RESAMPLING METHODS FOR COMPLEX SURVEYS

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Abstract Resampling methods for variance and confidence interval estimation include the jackknife, balanced repeated replication (BRR) and the bootstrap. In this article, we present some recent theoretical work on the jackknife with post-stratified weights and Fay's modification of BRR, under stratified multistage sampling.

1. Introduction

Standard sampling theory is largely devoted to estimation of mean square error (MSE) of unbiased or consistent estimators $\hat{Y}$ of a population total Y. An estimator of MSE, or a variance estimator, provides us with a measure of uncertainty in the estimator $\hat{Y}$. Also, the standard error of $\hat{Y}$ (i.e., square root of estimated MSE), denoted by $s(\hat{Y})$, may be used to construct normal theory confidence intervals $\{\hat{Y} \pm z_{\alpha/2}s(\hat{Y})\}$, where $z_{\alpha/2}$ is the upper $\alpha/2$-point of a $N(0,1)$ variable. These intervals cover the true total Y with a probability of approximately $1-\alpha$ in large samples.

For nonlinear statistics $\hat{\theta}$, such as a post-stratified estimator of Y, or ratio, regression and correlation coefficients, the well-known Taylor linearization method is often used (see Binder, 1983 for some general results). Resampling methods, such as the jackknife, balanced repeated replication (BRR) and the bootstrap are also being increasingly used. In fact, several agencies in the U.S.A. and Canada have adopted the jackknife or the BRR for variance estimation in large-scale surveys. Resampling methods employ a single standard error formula for all statistics $\hat{\theta}$, unlike the linearization method which requires the derivation of a separate formula for each statistic $\hat{\theta}$. Moreover, linearization can become cumbersome in handling post stratification and nonresponse adjustments, whereas it is relatively straightforward with resampling methods. For example, current software packages using the linearization (e.g., SUDAN and PC CARP) seem to handle only totals, means and ratios under post-stratification. As a result, they cannot handle statistical analyses such as linear regression and logistic regression with post-stratified weights, unlike resampling software such as WESREG and WESLOG developed by WESTAT.

Rao, Wu and Yue (1992) provide a review of some recent work on resampling methods for complex surveys. In this article, we supplement their review by providing some new theoretical results. In particular, we study the jackknife with post-stratified weights and Fay's modification of BRR under stratified multistage sampling. For simplicity, we assume complete response on all items.

2. Stratified Multistage Sampling

Large-scale surveys often employ stratified multistage designs with large number of strata L, and relatively few primary sampling units (or clusters), $n_h(\geq 2)$, sampled within each stratum. We assume that subsampling within sampled clusters is performed to ensure unbiased estimation of cluster totals, Y_{hi}, $i=1,\ldots,n_h$; $h=1,\ldots,L$.

From the specification of the sampling design, basic weights $w_{hik}(>0)$ attached to the sample elements (ultimate units) hik are obtained. Using these basic weights, an unbiased estimator of the total Y is of the form

$$\hat{Y} = \sum_{(hik)\in s} w_{hik}y_{hik}, \tag{2.1}$$

where s is the sample of elements and y_{hik} is the value of the characteristic of interest associated with (hik).

It is a common practice to sample clusters without replacement with probabilities proportional to sizes (pps). However, at the stage of variance estimation, the calculation are greatly simplified by treating the sample as if the clusters are sampled with replacement. This approximation generally leads to overestimation of variance of $\hat{Y}$, but the relative bias will be small if the first-stage sampling fractions are not large.

An estimator of variance of $\hat{Y}$ is simply given by

$$\text{var}(\hat{Y}) = \sum_{n=1}^{L} \frac{1}{n_h(n_h-1)} \sum_{i=1}^{n_h} (r_{hi} - \bar{r}_h)^2 = v(r_{hi}), \tag{2.2}$$

where $r_{hi} = \sum_k (n_h w_{hik}) y_{hik}$ and $\bar{r}_h = n_h^{-1} \sum_i r_{hi}$. The operator notation $v(r_{hi})$ denotes that var $(\hat{Y})$ depends only on the r_{hi}s.

Often the basic weights w_{hik} are subjected to post-stratification adjustment to ensure consistency with known totals of post-stratification variables. In the case of a single post-stratifier, the weights w_{hik} are ratio-adjusted to known population counts (e.g., projected census age-sex counts). Suppose the population is partitioned into C post-strata with known population counts ${}_cM$, $c = 1, \ldots, C$. We use the prescript c to denote post-strata. An estimator of M_c is

$${}_c\hat{M} = \sum_{(hik) \in {}_cs} w_{hik},$$

where ${}_cs$ is the set of sample elements belonging to c-th post-stratum. Similarly, an estimator of the post-stratum total ${}_cY$ is

$${}_c\hat{Y} = \sum_{(hik) \in {}_cs} w_{hik} y_{hik}.$$

Using ${}_c\hat{Y}$ and ${}_c\hat{M}$, we obtain a post-stratified estimator of Y as

$$\hat{Y}_{ps} = \sum_c ({}_cM / {}_c\hat{M}) {}_c\hat{Y}$$

which may be rewritten as

$$\hat{Y}_{ps} = \sum_c \sum_{(hik) \in {}_cs} {}_cw_{hik} y_{hik}, \tag{2.3}$$

where ${}_cw_{hik} = w_{hik}({}_cM / {}_c\hat{M})$ is the ratio- adjusted weight for $(hik) \in {}_cs$. If y_{hik} is taken as the indicator variable for a post-stratum, say c, then $\hat{Y}_{ps} = {}_cM$, thus ensuring consistency with the known total ${}_cM$.

A customary Taylor linearization variance estimator is given by (2.2) with r_{hi} changed to

$$\tilde{r}_{hi} = \sum_c \sum_{k \in {}_cs} (n_h w_{hik}) {}_ce_{hik},$$

where ${}_ce_{hik} = y_{hik} - {}_c\hat{Y} / {}_c\hat{M}$ for the k-th element in the (hi)-th cluster belonging to ${}_cs$; i.e.,

$$\text{var}(\hat{Y}_{ps}) = v(\tilde{r}_{hi}). \tag{2.4}$$

Rao (1985) proposed a "robust" linearization variance estimator using the ratio-adjusted weights ${}_cw_{hik}$:

$$\text{var}_R(\hat{Y}_{ps}) = v(r^*_{hi}), \tag{2.5}$$

where

$$r^*_{hi} = \sum_c \sum_{k \in {}_cs} (n_h \, {}_cw_{hik}) {}_ce_{hik}.$$

In the special case of simple random sampling, (2.5) reduces to a conditionally valid variance estimator, given the post-strata sample sizes ${}_cn$, unlike the customary variance estimator (2.4). Särndal, Swensson and Wretman (1989) justify (2.5) under a model-assisted frame work appropriate for unistage sampling. In the context of ratio estimation under a model-dependent framework, Royall and Cumberland (1981) demonstrated the "robustness" of variance estimators of the form (2.5) with $C = 1$ and the associated jackknife variance estimators.

To handle two or more post-stratifiers with known marginal population counts, the basic weights are "calibrated" through generalized regression as in the Canadian Labour Force Survey. Deville and Särndal (1992) develop a family of calibration estimators that includes the generalized regression estimator.

Using indicator auxiliary variables to denote the categories of post-stratifiers, a generalized regression estimator of Y is given by

$$\hat{Y}_r = \hat{Y} + (\mathbf{X} - \hat{\mathbf{X}})'\hat{\mathbf{B}},$$

where $\mathbf{X}$ is the vector of population totals of auxiliary variables $\mathbf{x}_{hik}$, $\hat{\mathbf{X}} = \sum_{(hik)\in s} w_{hik}\mathbf{x}_{hik}$ and $\hat{\mathbf{B}}$ is the vector of estimated regression coefficients:

$$\hat{\mathbf{B}} = \Big[\sum_{(hik)\in s} w_{hik}\mathbf{x}_{hik}\mathbf{x}'_{hik}\Big]^{-1} \Big[\sum_{(hik)\in s} w_{hik}\mathbf{x}_{hik}y_{hik}\Big].$$

It is readily verified that $\hat{\mathbf{X}}_r = \mathbf{X}$, thus ensuring consistency with known totals $\mathbf{X}$. The estimator $\hat{Y}_r$ may be rewritten as

$$\hat{Y}_r = \sum_{(hik)\in s} w^*_{hik}y_{hik} \tag{2.6}$$

with $w^*_{hik} = w_{hik}a_{hik}$,

$$a_{hik} = 1 + \mathbf{x}'_{hik}\hat{\mathbf{A}}^{-1}(\mathbf{X} - \hat{\mathbf{X}})$$

and

$$\hat{\mathbf{A}} = \sum_{(hik)\in s} w_{hik}\mathbf{V}_{hik}$$

with

$$\mathbf{V}_{hik} = \mathbf{x}_{hik}\mathbf{x}'_{hik}.$$

A customary Taylor linearization variance estimator is given by

$$\mathrm{var}(\hat{Y}_r) = v(\tilde{r}_{hi}) \tag{2.7}$$

with $\tilde{r}_{hi} = \sum_k (n_h w_{hik})e_{hik}$ and $e_{hik} = y_{hik} - \mathbf{x}'_{hik}\hat{\mathbf{B}}$. Note that $\hat{\mathbf{B}}$ may be written as $\hat{\mathbf{A}}^{-1}\hat{\mathbf{U}}$ with

$$\hat{\mathbf{U}} = \sum_{(hik)\in s} w_{hik}\mathbf{u}_{hik}$$

and $\mathbf{u}_{hik} = \mathbf{x}_{hik}y_{hik}$ so that $\hat{\mathbf{B}}$ may be computed using only the formula for the basic estimator $\hat{Y}$.

3. Jackknife with Post-Stratification

To introduce the jackknife method, we first consider the estimator $\hat{Y}$ with basic weights w_{hik}. We need to compute the estimator $\hat{Y}_{(gj)}$ for each (gj) obtained from the sample after omitting the data from the j-th sampled cluster in the g-th stratum ($j = 1,\ldots,n_g; g = 1,\ldots,L$). It is simply obtained from (2.1) by using the following weights $w_{hik(gj)}$ in place of w_{hik}:

$$w_{hik(gj)} = \begin{cases} 0 & \text{if } (hi) = (gj) \\ [n_g/(n_g-1)]w_{gik} & \text{if } h = g \text{ and } i \neq j \\ w_{hik} & \text{if } h \neq g. \end{cases} \tag{3.1}$$

A jackknife variance estimator is then given by

$$\mathrm{var}_J(\hat{Y}) = \sum_{g=1}^{L} \frac{n_g-1}{n_g} \sum_{i=1}^{n_g} \Big[\hat{Y}_{(gj)} - \hat{Y}\Big]^2. \tag{3.2}$$

For general statistics of the form $\hat{\theta} = g(\hat{Y})$, a jackknife variance estimate is simply obtained from (3.2) by replacing $\hat{Y}_{(gj)}$ and $\hat{Y}$ with $\hat{\theta}_{(gj)} = g[\hat{Y}_{(gj)}]$ and $\hat{\theta}$. In the linear case, $\hat{\theta} = \hat{Y}$, (2.6) reduced to the "correct" variance estimator (2.2).

Turning to the post-stratified estimator $\hat{Y}_{ps}$, we need to recalculate the post-stratification weights ${}_cw_{hik}$ each time a cluster (gj) is deleted. This is done by using the jackknife weights $w_{hik(gj)}$ to calculate

$${}_c\hat{M}_{(gj)} = \sum_{(hik)\in {}_cs} w_{hik(gj)}$$

and then using ${}_c\hat{M}_{(gj)}$ to get

$${}_cw_{(hik)gj} = \Big[{}_cM/{}_c\hat{M}_{(gj)}\Big]w_{hik(gj)}.$$

Substituting these post-stratification jackknife weights in (2.3) we get $\hat{Y}_{ps(gj)}$ for each (gj). The resulting jackknife variance estimator is

$$\mathrm{var}_J(\hat{Y}_{ps}) = \sum_{g=1}^{L} \frac{n_g-1}{n_g} \sum_{j=1}^{n_g} \Big[\hat{Y}_{ps(gj)} - \hat{Y}_{ps}\Big]^2. \tag{3.3}$$

By assuming that no survey weight w_{hik} is disproportionately large as the number of strata, L, increases (see Krewski and Rao, 1981), Yung and Rao (1994) obtained a linearized version of the jackknife variance estimator (3.3) which is identical to Rao's linearization variance estimator (2.5) (see also Valliant, 1993). In the important special case of $n_h = 2$ clusters per stratum, (3.3) and (2.5) are in fact equal to higher order terms, as L increases. This result generalizes Rao and Wu's (1985) result for the basic estimator $\hat{Y}$. It may be noted that no repeated sampling arguments are invoked in establishing these results, such as ${}_cM/{}_c\hat{M} \rightarrow_p 1$.

Since the jackknife variance estimator (3.3) is intuitively "robust", the above results suggest that the linearization variance estimator (2.5) is also "robust". Computationally, (2.5) is much simpler than (3.3) and can be implemented using software packages that use linearization, such as PC CARP and SUDAAN. However, as noted in Section 1, current software using linearization cannot handle statistical analyses with post-stratification weights, unlike resampling software.

Valliant (1993) proposes a model appropriate for stratified multistage sampling, and shows that (2.5) and (3.3) appropriately estimate the conditional (model) variance. He also presents simulation results, using data from the U.S. Current Population Survey, that support his theory.

Yung and Rao (1994) also consider a generalized regression estimator to handle several post-stratifiers, and obtain a jackknife variance estimator and its linearized version. For the jackknife method we need to recalculate the calibration weights w^*_{hik} each time a cluster (gj) is deleted. These weights are given by

$$w^*_{hik(gj)} = w_{hik(gj)} a_{hik(gj)}$$

with

$$a_{hik(gj)} = 1 + \mathbf{x}'_{hik}\hat{\mathbf{A}}^{-1}_{(gj)}(\mathbf{X} - \hat{\mathbf{X}}_{gj}),$$

$$\hat{\mathbf{A}}_{gj} = \sum_{(hik)\in s} w_{hik(gj)}\mathbf{V}_{hik}$$

and

$$\hat{\mathbf{X}}_{gj} = \sum_{(hik)\in s} w_{hik(gj)}\mathbf{x}_{hik}.$$

Denote the resulting generalized regression estimator as

$$\hat{Y}_{r(gj)} = \sum_{(hik)\in s} w^*_{hik(gj)} y_{hik}$$

$$= \hat{Y}_{(gj)} + (\mathbf{X} - \hat{\mathbf{X}}_{(gj)})'\hat{\mathbf{B}}_{(gj)},$$

where $\hat{\mathbf{B}}_{(gj)}$ is the vector of estimated regression coefficients when (gj)-th cluster is deleted.

The jackknife variance estimator of $\hat{Y}_r$ is

$$\text{var}_J(\hat{Y}_r) = \sum_{g=1}^{L} \frac{n_g - 1}{n_g} \sum_{j=1}^{n_g} \left[\hat{Y}_{r(gj)} - \hat{Y}_r\right]^2. \quad (3.4)$$

Linearizing (3.4), Yung and Rao (1994) obtained a robust linearization variance estimator

$$\text{var}_R(\hat{Y}_r) = v(r^*_{hi}) \quad (3.5)$$

with $r^*_{hi} = \sum_k (n_h w^*_{hik}) e_{hik}$. It is interesting to note that (3.5) is similar to the model-assisted variance estimator of Särndal et al. (1989) in the context of a superpopulation model appropriate for unistage sampling.

4. Fay's Modification of BRR

McCarthy (1969) proposed the method of BRR for the important special case of $n_h = 2$ clusters per stratum. A set of R balanced half-samples (replications) is formed by deleting one cluster from the sample in each stratum. This set may be defined by an $R \times L$ design matrix (δ^r_h), $1 \leq r \leq R$, $1 \leq h \leq L$ with $\delta^r_h = +1$ or -1 according as whether the first or second sample cluster in the h-th stratum is in the r-th half-sample, and $\sum_r \delta^r_h \delta^r_{h'} = 0$ for all $h \neq h'$, i.e, the columns of the matrix are orthogonal. A minimal set of R balanced half-samples may be constructed from Hadamard matrices ($L + 1 \leq R \leq L + 4$) by choosing any L columns, excluding the column of $+1's$.

Let $\hat{\theta}^{(r)}$ be the estimator of θ obtained from the r-th half-sample. Note that $\hat{\theta}^{(r)}$ is obtained from $\hat{\theta}$ by changing w_{hik} to $2w_{hik}$ or 0 according as the (hi)-th cluster is selected or not selected in the half-sample. (For simplicity, we consider only the basic weights w_{hik}). A BRR variance estimator of $\hat{\theta}$ is given by

$$\text{var}_{\text{BRR}}(\hat{\theta}) = \frac{1}{R}\sum_{r=1}^{R}[\hat{\theta}^{(r)} - \hat{\theta}]^2. \quad (4.1)$$

In the linear case, $\hat{\theta} = \hat{Y}$, (4.1) reduces to the "correct" variance estimator (2.2). Krewski and Rao (1981) establish the asymptotic consistency of $\text{var}_{\text{BRR}}(\hat{\theta})$ for smooth statistics $\hat{\theta} = g(\hat{Y})$, as L increases. Shao and Wu (1992) and Shao and Rao (1994) establish similar results for nonsmooth statistics, such as the median $\hat{M} = \hat{F}^{-1}\left(\frac{1}{2}\right)$ and the low income proportion $\hat{\theta} = \hat{F}^{-1}(\hat{M}/2)$, where $\hat{F}(t)$ is the estimator of the population distribution function $F(t)$:

$$\hat{F}(t) = \sum_{(hik)\in s} w_{hik} I(y_{hik} \le t) / \sum_{(hik)\in s} w_{hik}.$$

Limited simulation results (Rao et al., 1992) suggest that the delete-1 cluster jackknife might also perform well for nonsmooth statistics as the number of sample elements in a cluster increases, but no theoretical results are at present available.

A drawback of the BRR variance estimator (4.1) is that occasionally one or more replicate estimators $\hat{\theta}^{(r)}$ are undefined due to division by zero, e.g., in estimating the ratio of two domain totals. Judkins (1990) discusses other disadvantages of BRR. These disadvantages are primarily due to sharp perturbation of the weights w_{hik}; all weights are either multiplied by 2 or by 0. The jackknife avoids these problems by only dropping one sample cluster at a time.

Fay (see Dippo, Fay and Morganstein, 1983, Sec.4) proposes a compromise between the standard BRR and the jackknife by perturbing the weights by $1+\varepsilon$ and $1-\varepsilon$ for the half-sample and its compliment $(0 < \varepsilon \le 1)$. Thus, $\hat{\theta}^{(r)}(\varepsilon)$ is obtained from $\hat{\theta}$ by changing w_{hik} to $w_{hik}(\varepsilon) = (1+\varepsilon)w_{hik}$ if (hik)-th element is selected in the r-th half-sample or to $w_{hik}(\varepsilon) = (1-\varepsilon)\ w_{hik}$ if it is not selected. A modified BRR variance estimator is given by

$$\text{var}_{\text{BRR}(\varepsilon)}(\hat{\theta}) = \frac{1}{\varepsilon^2 R}\sum_{r=1}^{R}[\hat{\theta}^{(r)}(\varepsilon) - \hat{\theta}]^2. \quad (4.2)$$

Note that (4.2) reduces to the standard BRR variance estimator (4.1) when $\varepsilon = 1$.

Rao and Shao (1994) obtained the following theoretical results on the modified BRR.

Result 1. In the linear case, $\hat{\theta} = \hat{Y}$, $\text{var}_{\text{BRR}(\varepsilon)}$ $(\hat{Y}) = \text{var}(\hat{Y})$ for any ε.

Note 1. Judkins (1990) also established Result 1.

Result 2. Suppose $\hat{\theta} = g(\hat{Y})$ and assume that $g(\cdot)$ is continuously differentiable in a neighbourhood of $\hat{Y}$. Then

$$\lim_{\varepsilon\to 0^+} \text{var}_{\text{BRR}(\varepsilon)}(\hat{\theta}) = \text{var}_L(\hat{\theta}),$$

where $\text{var}_L(\hat{\theta})$ is the Taylor linearization variance estimator of $\hat{\theta}$.

Note 2. Result 2 is similar to the result in the i.i.d. case that the infinitesimal jackknife = Taylor linearization (Efron, 1982).

Result 2. Consider $\varepsilon = \varepsilon_n$, a function of n satisfying $0 < \varepsilon_n \le 1$ and let $\hat{g} = g(\hat{Y})$. Assume the following regularity conditions:

$(C.1)\ \max_{i,k} n_{hi}\tilde{w}_{hik} = O(n^{-1})$

$(C.2)\ 0 < \liminf[n\text{var}(\hat{Y})]$

$(C.3)\ \sum_h \sum_i \frac{1}{n_h} E|y_{hi} - E(y_{hi})|^4 = O(n^{-3}),$

where n_{hi} is the number of elements sampled from (hi)-th cluster, $y_{hi} = \sum_{k=1}^{n_{hi}} y_{hik}\tilde{w}_{hik}$, $n = \sum_h \sum_i n_{hi}$, $\tilde{w}_{hik} = w_{hik}/N_0$ and N_0 is the total number of elements in the population. Further, assume that $g(\cdot)$ is twice differentiable with $\nabla^2 g(\cdot) \ne 0$ in a neighbourhood of $\hat{Y}$. Then

$$\text{var}_{\text{BRR}(\varepsilon)}(\hat{\theta}) = \text{var}_L(\hat{\theta}) + \varepsilon_n O_p(n^{-3/2}) + \varepsilon_n^2 O_p(n^{-2}). \quad (4.3)$$

Note 3. C.1 means no survey weight is disproportionately large. C.3 is a Liapunov-type condition on the $2+\delta$ moment with $\delta = 2$.

Note 4. If we choose $\varepsilon_n = n^{-\frac{1}{2}}$, then it follows from (4.3) that

$$\text{var}_{\text{BRR}(\varepsilon)}(\hat{\theta}) = \text{var}_L(\hat{\theta}) + O_p(n^{-2}),$$

i.e., the two variance estimators are closer to each other compared to the case of fixed $\varepsilon_n = \varepsilon$.

In the case of sample quantils, $\hat{\theta} = \hat{F}^{-1}(p)$, $0 < p < 1$. consistency of the modified BRR variance estimator for any fixed $\varepsilon (0 < \varepsilon \leq 1)$ follows along the lines of Shao and Rao (1994), by noting that the weights $\tilde{w}_{hik}^{(r)}(\varepsilon) = w_{hik}^{(r)}(\varepsilon)/N_0$ satisfy the following conditions:

(1) $E\Big[\sum_{hik} \tilde{w}_{hik}^{(r)}(\varepsilon) I(y_{hik} \leq t)\Big] = F(t)$ for any r and t.

(2) $\max_{hik} n_{hi}\tilde{w}_{hik}^{(r)}(\varepsilon) = O(n^{-1})$ uniformly in r and ε.

(3) $0 \leq w_{hik}^{(r)}(\varepsilon)$. That is, for any weights satisfying (1)–(3), the proof in Shao and Rao (1994) goes through. Similar result hold in the case of low income proportion $\hat{\theta} = \hat{F}(\hat{M}/2)$ for any fixed $\varepsilon(0 < \varepsilon \leq 1)$.

Judkins (1990) conducted a simulation study on the modified BRR. His empirical results suggest that ε in the range of 0.5 to 0.7 is a good choice, and that with this choice the modified BRR works well for both smooth and nonsmooth statistics or when a few degrees of freedom are available for variance estimation. Our results provide theoretical support to these empirical results. Further work on the modified BRR, especially with post-stratified weights, would be useful.

5. Concluding Remarks

We assumed complete response on all items, but both unit and item nonresponse often occur in practice. Deterministic or hot deck imputation within imputation classes is often employed to handle item nonresponse. It is also a common practice to treat the imputed values as if they are true values and then compute the variance estimates using standard formulas. This procedure, however, could lead to serious underestimation of true variance when the proportion of missing values for an item is appreciable. Assuming uniform response within imputation classes, Rao and Shao (1992) obtained a consistent jackknife variance estimator for stratified multistage surveys by first adjusting the imputed values for each pseudo-replicate and then applying the standard jackknife formula. Rao (1993) obtained linearization versions of this jackknife variance estimator under different imputation schemes. The above results apply only to the basic weights, w_{hik}. A Ph.D. student, W. Yung, is currently extending these results to the important case of post-stratification cutting across imputation classes.

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