it to one or more provinces. A third step adjustment would consume additional computational resources, though the cost of these is dropping over time.

Some consideration will also be given to increasing the average size of the WAs. This might allow more constraints to be retained. Doing this, however, would increase the size of population/estimate differences for small areas.

5. Concluding Remarks

Poststratifying the initial weights, allowing positive weights less than 1 and reducing the number of constraints applied to the Census weights in 1996 significantly improved the performance of the Census estimates in terms of population/estimate consistency compared to previous Censuses. Reduction of the bias in the sample, additional adjustments to the initial weights to deal with biases in the sample plus consideration of other approaches to estimation, may allow further improvements to be made for the 2001

Canadian Census.

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APPENDIX A

WA and EA Level Constraints Applied to 1991 Census Weights
With Short Form Names for the Constraints
1996 Census First Run Constraints Flagged with a "*"
1996 Census Second Run Additional Constraints Flagged with a "#"

Person WA Level Constraints

*TOTPERS	-Total persons
*TPERGE15	-Total persons aged ≥ 15
*MALE	-Males
*MALEGE15	-Males aged ≥ 15
*AGE4	-Persons aged 0 to 4
*AGE9	-Persons aged 5 to 9
*AGE14	-Persons aged 10 to 14
*AGE19	-Persons aged 15 to 19
*AGE24	-Persons aged 20 to 24
*AGE29	-Persons aged 25 to 29
*AGE34	-Persons aged 30 to 34
*AGE39	-Persons aged 35 to 39
*AGE44	-Persons aged 40 to 44
*AGE49	-Persons aged 45 to 49
*AGE54	-Persons aged 50 to 54
*AGE59	-Persons aged 55 to 59
#AGE64	-Persons aged 60 to 64
*AGE74	-Persons aged 65 to 74
*AGE75P	-Persons aged ≥ 75
*MARRIED	-Married persons
*SINGLE	-Single persons
*DIVORCED	-Divorced persons
*WIDOWED	-Widowed persons
SEP	-Separated persons
CENFAM	-Census families
NONMEMB	-Non-members of census families
HUSBAND	-Husbands
CHILD	-Census family children
LONEPARF	-Lone-parent females

EA Level Constraints

*HHEACT	-Total households in EA
*PPEACT	-Total persons in EA

Household WA Level Constraints

*TOTHHLD	-Total households
OWNED	-Owned dwellings
MALEHM	-Households with male household maintainer
SINGDET	-Single detached dwellings
MOVABLE	-Movable dwellings
APT5PL	-Apartments in a building with ≥ 5 storeys
OTHDWLS	-All other types of dwellings
#HHSIZE1	-Households of size 1
*HHSIZE2	-Households of size 2
*HHSIZE3	-Households of size 3
*HHSIZE4	-Households of size 4
*HHSIZE5	-Households of size 5
HHSIZEG6	-Households of size ≥ 6
AGEHM24	-Households with household maintainer aged ≤ 24
AGEHM34	-Households with household maintainer aged 25 to 34
AGEHM44	-Households with household maintainer aged 35 to 44
AGEHM54	-Households with household maintainer aged 45 to 54
AGEHM64	-Households with household maintainer aged 55 to 64
AGEHM74	-Households with household maintainer aged 65 to 74
AGEHM75P	-Households with household maintainer aged ≥ 75
FAMCHLD0	-Census families with no children at home
FAMCHLD1	-Census families with one child at home
FAMCHLD2	-Census families with two children at home
FAMCHLD3	-Census families with three children at home
FAMCHGE4	-Census families with ≥ four children at home
AGECLE5	-Census families with all children at home aged ≤ 5
AGEC614	-Census families with all children at home aged 6 to 14
AGEC1517	-Census families with all children at home aged 15 to 17
AGEC014	-Census families with some children at home aged ≤ 5 and rest aged 6 to 14
AGEC617	-Census families with some children at home aged 6 to 14 and rest aged 15 to 17
AGECLE17	-Census families with all children at home aged ≤ 17
AGECGE18	-Census families with all children at home aged ≥ 18
AGEC1718	-Census families with some children at home aged ≤ 17 and rest aged ≥ 18

Table 3:1996 Census Weighting System - Canada Level - Initial Weights					ca3yemb2.xls
	Difference : Sample Estimate - Population Count				
	Discrepancies : (Difference/Population Count) *100				
Constraint	1996 Census First Run Discrepancy	1996 Census First Run Difference	1996 Census Second Run Difference	1996 Census First Run Difference Divided by Standard Error	
AGE4	0.85	15,779	11,108	**6.01**	
AGE9	0.97	18,705	14,812	**6.98**	
AGE14	0.92	17,918	13,813	**6.69**	
Total abs. value for children		**52,402**	**39,733**		
% in Terms of 2nd Run Total Abs. Val.		**132**	**100**		
AGE19	0.25	4,709	133	1.81	
AGE24	-1.32	(24,353)	(27,002)	**9.30**	
AGE29	-0.91	(17,831)	(20,772)	**6.57**	
AGE34	-0.17	(3,979)	(8,408)	1.30	
AGE39	-0.16	(3,924)	(8,971)	1.25	
AGE44	0.23	5,251	1,894	1.89	
AGE49	0.44	9,004	5,856	3.33	
AGE54	0.52	8,267	5,152	3.45	
AGE59	-0.17	(2,135)	(3,612)	0.87	
AGE64	0.22	2,533	1,317	1.29	
AGE74	0.23	4,582	3,896	1.79	
AGE75PL	-0.92	(11,408)	(8,960)	**5.21**	
Total abs. value for adults		**97,977**	**95,971**		
Total abs. value		**150,379**	**135,705**		
% in Terms of 2nd Run Total Abs. Val.		**111**	**100**		
Divorced	-0.85	(13,606)	(12,289)	**5.49**	
Married	0.79	91,338	57,639	**17.78**	
Single	-0.29	(37,340)	(56,579)	**6.93**	
Widowed	-0.91	(11,803)	(4,342)	**5.26**	
Separated	-0.82	(5,472)	(4,171)	**5.49**	
COMLAW YES	-0.08	(1,404)	(9,470)	0.55	
Total abs. value		**160,963**	**144,491**		
% in Terms of 2nd Run Total Abs. Val.		**111**	**100**		
Hhsize1	****	****	(1)		
Hhsize2	0.36	12,060	-	3.62	
Hhsize3	0.27	4,772	(2)	1.89	
Hhsize4	0.64	11,666	(12)	**4.51**	
hhsize5	0.43	3,170	(308)	1.90	
Hhsizeg6	****	****	(1,219)		
Hhsize16	-1.12	(32,728)	****		
Total abs. value		**64,396**	**1,542**		
Difference for the Hhld Size Constraint in Terms of Number of People					
Hhsize1	****	****	(1)		
Hhsize2	0.36	24,120	-		
Hhsize3	0.26	14,316	(7)		
Hhsize4	0.64	46,666	(48)		
hhsize5	0.43	15,850	(1,540)		
Hhsizeg6	****	****	(18,145)		
Hhsize16	-1.63	(77,834)	****		
Total abs. value		**178,785**	**19,742**		
% in Terms of 2nd Run Total Abs. Val.		**906**	**100**		
Male	-0.17	(22,868)	(45,949)	**4.08**	
Malege15	-0.45	(48,269)	(64,684)	**9.17**	
Tperge15	-0.13	(29,285)	(59,476)	**6.36**	
Tothhld	-0.01	1,060	(1,542)		
Totpers	0.08	23,117	(19,742)		
Total abs. value		**124,599**	**191,393**		
% in Terms of 2nd Run Total Abs. Val.		**65**	**100**		
Overall total absolute value of the differences					
		614,726	**491,331**		
% in Terms of 2nd Run Total Abs. Val.		**125**	**100**		

Table 4:1996 Census Weighting System - Canada Level - Final Two Step Regression Weights					ca3yemb3.xls
	Difference : Sample Estimate - Population Count				
	Discrepancies : (Difference/Population Count) *100				
	1996 Census	1996 Census	1996 Census	1991 Census	1986 Census
Constraint	First Run	First Run	Second Run		
	Discrepancy	Difference	Difference	Difference	Difference
AGE4	0.00	34	(208)	(2,151)	260
AGE9	0.01	168	(258)	(1,789)	(123)
AGE14	-0.01	(205)	462	3,925	(124)
Total abs. value for children		**407**	**928**	**7,865**	**507**
AGE19	0.10	1,890	1,853	8,705	(678)
AGE24	0.08	1,501	803	4,890	852
AGE29	0.00	(49)	105	(8,762)	(3,543)
AGE34	0.01	248	361	580	1,801
AGE39	0.02	493	320	(3,777)	(3,463)
AGE44	0.01	232	366	1,277	3,029
AGE49	0.09	1,743	971	2,665	(1,199)
AGE54	0.06	959	993	3,122	558
AGE59	-0.02	(201)	254	1,639	2,416
AGE64	-0.29	(3,380)	3,847	1,005	270
AGE74	-0.05	(1,056)	(662)	(4,312)	4,991
AGE75PL	-0.19	(2,377)	(9,207)	(7,169)	(5,048)
Total abs. value for adults		**14,128**	**19,743**	**47,903**	**27,848**
Total abs. value		**14,535**	**20,671**	**55,768**	**28,355**
% in Terms of 2nd Run Total Abs. Val.		**70**	**100**	**270**	**137**
Divorced	0.06	951	1,209	(4,131)	3,813
Married	0.00	10	73	4,926	(7,716)
Single	0.00	239	115	2,040	679
Widowed	-0.10	(1,338)	(1,387)	(6,695)	2,873
Separated	0.02	137	(10)	3,708	351
COMLAW YES	2.63	46,646	2,415	****	****
Total abs. value		**49,321**	**5,209**	**21,500**	**15,432**
% in Terms of 2nd Run Total Abs. Val.		**947**	**100**	**413**	**296**
Hhsize1	****	****	(4,750)	(14,572)	48
Hhsize2	-0.01	(172)	(1,666)	3,249	8,670
Hhsize3	0.06	1,106	871	6,226	(964)
Hhsize4	0.07	1,205	1,694	6,158	2,913
hhsize5	1.57	11,555	5,576	9,029	(3,221)
Hhsizeg6	****	****	(1,725)	(10,092)	(7,446)
Hhsize16	-0.47	(13,694)	****	****	****
Total abs. value		**27,732**	**16,282**	**49,326**	**23,262**
Difference for the Hhld Size Constraint in Terms of Number of People					
Hhsize1	****	****	(4,750)	(14,572)	48
Hhsize2	-0.01	(344)	(3,331)	6,498	17,340
Hhsize3	0.06	3,317	2,614	18,678	(2,892)
Hhsize4	0.07	4,822	6,776	24,632	11,652
hhsize5	1.57	57,775	27,879	45,145	(16,105)
Hhsizeg6	****	****	(29,187)	(80,534)	(10,043)
Hhsize16	-1.38	(65,570)	****	****	****
Total abs. value		**131,826**	**74,537**	**190,059**	**58,080**
% in Terms of 2nd Run Total Abs. Val.		**177**	**100**	**255**	**78**
Male	0.00	-	15	(397)	(13)
Malege15	0.00	(213)	(276)	(1,023)	(38)
Tperge15	0.00	3	3	(137)	(13)
Tothhld	0.00	-	-	(1)	-
Totpers	0.00	-	-	(153)	-
Total abs. value		**216**	**294**	**1,711**	**64**
Overall total absolute value of the differences				In this overall total absolute value of the differences, the household size constraints are considered in terms of number of people.	
Total abs. value		**195,899**	**100,710**	**269,038**	**101,931**
% in Terms of 2nd Run Total Abs. Val.		**195**	**100**	**267**	**101**