ISSUES IN COMPLEX SAMPLING INVOLVING LATENT VARIABLES

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Abstract

In this paper we describe three alternative approaches to the estimation in large scale complex sampling situations involving latent variables, as typified by NAEP: Direct estimation, plausible values, and errors-in-variables regression. The advantages and disadvantages of each are discussed, emphasizing that different problems may require different approaches. We illustrate these approaches with an example, and discuss the implications for the future design of NAEP materials.

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Introduction

The plausible values technique developed as part of the National Assessment of Educational Progress (NAEP) program represent an important breakthrough in testing technology. It allows testing programs such as the NAEP and the forthcoming Third International Mathematics and Science Study to cover a much broader range of content that would otherwise be possible. Further, plausible values are very flexible, in the sense that they can be used by a secondary data analyst to "correctly" estimate the parameters of any statistical model that is consistent with the population model that is used in the generation of the plausible values. That model is a standard multivariate linear regression model that uses latent ability as the outcome variable and a large array of independent variables.

While plausible values are straightforward to use in secondary analysis, a number of substantive researchers are arguing that there use is excessively tedious. The program of research that we are undertaking is primarily concerned with two issues:

1. Is the population model used in the generation of plausible values appropriate when the plausible values are used in multi-level regression models, and
2. Do other approaches provide viable alternatives, at least in the context of linear multi-level regression?

In this paper we examine three alternative approaches that have been proposed for analyzing complex sample data from multilevel contexts that include substantial measurement error components in a dependent variable—in this case student "abilities" that are derived from a set of item responses.

The Item Response Model

For the item response model we will restrict our attention to the random coefficients multinomial logit (RCML) recently described by Adams and Wilson (in press). We use the RCML because it is a very general from of the Rasch model. In addition to this generality, the RCML inherits the fundamental measurement properties of the Rasch family. The general principles of our approach could, however, be applied with any item response model. As the item response model is not the focus of this paper, we will not describe the RCML in detail here, but direct the reader to Adams and Wilson (in press) instead.

We write the response vector probability model as

$$\Pr(\mathbf{X}=\mathbf{x}|\theta) \quad = \quad f_{\mathbf{x}}(\mathbf{x};\xi|\theta), \qquad (1)$$

where θ is person ability, ξ is a vector of item parameters, $\mathbf{X}$ is a vector random variable, and $\mathbf{x}$ is a realization of that random variable.

The Population Model

For the population model we will be specifying a two level linear model with normal distribution assumptions. In describing these models we will use the language of students nested within schools, as this is the focus of the NAEP context, although like all multi-level models they can be applied more generally. If we consider the case of S schools indexed $s=1,\dots,S$ with N_S students sampled from school s and θ_{ns} as the latent ability of person n in school s then a within school model is:

$$\theta_{ns} \quad = \quad \mathbf{Y}'_{ns}\beta_s + E_{ns}\,, \qquad (2)$$

where $\mathbf{Y}'_{ns}$ is a vector of u fixed predictors for person n in school s, β_S is a vector of u regression coefficients for school s and E_{ns} is a random error independent of $\mathbf{Y}'_{ns}$ with

$$E_{ns} \overset{iid}{\sim} N\left(0,\sigma^2\right). \qquad (3)$$

At the between school level we model variation in the school level regression coefficients with

$$\beta_s = \mathbf{W}_s \gamma + \mathbf{U}_s , \tag{4}$$

where γ, is a vector of uv regression coefficients, $\mathbf{W}_s = \omega_s' \otimes \mathbf{I}_u$ with ω_s' a vector of v school level regression coefficients and $\mathbf{I}_u$ is an identity matrix of dimension u.; and $\mathbf{U}_S$ is a random error term independent of $\mathbf{W}_S$ with

$$\mathbf{U}_s \overset{iid}{\sim} MVN(0, \Sigma). \tag{5}$$

In the present context it might be useful to think of the between schools variables $\mathbf{W}_S$ as being school characteristics such as "school size" or "average student SES".

It follows from (2) and (3) that

$$f_\theta(\theta_{ns}; \mathbf{Y}_{ns}, \sigma^2 | \beta_s) = (2\pi\sigma^2)^{-1/2} \times \exp\left[-\frac{1}{2\sigma^2}(\theta_{ns} - \mathbf{Y}_{ns}'\beta_s)'(\theta_{ns} - \mathbf{Y}_{ns}'\beta_s)\right], \tag{6}$$

which for convenience we will write as $f_\theta(\theta_{ns} | \beta_s)$;

and from (4) and (5) that

$$f_\beta(\beta_s; \mathbf{W}_s, \Sigma, \gamma) = |\Sigma|^{-1/2}(2\pi)^{-u/2} \times \exp\left[-\frac{1}{2}(\beta_s - \mathbf{W}_s\gamma)'\Sigma(\beta_s - \mathbf{W}_s\gamma)\right] \tag{7}$$

which for convenience we will write as $f_\beta(\beta_s)$.

Estimating the model

Plausible Values.

First we can use the NAEP plausible value methods developed at ETS by Mislevy and his colleagues (Mislevy , 1991; Mislevy, Beaton, Kaplan, and Sheehan, 1992). The plausible values approach is a multi-step approach. First a measurement model must be fit to the student item responses to produce item parameter estimates. A regression model with many independent variables is then combined with the measurement model to provide a posterior ability distribution for each student[1] (Johnson, Mazzeo and Kline, 1993). To ease the computation burden NAEP uses a two step approach where the parameters are first estimated without the use of conditioning variables and, in a second phase, the item parameters are fixed at their estimated values while the population parameters are estimated as an intermediate step in the generation of plausible values (Thomas, 1992). Multiple sets of random draws are then taken from the posteriors to produce the so called "plausible values" (5 is the standard number used by NAEP). Multiple runs of standard (including multi-level) regression software are then used to analysis the data using each of the sets of plausible values. The results of these multiple runs are then combined (see Zwick, 1992) to produce both an estimate and a standard error that reflects measurement error.

Thomas (1992) has reported on the accuracy of asymptotic corrections in evaluating the posterior, and has reported excellent results in standard (i.e., not multilevel) analyses. However, when we consider multilevel situations like the one described above, we have been concerned with another aspect of the plausible value generation--the disaggregation of school-level variables in the conditioning procedure. While it is plausible to expect that this will have no undue effects on a standard analysis, it would be reasonable to expect that there might be some consequences in a multilevel analysis. For example, at a recent professional meeting several researchers reported finding odd results when applying multilevel models to NAEP plausible values (Atash, Burgdorf, Chaney, and Williams (1995); Kaufman, Wilson & Adams, 1995; Lee, Croninger, and Smith (1995)).

Errors in Variables Regression

A second possible method for using latent outcome variables that have large error components in a hierarchical context is to develop a multi-level errors in variables regression model. This method is an extension of the so-called "v-known" approach developed by Raudenbush and Bryk (1985) and implemented in their computer program HLM (Bryk, Raudenbush, Selzter and Congdon, 1988). The method proceeds by adding an additional level to the bottom of the usual multi-level hierarchy. This level can be regarded as a measurement level (or model) since it represents how the latent ability estimate (or prediction) is related to its *true* value.

Suppose again that θ_{ns} is the *true* latent ability of person n in school s and that we have access to φ_{ns} a prediction of that ability where φ_{ns} is independently drawn from a normal "population" of possible values with mean θ_{ns} and known variance τ_{ns}^2. That is

$$\varphi_{ns} \overset{id}{\sim} N(\theta_{ns}, \tau_{ns}^2) \text{ or}$$

$$f(\varphi_{ns}; \tau_{ns}^2 | \theta_{ns}) \equiv f(\varphi_{ns} | \theta_{ns}) = (2\pi\tau_{ns}^2)^{-1/2} \exp\left\{-\frac{1}{2\tau_{ns}^2}(\varphi_{ns} - \theta_{ns})^2\right\} \tag{8}$$

We call (8) the within student, or measurement model. In specifying the between student within school and between school models we again use the normal hierarchy as above. Our efforts to apply the errors-in-variables idea to the problem of multilevel models have been documented in a series of papers. First, for a two-level model in Adams, Wilson and Wu (1992), then for a three-level model in Wilson and Adams (1994) and

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[1] In this context the independent variables are often called conditioning variables.

Adams and Wilson (1994). These studies revealed that the EAP estimator we have been using was not appropriate for the errors-in-variable approach, so we have concentrated our efforts recently on finding an unbiased estimator of the person ability. We have found that the so-called "Warm" estimator (Warm, 1989) solves at least some problems for dichotomous items (Roberts & Adams, 1995a; Adams, Roberts & Wilson, 1995), and have generalized it for polytomous items (Roberts & Adams, 1995b).

Estimating the Model in a Single Step

The third approach that we consider is the direct solution of the multi-level item response model (Adams, Wilson and Wu, 1993). Multi-level item response models combine the regression model of interest with the measurement model to produce a non-linear multi-level model that allows the direct estimation of population characteristics (e.g. variance components and regression coefficients) from item responses. This procedure should be seen as contrasting with the more typical usage of multi-level models in psycho-social applications, where the basic data analyzed consists of either raw or scaled scores. The use of such models for single level regression has was proposed by Mislevy (1984) and explored by Adams, Wilson and Wu (1993); in this paper we illustrate the extension the approach to multi-level regression. Because this approach produces estimates of all of the parameters of interest in a single analysis we call it a one-step procedure.

To estimate the model we are currently using a full maximum likelihood (or MLF) method. MLF is the standard approach to estimating item response models although restricted maximum likelihood (MLR) is generally preferred for the estimation of multi-level models. See Dempster, Rubin and Tsutakawa (1981) for discussion of the differences between MLF and MLR.

If we let $\{\mathbf{x}\}_s$ be the collection of all item response data for the students in school s and let $\{\mathbf{x}\}$ be the collection of item responses for all sampled students then it follows from (1), (6) and (7) that the likelihood of ξ, γ, σ^2 and Σ given $\{\mathbf{x}\}$ is

$$\Lambda\left(\xi,\gamma,\sigma^2,\Sigma \mid \{\mathbf{x}\}\right) = \prod_{s=1}^{S} f_{\mathbf{x}}\left(\{\mathbf{x}\}_s\right)$$

$$= \prod_{s=1}^{S} \int_{\beta_s} f_{\mathbf{x}}\left(\{\mathbf{x}\}_s \mid \beta_s\right) f_\beta(\beta_s)\, d\beta_s$$

$$= \prod_{s=1}^{S} \int_{\beta_s} \prod_{n=1}^{N_s} \int_{\theta_{ns}} f_{\mathbf{x}}(\mathbf{x} \mid \theta_{ns}) f_\theta(\theta_{ns} \mid \beta_s) f_\beta(\beta_s)\, d\theta_{ns}\, d\beta_s \quad (9)$$

where

$$f_{\mathbf{x}}\left(\{\mathbf{x}\}_s \mid \beta_s\right) \equiv f_{\mathbf{x}}\left(\{\mathbf{x}\}_s ; \xi,\gamma,\sigma^2,\Sigma \mid \beta_s\right)$$

$$= \prod_{n=1}^{N_s} \int_{\theta_{ns}} f_{\mathbf{x}}(\mathbf{x}_{ns} \mid \theta_{ns}) f_\theta(\theta_{ns} \mid \beta_s)\, d\theta_{ns}\,. \quad (10)$$

The log likelihood is then

$$\lambda \equiv \lambda\left(\xi,\gamma,\sigma^2,\Sigma \mid \{\mathbf{x}\}\right)$$

$$= \sum_{s=1}^{S} \log\left[\int_{\beta_s} f_{\mathbf{x}}\left(\{\mathbf{x}\}_s \mid \beta_s\right) f_\beta(\beta_s)\, d\beta_s\right]. \quad (11)$$

The computation necessary for maximizing this likelihood is extensive so we will illustrate the method using models that involve a limited number of school and student level variables.

A Demonstration

Consider a multilevel example of the following form:

$$\theta_{ns} = \beta_{0s} + \beta_{1s} Y_{1ns} + \varepsilon_{ns} \quad (12)$$

where, θ_{ns} refers to the ability of student n in school s, and, in order to make the example concrete, we consider Y_1 to be the student's gender (say, 0=male, 1=female). ε is assumed distributed as $N(0,\sigma^2)$. Suppose, furthermore, that these individual predictors are influenced by school-level variables of the following form:

$$\begin{aligned} \beta_{0s} &= \gamma_{00} + \gamma_{01} W_{1s} + U_{0s} \\ \beta_{1s} &= \gamma_{10} + \gamma_{11} W_{1s} + U_{1s} \end{aligned} \quad (13)$$

where, in order to make the example concrete, we might label W_1 as "Teacher Qualifications",. Assume that the school-level error variables are distributed thus:

$$\begin{bmatrix} U_{0s} \\ U_{1s} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{00} & 0 \\ 0 & \sigma_{11} \end{bmatrix}\right). \quad (14)$$

The simulated sample.

We chose values for the variables for the structural parameters--γ_{00}, γ_{01}, γ_{10}, γ_{11}, σ_{00}, σ_{11}, and σ^2-- in order to generate, through (12), (13) and (14), reasonable values of θ_{ns} according to our experience. These values are shown in the top row of Table 1. We generated θ_{ns} values for each of 30 students in 100 schools, with gender split evenly in each school, and with W_{1s} (Teacher qualifications) distributed normally with mean 0 and variance 1. In the second row of Table 1 are the estimates obtained by applying the HLM program (Raudenbush et al, 1988) to the generated values. These are provided to give some idea of the actual sample that was obtained, and should be the bais of comparison of the estimates. To generate the results in the remainder of the table, we used the generated θ values to in turn generate responses to four series of items according to the RCML model referred to above. The items are assumed to be dichotomous with item difficulties approximately uniformly distributed from -3.0 to 3.0. The number of items varied from 5 to 40. Typically, we would expect to see some improvement of variaance estimation results as the number of items increases.

For the plausible values, the RCML model was used with school variates disaggregated to the student level following the procedure used by NAEP. To make his more exact, note that the total equation implied by (12) and (13) is:

$$\theta_{ns} = \gamma_{00} + \gamma_{01}W_{1s} + (\gamma_{10} + \gamma_{11}W_{1s})Y_{1ns} + U_{0s} + Y_{1ns}U_{1s} + \varepsilon_{ns} \quad (15)$$

The effect of disaggregating the conditioning variables is to alter this equation to the following:

$$\theta_{ns} = \gamma_{00} + \gamma_{01}W_{1s} + (\gamma_{10} + \gamma_{11}W_{1s})Y_{1ns} + V_{ns} \quad (16)$$

Thus, one of the questions that we are interested in for this analysis is whether, when generating plausible values, the use of an equation of the form (16) rather than (15) might be a source of problems.

Results.

Plausible Values. Examination of columns 2 to 5 of the Plausible Values panel of Table 1 shows that the estimates of the school-level weights ("gammas") straddle the generated value, even with only five items. However, this is not the case for the estimates of variance. Looking now at columns 6 and 7, it is evident that all of the school level variance estimates are low, with improvement as the number of items increase, ranging from an approximate 80% underestimate for 5 items, to an approximate 15-40% underestimate for 40 items. The student level variance, in column 8, is consistently overestimated, with, once again, improvement for greater numbers of items.

Errors in Variables. Examination of the gamma estimates show a similar story as for the plausible values. The school level variances are considerably better, with the worst case here, for 5 items, being just about as good as the best case for the plausible values. Once again, they are underestimates, with the underestimation tending to decrease as the number of items increase. The underestimation ranges from about 15-30% for 5 items down to about 10-20% for 40 items. In contrast to the plausible values results, the results for the student level variance indicate that errors in variables approach results in an underestimation, with improvement as the number of items increase. The underestimation for 5 items is by about 80%, reducing to less than 10% for 40 items.

Direct Estimation. Examination of the gamma estimates show a similar story as for the plausible values and errors in variables. The school level variances show a different pattern for the two cases, an underestimate in one case, and an overestimate in the other. Both tend to improve as the number of items increase. The student variance is somewhat overestimated, without a clear trend by number of items; it is overall the best result of the three.

Discussion

Probably the most important results are those for plausible values: they are altogether quite disturbing, as this is our current "state-of-the-art". The underestimation of school level variance indicates that if we were to try and use these plausible values for secondary multilevel analysis, we would most likely find far fewer school level effects than is actually the case. This would be a possible explanation of the troublesome results mentioned earlier for HLM analyses of NAEP data during a symposium on "Multilevel modeling with NAEP data".. One way to look at the situation is to consider the variance-covariance matrix for the multilevel model given by (12) and (13). This is a block diagonal matrix, with the size of the blocks given by the number of students per school. Equation (16) would be the appropriate one if the variance-covariance matrix were diagonal. The conditioning used by NAEP maximizes the number of conditioning variables, in the hope of reducing equations (13) to "fixed" equations (i.e., reducing the variance of U_{is} to virtually zero). This will tend to make the values in the diagonal blocks constant, but it will not make the variance-covariance matrix itself diagonal. Using fewer students per school will tend to make the block diagonal matrix more like a diagonal matrix, but will only do so exactly when there is just one student per school for that outcome variable. Indeed, because of the matrix sampling design used by NAEP, there are relatively few students per outcome variable, but never just one except by accident.

One alternative strategy here would be to include the schools themselves as dummy variables in the multilevel model. We have tried this out with a similar, but slightly more complex, model (Adams, 1995). The results indicate that conditioning on the school dummy variables works well on the understimation of the intercept variance (σ_{00}), and on the student level variance (σ^2), but not very well on the other school level variance (in this case σ_{11}). This can be greatly improved by conditioning also on all the interactions between schools nad the school-level variables. However, although this is an interesting theoretical result, it is one that would be rather difficult to implement in a NAEP-like situation with many hundreds of schools. Note that these results are for a situation somewhat different to that for NAEP--the measurement model is different, we have drastically simplified the between school model, and our procedures are somewhat simpler than that for NAEP because of the simpler situation. However, we see no reason why the results should not extend to that more complicated situation. Another approach would be to cleave more closely to the original idea as expressed by Mislevy (1991), where he suggests that one should, instead of disaggregating of the school level variables, construct the correct multi-level model for the joint posteriors and draw the plausible values from those. We plan to carry out studies of the feasibility of this in the near future.

The results for the errors in variables approach are somewhat better than for the plausible values approach, at least for the school level variances. But they are still underestimated. And the results for the student level variance is quite poor for fewer items. We associate these difficulties with the problem of getting unbiased estimates of the student abilities when there are few items. The Warm estimator has helped with this, but further work is needed.

The results for the direct estimation method are rather disappointing. We would expect these to be the best available, but they are not. Clearly, further work is needed on improving our methods and computational techniques for this approach.

From these demonstrations, we can garner some points regarding the design and analysis of NAEP and other NAEP-like sample surveys. First, things certainly get better with increasing numbers of items, no matter how you analyse them. Thus, any technique that accomplishes that is worth trying out. Although we have only used dichotomous items in this simulation, we can note that polytomous items typically provide more information than dichotomous items (see, for example, Wilson and Wang (in press), where polytomous items provided between 3 and 4 times as much information as dichotomous items). Thus, replacement of dichotomous items with polytomous ones, where other costs are held constant, would be a good strategy. Unfortunately, the typical use of polytomous items in education has been with performance assessments which are considerably more expensive than are the multiple choice dichotomous item. Perhaps we need to provide more impetus for the development of machine-scorable polytomous items. A second stategy to increase the effective number of items is to take advantage of the correlations between the subscales which are typically used in such surveys. A multidimensional measurement model will allow student information on items on other subscales to contribute to the results for the subscale of interest, thereby increasing the effective number of items on any given subscale (this is the current strategy of choice for the Third International Mathematics and Science Study--Adams (1995)). Second, use of schools as conditioning variables can help--any strategy that fulfilled the purposes of the survey and reduced the number of schools so that it became feasible to do so, would be useful. Clearly this would need some consideration of the pros and cons, as reducing the number of schools and increasing the number of students per school will have a serious effect on the efficiency of the survey. However, for those surveys that have the functions of the school as focus (which are the ones where multilevel modeling is most important anyway), we will need to ensure that there is enough information collected inside each school to justify the rich and sophisticated analyses possible with multi-level modeling.

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Table 1
Results of the Demonstration

# Items	γ_{00}	γ_{01}	γ_{10}	γ_{11}	σ_{00}	σ_{11}	σ^2
Generating Value							
	0	0.3	0.3	0.1	0.2	0.2	0.8
Generated Value							
	0.0391	0.3081	0.2217	0.0573	0.1882	0.2512	0.7907
Plausible Values							
5	0.0455	0.3277	0.2541	0.0348	0.0335	0.0289	0.9639
10	0.0509	0.3213	0.2292	0.0356	0.0675	0.0441	0.9353
20	0.0513	0.2893	0.2195	0.0669	0.0994	0.0900	0.8977
40	0.0331	0.2851	0.2362	0.0706	0.1322	0.1477	0.8382
Errors-in-Variables							
5	0.0316	0.2710	0.2034	0.0263	0.1397	0.1722	0.1644
10	0.0403	0.2709	0.1918	0.0402	0.1439	0.1227	0.4126
20	0.0473	0.2643	0.2065	0.0723	0.1563	0.1739	0.6530
40	0.0286	0.2724	0.2315	0.0736	0.1658	0.2042	0.7166
Direct Estimation							
5	0.0347	0.3231	0.2660	0.0500	0.0001	0.6944	0.8953
10	0.0486	0.3187	0.2302	0.0411	0.1098	0.4494	0.7936
20	0.0502	0.2895	0.2220	0.0695	0.1013	0.4110	0.8068
40	0.0274	0.2907	0.2436	0.0658	0.1111	0.4050	0.8139