ESTIMATION AND ANALYSIS OF DESIRED FAMILY SIZE WITH WFS DATA

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KEY WORDS: Desired family size, Fertility, Synthetic Cohort, World Fertility Survey

1. INTRODUCTION

The development of all probability models of desired family size is based on the synthetic, stationary fertility framework (Udry & Chase, 1973; Pullum, 1979; Lightbourne, 1977; Rodriguez & Trussell, 1981 and Nour, 1983). In the present paper, parameters and assumptions of the synthetic fertility population are discussed. Our main objective is to present a new procedure that yields the estimates of (i) the largest parity level, (ii) the mean of desired family size, (iii) the fertility preference implementation index; and (iv) marginal and joint distributions of these estimates. The proposed procedure has the following advantages: (i) the parameters can be estimated separately with closed form expressions that do not require the use of numerical algorithms, (ii) the estimates are consistent, and (iii) the same expressions for the estimators can be obtained under either the moment method or the maximum likelihood estimation technique. Some simple examples are used to demonstrate the application of the proposed procedure.

2. THE STATIONARY FERTILITY MODEL

Suppose that the fertility survey covers the entire population and the fertility behaviours of the population are unchanged during the time under investigation.

2.1. Notations

i = Parity index (i = 0, 1, ...)

Variables and parameters:

K = Maximum possible family size
X = Actual family size
Y = Desired family size
ε = Proportion of women, out of the total fertile population, who have fully implemented their fertility preferences = The fertility preference implementation index.

The population means of X, Y and the conditional mean of Y given X, $\mu_{Y|X}$, are denoted as μ_X, μ_Y and $\mu_{Y|X}$; and their estimates as μ_X, μ_Y, $\mu_{Y|X;}$ respectively. The estimates of K and ε are denoted as k and ε, respectively.

Statistics:

n = Total number of women
n_i = Number of women of parity i
l_i = Number of women of parity i who wanted their last child
m_i = Number of women of parity i who want more children

Initial constraints:

$$n = \Sigma^{k+1}_{i=0} n_i \quad (2.1)$$
$$l_0 = n_0 \quad (2.2)$$
$$m_k = 0 \quad (2.3)$$
$$n_i \geq l_i \geq mi \quad (2.4)$$
$$0 \leq X \leq k \text{ and } 0 \leq Y \leq k \quad (2.5)$$

2.2. Basic Assumptions

(A1) The distribution of Y, P(Y=i), is representative of the ith desired family size of the synthetic population. At each parity level, the desired family size (Y) is not dependent of ε, the implementation index.
(A2) The same implementation index (ε) applies to all members of the synthetic population. The actual family size (X) at each parity level is dependent on ε.
(A3) The parity levels (i = 0, 1, ..., K) represent K equal time intervals between births.
(A4) Values of n_i, m_i and l_i are reported accurately.
(A5) The maximum number of family size (K) is finite.

3. THE RELATIONSHIP BETWEEN ACTUAL FAMILY SIZE (X) AND DESIRED FAMILY SIZE (Y)

In Nour (1983), the number of women having i children (n_i) is a sum of three subgroups that can be summarized succinctly as:

$$n_i = n(1-\hat{\varepsilon}) + n\hat{\varepsilon}\cdot p(Y>i) + n\hat{\varepsilon}(k+1-i)\cdot p(Y=i).$$

In the first group, there are $n(1-\hat{\varepsilon})$ women, out of n_i, who do not implement the assumed fertility preference and move on to the next parity level. In the second group, there are $n\hat{\varepsilon}\cdot p(Y>i)$ women who have not fully implemented their fertility preference and also move on. Finally, the are $n\hat{\varepsilon}(k+1-i)\cdot p(Y=i)$ women in the third group who have obtained their desired family size and stay at the ith parity level. In Panel a of Figure 1, the components of these subgroups are used to count, at the ith parity level, the number of women who liked their last child (l_i) and the number of those who want more children (m_i). Once the observed values of l_i and m_i have been collected, they are in turn used to determine $p(Y=i)$.

Insert Figure 1 about here

In the present paper, the estimation procedure is constructed on the partition of the total fertile population into subgroups basing on the relative comparison of actual family size (X) against desired family size (Y) as depicted in Panel b of Figure 1. The formulation under this framework is explained below.

3.1. Grouping of women in the joint X and Y Space

The entire fertile population can be partitioned into three subgroups depending on the relative magnitudes of X and Y as follows:

Group 1 ($X< Y$): with the probability of membership equal to $\pi_1 = P(X < Y \mid \varepsilon)$

Group 2 ($X=Y$): with the probability of membership equal to $\pi_2 = P(X = Y \mid \varepsilon)$

Group 3 ($X>Y$): with the probability of membership equal to $\pi_3 = P(X > Y \mid \varepsilon)$.

Lemma 1. The probabilities of group memberships in the (X,Y)-space are specified as:

$$\pi_1 = \mu_Y/(K + 1) \quad (3.2)$$

$$\pi_2 = \{1/(K + 1)\}\{(K - \mu_Y)\varepsilon + 1\} \quad (3.3)$$

$$\pi_3 = (K - \mu_Y)(1 - \varepsilon)/(K + 1) \quad (3.4)$$

Proof. Basing on the relationship between Y and X specified by Eq. (20) in Nour (1983), and due to the constraint of $\Sigma\pi_i = 1$, the following results can be obtained:

$$\pi_1 = \Sigma^K_{i=0} P(X<Y \mid Y=i,\varepsilon)\cdot P(Y=i)$$

$$= \Sigma^K_{i=0} \{i/(K+1)\}\cdot P(Y=i)$$

$$\pi_2 = \Sigma^K_{i=0} P(X=Y \mid Y=i,\varepsilon)\cdot P(Y=i)$$
$$= \Sigma^K_{i=0}\{\varepsilon(1 - i/(K+1)) + (1-\varepsilon)/(K+1)\}\cdot P(Y=i)$$

$$\pi_3 = \Sigma^K_{i=0} P(X>Y \mid Y=i,\varepsilon)\cdot P(Y=i)$$

$$= \Sigma^K_{i=0} (K-i)(1-\varepsilon)/(K+1))\cdot P(Y=i)$$

Upon simplifying the above equations, the expressions (3.2) to (3.4) hold.■
(See Panel b, Figure 1). As an outcome of this grouping, the following result can be obtained.

Lemma 3. The methods of moment and maximum likelihood estimation
yield the same estimates for π_i, $\{i =1, 2, 3\}$, that can be specified as:

$$\hat{\pi}_1 = \Sigma^K_{i=0} m_i/\Sigma^K_{i=0} n_i, \quad (3.6)$$

$$\hat{\pi}_2 = \Sigma^K_{i=0} (l_i - m_i)/\Sigma^K_{i=0} n_i, \text{ and} \quad (3.7)$$

$$\hat{\pi}_3 = \Sigma^K_{i=0} (n_i - l_i)/\Sigma^K_{i=0} n_i. \quad (3.8)$$

Proof. From the partitioning of the (X,Y)-space, we have $\pi_1 = E[\Sigma^K_{i=0} m_i]/n$, $\pi_2 = E[\Sigma^K_{i=0} (l_i - m_i)]/n$ and $\Sigma^K_{i=0} \pi_i = 1$. Therefore, the moment estimates of

π_i, {i = 1, 2, 3}, are derived as given in (3.6) to (3.8), respectively.

To obtain the maximum likelihood estimates, consider the following likelihood function:

$$\mathcal{L}(l_i, m_i, n_i) = \frac{(\Sigma^K_{i=0} n_i)!\{\pi_1^{\Sigma m_i} \pi_2^{\Sigma l_i - m_i} \pi_3^{\Sigma n_i - l_i}}{(\Sigma^K_{i=0} m_i)!\{\Sigma^K_{i=0}(l_i - m_i)\}!\{\Sigma^K_{i=0}(n_i - l_i)\}!}. \quad (3.9)$$

The maximum likelihood estimates for π_1 and π_3 can be obtained, as given in (3.6) and (3.7), by solving the derivatives of the corresponding log-likelihood equations simultaneously for them:

$$\frac{\Sigma^K_{i=0} m_i}{\pi_1} - \frac{\Sigma^K_{i=0}(n_i - l_i)}{1-\pi_1-\pi_2} = 0$$

$$\frac{\Sigma^K_{i=0}(l_i - m_i)}{\pi_1} - \frac{\Sigma^K_{i=0}(n_i - l_i)}{1-\pi_1-\pi_2} = 0.$$

The maximum likelihood estimate of π_3 can be specified as in (3.8) since $\pi_1 + \pi_2 + \pi_3 = 1$.

4. ESTIMATING μ_Y AND ε

Basing on the grouping of women in the joint (X,Y)-space discussed above, new estimates for μ_Y and ε are derived such that their limiting distributions can be developed.

Theorem 1. The estimates of μ_Y and ε, by both methods of moment and maximum likelihood estimation, are:

$$\hat{\mu}_Y = \frac{\Sigma^K_{i=0} m_i}{\Sigma^K_{i=0} n_i}(k+1), \quad (4.1)$$

$$\hat{\varepsilon} = \frac{\Sigma^K_{i=0} l_i - m_i) - (1/[k+1])(\Sigma^K_{i=0} n_i)}{\Sigma^K_{i=0} n_i[1 - \{1/(k+1)\}] - \Sigma^K_{i=0} m_i}, \quad (4.2)$$

respectively.

Proof. The expressions for μ_Y and ε can be obtained from (3.2) and (3.4), respectively, as:

$\mu_Y = \pi_1(K+1)$, and $\varepsilon = 1 - \{(K+1)/(K-\mu_Y)\}\pi_3$.

From the above results, upon substituting K by k, π_1 by $\hat{\pi}_1$ in (3.6) and π_3 by $\hat{\pi}_3$ in (3.8), the moment estimates of μ_Y and ε given in (4.1) and (4.2), respectively, hold. ■

In the proposed procedure, values of μ_Y and ε can be obtained without the computation of p(Y = i). Moreover, the estimate of ε as given in (4.2) is an improvement from the work of Nour (1983) since it is given as a simple, closed-form expression that is the same in both moment and maximum likelihood estimation methods. Moreover, its upper bound is always at most equal to one. The last property is explained in Appendix II.

5. LIMITING DISTRIBUTIONS OF $\hat{\mu}_Y$ and $\hat{\varepsilon}$

In most cross-national surveys, the sample sizes (n) are very large. Therefore, for statistical testing purposes, it is necessary to derive the asymptotic distributions of estimates of μ_Y and ε.

Theorem 2. As n becomes sufficiently large, the asymptotic distribution of μ_Y is obtained as:

$$1/2(\hat{\mu}_Y - \mu_Y) \sim AN(0, (K+1)^2\pi_1(1-\pi_1)) \quad (5.1)$$

Proof. Since $\{m_i, l_i - m_i, n_i - l_i\}$ has a multinomial distribution with parameter (π_1, π_2, π_3), as $n \rightarrow \infty$, from a well-known result (Serfling, 1980, Theorem 1.9.1B, p.108), it can be shown that,

$$\begin{bmatrix} n(\hat{\pi}_1 - \pi_1)/(n\pi_1)^{1/2} \\ n(\pi_2 - \pi_2)/(n\pi_2)^{1/2} \\ n(\pi_3 - \pi_3)/(n\pi_3)^{1/2} \end{bmatrix} \sim AN\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1-\pi_1, & (\pi_1\pi_2)^{1/2}, & \pi_1\pi_3^{1/2} \\ (\pi_2\pi_1)^{1/2}, & 1-\pi_2, & (\pi_2\pi_3)^{1/2} \\ (\pi_1\pi_3)^{1/2}, & (\pi_2\pi_3)^{1/2}, & 1-\pi_3 \end{bmatrix} \right) \quad (5.2)$$

From (3.6) and (4.1), we have $\mu_Y = (k+1)\pi_1$, which is a consistent estimate of $\mu_Y = (K+1)\pi_1$. Therefore, due to (5.2) and Theorem A (Serfling, 1980, p. 118), the result (5.1) holds■

Theorem 3. The asymptotic distribution of ε is specified as:

$$n^{1/2}(\hat{\varepsilon}-\varepsilon) \sim AN(0,\sigma^2_\varepsilon) \quad (5.3)$$

where

$$\sigma^2_\varepsilon = \{[(K+1)\pi_2 - 1]^2/A^2\}(K+1)^2\pi_1(1-\pi_1) + \{(K+1)^2/A\}\pi_2(1-\pi_2) + \{2(K+1)^2/A\}[(K+1)\pi_2 - 1]\pi_1\pi_2 \quad (5.4)$$

and,

$$A = [K - (K+1)\pi_1]^2. \quad (5.5)$$

Proof. The expression for ε in (4.2) can be rewritten as,

$$\hat{\varepsilon} = \frac{(k+1)\pi_2 - 1}{k - (k+1)\pi_1} = g(\hat{\pi}_1, \hat{\pi}_2)$$

Therefore, we have:

$$n^{1/2}(\hat{\varepsilon}-\varepsilon) \approx \partial g/\partial\hat{\pi}_1|_{(\pi1,\pi2)} n^{1/2}(\hat{\pi}_1-\pi_1) + \partial g/\partial\hat{\pi}_2|_{(\pi1,\pi2)} n^{1/2}(\hat{\pi}_2-\pi_2)$$

which in turn yields the following equation:

$$\mathrm{Var}\{n^{1/2}(\hat{\varepsilon}-\varepsilon)\} \approx (\partial g/\partial\hat{\pi}_1)^2 \mathrm{Var}[n^{1/2}(\hat{\pi}_1-\pi_1)] + (\partial g/\partial\hat{\pi}_2)^2 \mathrm{Var}[n^{1/2}(\hat{\pi}_1-\pi_2)] + O(n^{-1/2}) + 2[(\partial g/\partial\hat{\pi}_1)(\partial g/\partial\hat{\pi}_2)]\mathrm{Cov}\{n^{1/2}(\hat{\pi}_1-\pi_1), n^{1/2}(\hat{\pi}_2-\pi_2)\}. \quad (5.6)$$

The components of (5.6) can be specified below. As $n \to \infty$, we have,

$$\mathrm{Var}\{n^{1/2}(\hat{\pi}_i-\pi_i)\} \to \pi_i(1-\pi_i) \quad \text{for } i=1, 2 \text{ (from Eq. (5.2))},$$

$$\mathrm{Cov}\{n^{1/2}(\hat{\pi}_1-\pi_1), n^{1/2}(\pi_2-\pi_2)\} \to \pi_1\pi_2 \quad \text{(from Eq. (5.2))}$$

$$\partial g/\partial\hat{\pi}_1|_{(\pi1,\pi2)} = \{[(k+1)\pi_1-1]/A\}(k+1), \text{ and}$$

$$\partial g/\partial\hat{\pi}_2|_{(\pi1,\pi2)} = (k+1)/A,$$

where A is defined as in (5.5). By substituting the above expressions into (5.6), the result in (5.4) holds. ■

Theorem 4. The asymptotic joint distribution of μ_Y and ε is of the form:

$$\begin{bmatrix} n^{1/2}(\hat{\mu}_Y-\mu_Y) \\ n^{1/2}(\hat{\varepsilon}-\varepsilon) \end{bmatrix} \sim AN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} (K+1)^2\pi_1\pi_2 & B_{12} \\ B_{12} & \sigma^2_\varepsilon \end{bmatrix}\right) \quad (5.7)$$

where

$$B_{12} = [\{(K+1)\pi_2 -1\}/A](K+1)^2\pi_1(1-\pi_1) + [1/\{K-(K+1)\pi_1\}](K+1)^2\pi_1\pi_2 \quad (5.8)$$

Proof. It only requires to show that B_{12} is the relevant covariance. This is true since

$$\mathrm{Cov}[n^{1/2}(\hat{\mu}_Y-\mu_Y), n^{1/2}(\hat{\varepsilon}-\varepsilon)] = (K+1)\,\partial g/\partial\hat{\pi}_1|_{(\pi1,\pi2)}\mathrm{Var}[n(\hat{\pi}_1-\pi_1)] + (K+1)\partial g/\partial\hat{\pi}_2|_{(\pi1,\pi2)}\mathrm{Cov}\{n^{1/2}(\hat{\pi}_1-\pi_1), n^{1/2}(\hat{\pi}_2-\pi_2)\} \quad (5.9)$$

Upon simplification, Eq. (5.9) becomes B_{12} as given above. ■

6. EXAMPLES

The procedures discussed in this paper are applied to a set of hypothetical data as well as to the empirical data of Sri Lanka (Nour, 1983, Table 2, p.320). The hypothetical populations are considered for studying properties of the five existing probability models and the proposed procedure whereas the empirical data are used for illustrating the steps involving in the estimation process.

Let N_i, M_i and L_i denote the population values of n_i, m_i and l_i, respectively. Suppose that these population values are given, it is possible to obtain the marginal distributions of P(Y=i) according to the five existing models as well as the means, standard deviations, and relevant confidence intervals, for Y and ε under the proposed procedure. In Table 1, five hypothetical populations under consideration are characterized by the values of the preference implementation index (ε = .00, .25, .50, .75 and 1.00) and μ_Y is set at 1.85 in all configurations.

Insert Table 1 about here

In all cases, the procedures of Nour(1983) and the proposed method successfully reproduce the assumed values of μ_Y and ε. Except when $\varepsilon = 0$, values of μ_Y are under-reported under the first three probability models and over-reported by Model 4 (Rodriguez & Trussell). The confidence intervals for μ_Y in the proposed method contain the true value of μ_Y as well as those values obtained under Model 4 (Rodriguez & Trussell). As another observation, Cov(μ_Y, ε)increase with the magnitude of ε.

For the Sri Lanka data set, first of all, an estimate of K has to be selected because values of n_i, l_i and m_i are quite small for large parity levels. Since p(X=k) = .0137, .0326 and .0690 for k = 10, 9 and 8, respectively, the estimate of K can be set at k = 10 for $\alpha = .01$ or k = 8 for $\alpha = .05$. To illustrate how different values of k can influence the estimates of μ_Y and ε, two values of k (14 and 8) have been chosen. For k = 14, estimates of P(Y=i), μ_Y, σ_Y, ε, σ_ε and the relevant confidence intervals are reported in Table 2.

Insert Table 2 about here

Comparing to the more recent methods, the first three estimation methods (Udry et al., Pullum and Lightbourne) yield smaller values of p(Y=i) for $i \geq 3$ and larger values of p(Y = i) for smaller parity levels. The estimates of P(Y=i) are virtually equal to zero for i > 10 in all cases and for i > 6 under the first three estimation models. The estimates derived by Model 4 (Rodriguez & Trussell) and Model 5 (Nour) are quite similar. In most cases, they are identical up to two decimal points.

Estimates for the mean and standard deviation of Y are also reported in Table 2. Under the proposed procedure, it is possible to compute the standard deviation for $\hat{\varepsilon}$ (by (5.4)) as well as the confidence intervals for μ_Y and ε. As expected, values of $\hat{\mu}_Y$ under the first three estimation methods (Udry et al., Pullum and Lightbourne) are biased downwards. Comparing to Nour's (1983) results, the estimates of μ_Y and ε under the proposed procedure are larger and the standard deviation for Y is substantially smaller.

In all estimation methods, p(Y=i) $\approx$ 0 for i > 8. Hence, values of p(Y=i) are recomputed in Table 3 by setting k = 8+. This change of parity range does not affect values of p(Y=i) and μ_Y under the first three estimation models (Udry et al., Pullum and Lightbourne) at all. For other models, there are small changes in all values across parity levels. For i > 5, p(Y=i) increases in Model 4 (Rodriguez & Trussell) and Model 5 (Nour). Values of $\hat{\mu}_Y$ and $\hat{\varepsilon}$ under the proposed model are smaller than those under Model 4 (Rodriguez & Trussell) and Model 5 (Nour). In both Tables 2 and 3, the confidence intervals for μ_Y and ε obtained in the proposed model do not contain the relevant estimates derived under the existing estimation models.

Insert Table 3 about here

7. CONCLUSIONS

It has been shown in Theorem 4 that the estimates of μ_Y and ε under the proposed procedure are consistent. By means of hypothetical data, both Nour's (1983) and our methods can reproduce the assumed values of μ_Y and ε in all configurations under consideration. Whereas sample properties of Nour's (1983) estimates have not been investigated, limiting distributions of our estimates are derived. Since the size of most national surveys is substantially large, the marginal and joint asymptotic distributions of the estimates of μ_Y and ε are relevant and practical. These distributions facilitate statistical inferences involving desired family

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APPENDIX I: UPPER BOUNDS OF ε

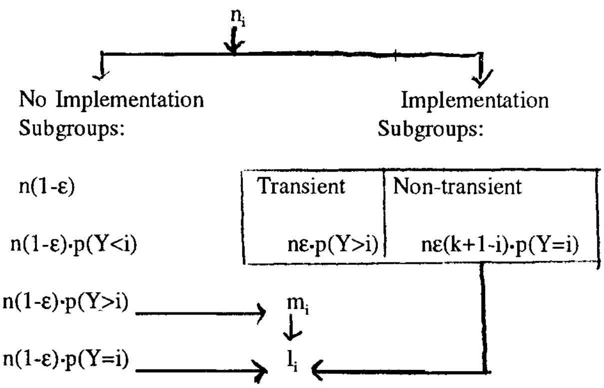
In the proposed procedure, the expression of ε in (4.1) can be ewritten as,

$$\varepsilon = \frac{\Sigma^{K}_{i=0}(l_i-m_i) - (1/(k+1))\Sigma^{K}_{i=0}n_i}{\Sigma^{K}_{i=0}(n_i-m_i) - (1/(k+1))\Sigma^{K}_{i=0}n_i}. \qquad (I.1)$$

The upper bound of ε is ≤ 1 since $\Sigma^{K}_{i=0}(l_i-m_i) \leq \Sigma^{K}_{i=0}(n_i-m_i)$.■

Figure 1. Functional Relationships Between X and Y

Panel a. Grouping of Women in the ith Parity Level (X=i) under Model 5 (Nour)



Panel b. Grouping of Women in the Fertile Population under the Proposed Model

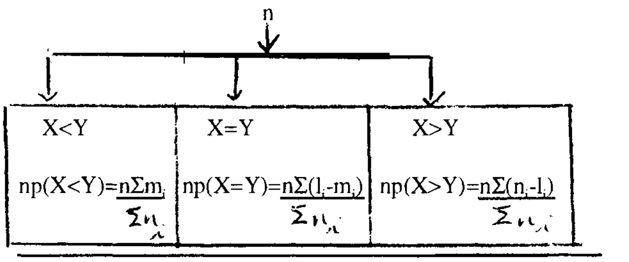


Table 1. Predetermined and Derived Parameters of Five Synthetic Fertility Populations (500 Women)

Assumed Parameters						Values of P(Y=i) as determined by the probability models of				
ε	i	P(Y=i)	N_i	M_i	L_i	Udry	Pullum	Lightbourne	Rodriguez	Nour
.00	0	.10	100	90	100.0	.10	.10	.10	.10	.10
	1	.20	100	70	90.0	.27	.20	.20	.20	.20
	2	.50	100	20	70.0	.50	.50	.50	.50	.50
	3	.15	100	5	20.0	.12	.15	.15	.15	.15
	4	.05	100	.0	5.0	.01	.05	.05	.05	.05
Existing procedures:					μ_Y	1.66	1.85	1.85	1.85	1.85
					σ^2_Y	.70	.93	.93	.93	.93

Proposed procedure:

μ_Y = 1.85, σ_Y = 0.012, $CI_{.95}$ for μ_Y = (1.638, 2.061)
ε = 0.00, σ_ε = 0.002, $CI_{.95}$ for ε = (-0.081, 0.081)
B_{12} = 0.002, Cov(μ_Y, ε) = 0.0001

Assumed Parameters						Values of P(Y=i) as determined by the probability models of				
ε	i	P(Y=i)	N_i	M_i	L_i	Udry	Pullum	Lightbourne	Rodriguez	Nour
.25	0	.10	110.00	90	110.00	.18	.18	.18	.07	.10
	1	.20	112.50	70	105.00	.31	.28	.20	.19	.20
	2	.50	117.50	20	95.00	.42	.43	.45	.53	.50
	3	.15	83.75	5	23.75	.08	.09	.11	.17	.15
	4	.05	76.25	0	5.00	.00	.02	.06	.05	.05
Exisitng procedures:					μ_Y	1.42	1.50	1.67	1.93	1.85
					σ^2_Y	..80	..94	1.16	..82	..93

Proposed procedure:

μ_Y = 1.85, σ_Y = 0.108, $CI_{.95}$ for μ_Y = (1.645, 2.070)
ε = 0.25, σ_ε = 0.062, $CI_{.95}$ for ε = (0.129, 0.371)
B_{12} = 0.684, Cov(μ_Y, ε) = 0.031

Assumed Parameters						Values of P(Y=i) as determined by the probability models of				
ε	i	P(Y=i)	N_i	M_i	L_i	Udry	Pullum	Lightbourne	Rodriguez	Nour
.50	0	.10	120.0	90	120.0	.25	.25	.25	.07	.10
	1	.20	125.0	70	120.0	.33	.31	.19	.18	.20
	2	.50	135.0	20	120.0	.36	.36	.41	.52	.50
	3	.15	67.5	5	.27.5	.06	.06	.07	.18	.15
	4	.05	52.5	.0	5.0	.00	.01	.07	.04	.05

		Udry	Pullum	Lightbourne	Rodriguez	Nour
Existing procedure:	μ_Y	1.24	1.27	1.53	1.94	1.85
	σ^2_Y	.82	.90	1.34	.83	.93

Proposed procedure:

$\mu_Y = 1.85$, $\sigma_Y = 0.108$, $CI_{.95}$ for $\mu_Y = (1.642, 2.066)$
$\varepsilon = 0.50$, $\sigma_\varepsilon = 0.083$, $CI_{.95}$ for $\varepsilon = (0.338, 0.663)$
$B_{12} = 1.363$, $Cov(\mu_Y, \varepsilon) = 0.061$

Assumed Parameters						Values of P(Y=i) as determined by the probability models of				
ε	i	P(Y=i)	N_i	M_i	L_i	Udry	Pullum	Lightbourne	Rodriguez	Nour
.75	0	.10	130.00	90	130.00	.31	.31	.31	.08	.10
	1	.20	137.50	70	135.00	.34	.33	.18	.18	.20
	2	.50	152.50	20	145.00	.31	.31	.38	.50	.50
	3	.15	51.25	5	31.25	.04	.04	.03	.18	.15
	4	.05	28.75	0	5.00	.00	.01	.09	.04	.05

		Udry	Pullum	Lightbourne	Rodriguez	Nour
Existing procedure:	μ_Y	1.10	1.11	1.43	1.92	1.85
	σ^2_Y	.81	.84	1.51	.50	.93

Proposed procedure:

$\mu_Y = 1.85$, $\sigma_Y = 0.108$, $CI_{.95}$ for $\mu_Y = (1.645, 2.070)$
$\varepsilon = 0.75$, $\sigma_\varepsilon = 0.105$, $CI_{.95}$ for $\varepsilon = (0.550, 0.963)$
$B_{12} = 2.065$, $Cov(\mu_Y, \varepsilon) = 0.092$

Assumed Parameters						Values of P(Y=i) as determined by the probability models of				
ε	i	P(Y=i)	N_i	M_i	L_i	Udry	Pullum	Lightbourne	Rodriguez	Nour
1.0	0	.10	140	90	140	.36	.36	.36	.10	.10
	1	.20	150	70	150	.34	.34	.18	.20	.20
	2	.50	170	20	170	.26	.26	.35	.50	.50
	3	.15	35	5	35	.03	.03	.10	.15	.15
	4	.05	5	0	5	.00	.00	.02	.05	.05

		Udry	Pullum	Lightbourne	Rodriguez	Nour
Existing procedures:	μ_Y	0.98	0.98	1.52	1.85	1.85
	σ^2_Y	.79	.79	1.77	.93	.93

Proposed procedure:

$\mu_Y = 1.85$, $\sigma_Y = 0.108$, $CI_{.95}$ for $\mu_Y = (1.638, 2.062)$
$\varepsilon = 1.00$, $\sigma_\varepsilon = 0.126$, $CI_{.95}$ for $\varepsilon = (0.753, 1.247)$
$B_{12} = 2.716$, $Cov(\mu_Y, \varepsilon) = 0.121$

Notes:

1. The results for $\varepsilon = 0$ and $\varepsilon = 1$ are similar to those reported in Rodriguez & Trussell (1981) whereas the results for other cases are the same as those reported in Nour (1983).
2. Under the five existing models, the mean and standard deviation of Y are computed as $\mu_Y = \Sigma iP(Y=i)$ and $\sigma^2_Y = \Sigma i^2 P(Y=i) - (\mu_Y)^2$
3. Under the proposed procedure, the estimates are computed according to results of Theorems 2, 3 and 4.

Table 2. Marginal Distribution of Desired Family Size Based on Data for Sri Lanka (15 parity levels)

Parity	n_i	l_i	mi	$\frac{n_i}{n}$	Estimates of P(Y=i): Udry	Pullum	Lightbourne	Rodriguez	Nour
0	536	536	505	.101	.0578	.0578	.0578	.0195	.0145
1	894	865	724	.168	.1792	.1536	.1324	.0735	.0699
2	883	749	413	.166	.4061	.3538	.3421	.1602	.1779
3	811	594	220	.152	.2601	.2738	.1964	.2137	.2124
4	648	360	87	.122	.0838	.1221	.1370	.1797	.1671
5	535	244	50	.100	.0118	.0309	.0408	.1343	.1289
6	359	133	20	.067	.0011	.0068	.0378	.0885	.0820
7	290	101	14	.054	.0001	.0010	.0074	.0655	.0696
8	196	.48	20	.037	.0000	.0001	.0537	.0195	.0250
9	101	40	4	.019	.0000	.0001	.0624	.0313	.0362
10	41	8	0	.008	.0000	.0000	.0396	.0085	.0092
11	.18	4	0	.003	.0000	.0000	.0000	.0045	.0054
12	12	.0	0	.002	.0000	.0000	.0000	.0000	.0000
13	1	1	0	.000	.0000	.0000	.0000	.0013	.0021
14	1	0	0	.000	.0000	.0000	.0000	.0000	.0000

		Udry	Pullum	Lightbourne	Rodriguez	Nour
Existing procedures:	μ_Y	2.173	2.375	3.823	4.003	4.059
	σ_Y	1.194	1.427	2.457	2.152	2.221
	ε					.360

Proposed procedure:

$\mu_Y = 5.793$, $\sigma_Y = 0.100$, $CI_{.95}$ for $\mu_Y = (5.597, 5.989)$
$\varepsilon = 0.436$, $\sigma_\varepsilon = 0.026$, $CI_{.95}$ for $\varepsilon = (0.385, 0.488)$
$B_{12} = 2.835$, $Cov(\mu_Y, \varepsilon) = 0.039$

Source:

Data for n_i, l_i, m_i are reported in Table 2, Nour (1983)

Table 3. Marginal Distribution of Desired Family Size Based on Data for Sri Lanka (9 parity levels)

Parity	n_i	l_i	mi	$\frac{n_i}{n}$	Udry	Pullum	Lightbourne	Rodriquez	Nour
					Estimates of P(Y=i)				
0	536	536	505	.101	.0578	.0578	.0578	.0195	.0119
1	894	865	724	.168	.1792	.1536	.1324	.0737	.0598
2	883	749	413	.166	.4061	.3538	.3421	.1610	.1579
3	811	594	220	.152	.2601	.2738	.1964	.2162	.1968
4	648	360	87	.122	.0838	.1221	.1370	.1839	.1632
5	535	244	50	.100	.0118	.0309	.0408	.1406	.1343
6	359	133	20	.067	.0011	.0068	.0378	.0975	.0929
7	290	101	14	.054	.0001	.0010	.0074	.0791	.0880
8+	196	48	20	.037	.0000	.0001	.0537	.0155	.0381
Existing procedures:				μ_Y	2.173	2.375	2.866	3.746	3.769
				σ_Y	1.050	1.069	2.290	1.863	2.079
				ε					.428

Proposed procedure:

μ_Y = 3.476, σ_Y = 0.060, $CI_{.95}$ for μ_Y = (3.358, 3.594)
ε = 0.368, σ_ε = 0.021, $CI_{.95}$ for ε = (0.326, 0.410)
B_{12} = 1.564, Cov(μ_Y, ε) = 0.021