1. INTRODUCTION

The development of all probability models of desired family size is based on the synthetic, stationary fertility framework (Udry & Chase, 1973; Pullum, 1979; Lightbourne, 1977; Rodriguez & Trussell, 1981 and Nour, 1983). In the present paper, parameters and assumptions of the synthetic fertility population are discussed. Our main objective is to present a new procedure that yields the estimates of (i) the largest parity level, (ii) the mean of desired family size, (iii) the fertility preference implementation index; and (iv) marginal and joint distributions of these estimates. The proposed procedure has the following advantages: (i) the parameters can be estimated separately with closed form expressions that do not require the use of numerical algorithms, (ii) the estimates are consistent, and (iii) the same expressions for the estimators can be obtained under either the moment method or the maximum likelihood estimation technique. Some simple examples are used to demonstrate the application of the proposed procedure.

2. THE STATIONARY FERTILITY MODEL

Suppose that the fertility survey covers the entire population and the fertility behaviours of the population are unchanged during the time under investigation.

2.1. Notations

- \( i \) = Parity index \((i = 0, 1, \ldots)\)

Variables and parameters:
- \( K \) = Maximum possible family size
- \( X \) = Actual family size
- \( Y \) = Desired family size
- \( \varepsilon \) = Proportion of women, out of the total fertile population, who have fully implemented their fertility preferences = The fertility preference implementation index.

The population means of \( X, Y \) and the conditional mean of \( Y \) given \( X \), \( \mu_{Y|X} \), are denoted as \( \mu_X, \mu_Y \) and \( \mu_{Y|X} \); and their estimates as \( \hat{\mu}_X, \hat{\mu}_Y, \hat{\mu}_{Y|X} \) respectively. The estimates of \( K \) and \( \varepsilon \) are denoted as \( k \) and \( \varepsilon \), respectively.

Statistics:
- \( n = \) Total number of women
- \( n_i = \) Number of women of parity \( i \)
- \( l_i = \) Number of women of parity \( i \) who wanted their last child
- \( m_i = \) Number of women of parity \( i \) who want more children

Initial constraints:

\[ n = \sum_{i=0}^{K} n_i \]  
\[ l_0 = n_0 \]  
\[ m_k = 0 \]  
\[ n_i \geq l_i \geq m_i \]  
\[ 0 < X < K \] and \[ 0 < Y < k \]

2.2. Basic Assumptions

(A1) The distribution of \( Y, P(Y=i) \), is representative of the \( i \)th desired family size of the synthetic population. At each parity level, the desired family size \( Y \) is not dependent of \( \varepsilon \), the implementation index.

(A2) The same implementation index \( \varepsilon \) applies to all members of the synthetic population. The actual family size \( X \) at each parity level is dependent on \( \varepsilon \).

(A3) The parity levels \((i = 0, 1, \ldots, K)\) represent \( K \) equal time intervals between births.

(A4) Values of \( n_i, m_i \) and \( l_i \) are reported accurately.

(A5) The maximum number of family size \( K \) is finite.
3. THE RELATIONSHIP BETWEEN ACTUAL FAMILY SIZE (X) AND DESIRED FAMILY SIZE (Y)

In Nour (1983), the number of women having i children \( n_i \) is a sum of three subgroups that can be summarized succinctly as:

\[
n_i = n(1-e) + n_0e(Y>i) + n_3e(k+1-i)e(Y=i).
\]

In the first group, there are \( n(1-e) \) women, out of \( n \), who do not implement the assumed fertility preference and move on to the next parity level. In the second group, there are \( n_0e(Y>i) \) women who have not fully implemented their fertility preference and also move on. Finally, the are \( n_3e(k+1-i)e(Y=i) \) women in the third group who have obtained their desired family size and stay at the ith parity level. In Panel a of Figure 1, the components of these subgroups are used to count, at the ith parity level, the number of women who liked their last child \( (li) \) and the number of those who want more children \( (m_1) \). Once the observed values of \( l_i \) and \( m_1 \) have been collected, they are in turn used to determine \( p(Y=i) \).

In the present paper, the estimation procedure is constructed on the partition of the total fertile population into subgroups basing on the relative comparison of actual family size (X) against desired family size (Y) as depicted in Panel b of Figure 1. The formulation under this framework is explained below.

3.1. Grouping of women in the joint X and Y Space

The entire fertile population can be partitioned into three subgroups depending on the relative magnitudes of X and Y as follows:

Group 1 (X<Y): with the probability of membership equal to \( \pi_1 = P(X < Y | \epsilon) \)

Group 2 (X=Y): with the probability of membership equal to \( \pi_2 = P(X = Y | \epsilon) \)

Group 3 (X>Y): with the probability of membership equal to \( \pi_3 = P(X > Y | \epsilon) \).

Lemma 1. The probabilities of group memberships in the (X,Y)-space are specified as:

\[
\pi_1 = \mu_Y/(K + 1) \quad (3.2)
\]

\[
\pi_2 = \left(1/(K + 1)\right)\{(K - \mu_Y)e + 1\} \quad (3.3)
\]

\[
\pi_3 = (K - \mu_Y)(1 - e)/(K + 1) \quad (3.4)
\]

Proof. Basing on the relationship between Y and X specified by Eq. (20) in Nour (1983), and due to the constraint of \( \Sigma \pi_i = 1 \), the following results can be obtained:

\[
\pi_1 = \sum_{i=0}^{K} P(X<Y | Y=i, \epsilon)P(Y=i)
= \sum_{i=0}^{K} \left(i/(K+1)\right)P(Y=i)
\]

\[
\pi_2 = \sum_{i=0}^{K} P(X=Y | Y=i, \epsilon)P(Y=i)
= \sum_{i=0}^{K} \left(\epsilon(1-i/(K+1)) + (1-\epsilon)/(K+1)\right)P(Y=i)
\]

\[
\pi_3 = \sum_{i=0}^{K} P(X>Y | Y=i, \epsilon)P(Y=i)
= \sum_{i=0}^{K} \left(K-i\right)(1-\epsilon)/(K+1)P(Y=i)
\]

Upon simplifying the above equations, the expressions (3.2) to (3.4) hold. (See Panel b, Figure 1). As an outcome of this grouping, the following result can be obtained.

Lemma 3. The methods of moment and maximum likelihood estimation yield the same estimates for \( \pi_i \), \( i = 1, 2, 3 \), that can be specified as:

\[
\hat{\pi}_1 = \sum_{i=0}^{K} \frac{m_i}{\sum_{i=0}^{K} n_i}, \quad (3.6)
\]

\[
\hat{\pi}_2 = \sum_{i=0}^{K} \frac{(l_i - m_i)}{\sum_{i=0}^{K} n_i}, \quad (3.7)
\]

\[
\hat{\pi}_3 = \sum_{i=0}^{K} \frac{(n_i - l_i)}{\sum_{i=0}^{K} n_i}, \quad (3.8)
\]

Proof. From the partitioning of the (X,Y)-space, we have \( \pi_1 = E[\sum_{i=0}^{K} m_i]/n \), \( \pi_2 = E[\sum_{i=0}^{K} (l_i - m_i)]/n \) and \( \sum_{i=0}^{K} \pi_i = 1 \). Therefore, the moment estimates of
\( \pi_i, \{i = 1, 2, 3\}, \) are derived as given in (3.6) to (3.8), respectively.

To obtain the maximum likelihood estimates, consider the following likelihood function:

\[ \log L(l_i, m_i, n_i) = \frac{(\sum_{i=0}^{K} m_i)!(\sum_{i=0}^{K}(1-m_i))!}{(\sum_{i=0}^{K}(l_i-m_i))!} \cdot \frac{\sum_{i=0}^{K}(n_i-1)}{1-\pi_1-\pi_2} = 0 \]

The maximum likelihood estimates for \( \pi_1 \) and \( \pi_3 \) can be obtained, as given in (3.6) and (3.7), by solving the derivatives of the corresponding log-likelihood equations simultaneously for them:

\[ \frac{\sum_{i=0}^{K} m_i}{\pi_1} = \frac{\sum_{i=0}^{K}(n_i-1)}{1-\pi_1-\pi_2} = 0 \]

The maximum likelihood estimate of \( \pi_2 \) can be specified as in (3.8) since \( \pi_1 + \pi_2 + \pi_3 = 1 \).

4. ESTIMATING \( \mu_Y \) AND \( \varepsilon \)

Basing on the grouping of women in the joint \((X,Y)\)-space discussed above, new estimates for \( \mu_Y \) and \( \varepsilon \) are derived such that their limiting distributions can be developed.

**Theorem 1.** The estimates of \( \mu_Y \) and \( \varepsilon \), by both methods of moment and maximum likelihood estimation, are:

\[ \hat{\mu}_Y = \frac{\sum_{i=0}^{K} m_i}{\sum_{i=0}^{K} n_i} (k+1), \quad (4.1) \]

\[ \hat{\varepsilon} = \frac{\sum_{i=0}^{K} (1-m_i) - (1/(k+1)) \sum_{i=0}^{K} n_i}{\sum_{i=0}^{K} n_i[1 - (1/(k+1))] - \sum_{i=0}^{K} m_i}, \quad (4.2) \]

respectively.

**Proof.** The expressions for \( \mu_Y \) and \( \varepsilon \) can be obtained from (3.2) and (3.4), respectively, as:

\[ \mu_Y = \pi_1(K+1), \quad \varepsilon = 1 - \{(K+1)/(K-\mu_Y)\} \pi_3. \]

From the above results, upon substituting \( K \) by \( k \), \( \pi_1 \) by \( \hat{\pi}_1 \) in (3.6) and \( \pi_3 \) by \( \hat{\pi}_3 \) in (3.8), the moment estimates of \( \mu_Y \) and \( \varepsilon \) given in (4.1) and (4.2), respectively, hold.

In the proposed procedure, values of \( \mu_Y \) and \( \varepsilon \) can be obtained without the computation of \( p(Y = i) \). Moreover, the estimate of \( \varepsilon \) as given in (4.2) is an improvement from the work of Nour (1983) since it is given as a simple, closed-form expression that is the same in both moment and maximum likelihood estimation methods. Moreover, its upper bound is always at most equal to one. The last property is explained in Appendix II.

5. LIMITING DISTRIBUTIONS OF \( \hat{\mu}_Y \) AND \( \hat{\varepsilon} \)

In most cross-national surveys, the sample sizes \( (n) \) are very large. Therefore, for statistical testing purposes, it is necessary to derive the asymptotic distributions of estimates of \( \mu_Y \) and \( \varepsilon \).

**Theorem 2.** As \( n \) becomes sufficiently large, the asymptotic distribution of \( \hat{\mu}_Y \) is obtained as:

\[ 1/2(\hat{\mu}_Y - \mu_Y) \sim \text{AN}(0, (K+1)^2/(n(k-1))) \quad (5.1) \]

**Proof.** Since \( \{m_i, l_i-m_i, n_i-1\} \) has a multinomial distribution with parameter \((\pi_1, \pi_2, \pi_3)\), as \( n \to \infty \), from a well-known result (Serfling, 1980, Theorem 1.9.1B, p.108), it can be shown that,

\[ \begin{bmatrix} n(n_i-n_i)(n_i)^{1/2} \\ n(n_i-n_i)(n_i)^{1/2} \end{bmatrix} \sim \text{AN} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} (1-\pi_1)(\pi_1)^{1/2}, (1-\pi_2)(\pi_2)^{1/2} \\ (1-\pi_3)(\pi_3)^{1/2}, (1-\pi_3)(\pi_3)^{1/2} \end{bmatrix} \quad (5.2) \]

From (3.6) and (4.1), we have \( \mu_Y = (k+1)\pi_1 \), which is a consistent estimate of \( \mu_Y = (K+1)\pi_1 \). Therefore, due to (5.2) and Theorem A (Serfling, 1980, p. 118), the result (5.1) holds.
Theorem 3. The asymptotic distribution of $e$ is specified as:

$$n^{1/2}(\hat{e} - e) \sim \text{AN}(0, \sigma^2_e) \quad (5.3)$$

where

$$\sigma^2_e = \frac{\{(K+1)\pi_i - 1\}^2/A^2 \{(K+1)^2 \pi_i (1-\pi_i) \}}{(K+1)^2/A \pi_i (1-\pi_i) + 2 \{(K+1)^2/A \} \{2(K+1)^2/A \} \{(K+1)\pi_i - 1\} \pi_i \pi_2}$$

and,

$$A = \left[K - \frac{(K+1)\pi_i}{1} \right]^2. \quad (5.5)$$

Proof. The expression for $e$ in (4.2) can be rewritten as,

$$\hat{e} = \frac{(k+1)\pi_i - 1}{k - (k+1)\pi_i} = g(\hat{\pi}_1, \hat{\pi}_2)$$

Therefore, we have:

$$n^{1/2}(\hat{e} - e) = \partial g/\partial \hat{\pi}_1|_{(1,0)} n^{1/2}(\hat{\pi}_1 - \pi_1)$$

$$+ \partial g/\partial \hat{\pi}_2|_{(1,0)} n^{1/2}(\hat{\pi}_2 - \pi_2)$$

which in turn yields the following equation:

$$\text{Var}[n^{1/2}(\hat{e} - e)] = \left(\partial g/\partial \hat{\pi}_1\right)^2 \text{Var}[n^{1/2}(\hat{\pi}_1 - \pi_1)]$$

$$+ \left(\partial g/\partial \hat{\pi}_2\right)^2 \text{Var}[n^{1/2}(\hat{\pi}_2 - \pi_2)] + \mathcal{O}(n^{1/2})$$

$$+ 2\left(\partial g/\partial \hat{\pi}_1\right)\left(\partial g/\partial \hat{\pi}_2\right)\text{Cov}[n^{1/2}(\hat{\pi}_1 - \pi_1), n^{1/2}(\hat{\pi}_2 - \pi_2)]. \quad (5.6)$$

The components of (5.6) can be specified below.

As $n \to \infty$, we have,

$$\text{Var}[n^{1/2}(\hat{\pi}_i - \pi_i)] \to \pi_i (1-\pi_i) \quad \text{for } i=1, 2 \text{ (from Eq. (5.2)),}$$

$$\text{Cov}[n^{1/2}(\hat{\pi}_1 - \pi_1), n^{1/2}(\hat{\pi}_2 - \pi_2)] \to \pi_1 \pi_2 \quad \text{from Eq. (5.2))}$$

$$\partial g/\partial \hat{\pi}_1|_{(1,0)} = \frac{\{(k+1)\pi_i - 1\}/A}{(k+1)}, \text{ and}$$

$$\partial g/\partial \hat{\pi}_2|_{(1,0)} = \frac{(k+1)}{A},$$

where $A$ is defined as in (5.5). By substituting the above expressions into (5.6), the result in (5.4) holds. ■

Theorem 4. The asymptotic joint distribution of $e$ and $e$ is of the form:

$$\begin{bmatrix} n^{1/2}(\hat{e} - e) \\ n^{1/2}(\hat{\mu}_y - \mu_y) \end{bmatrix} \sim \text{AN}\left(0, \begin{bmatrix} \sigma^2_e & B_{12} \\ B_{12} & \sigma^2_{\mu} \end{bmatrix}\right). \quad (5.7)$$

where

$$B_{12} = \left[\{(K+1)\pi_i - 1\}/A \right] \{(K+1)^2 \pi_i (1-\pi_i) \} + \left[1/(K-\right.$$}

$$(K+1)\pi_i \right) \{(K+1)\pi_i \} \{(K+1)^2 \pi_i \pi_2 \} \quad (5.8)$$

Proof. It only requires to show that $B_{12}$ is the relevant covariance. This is true since

$$\text{Cov}[n^{1/2}(\hat{e} - e), n^{1/2}(\hat{\mu}_y - \mu_y)]$$

$$= (K+1) \left(\partial g/\partial \hat{\pi}_1|_{(1,0)} \right) \text{Var}[n^{1/2}(\hat{\pi}_1 - \pi_1)]$$

$$+ (K+1) \left(\partial g/\partial \hat{\pi}_2|_{(1,0)} \right) \text{Cov}[n^{1/2}(\hat{\pi}_1 - \pi_1), n^{1/2}(\hat{\pi}_2 - \pi_2)]. \quad (5.9)$$

Upon simplification, Eq. (5.9) becomes $B_{12}$ as given above. ■

6. EXAMPLES

The procedures discussed in this paper are applied to a set of hypothetical data as well as to the empirical data of Sri Lanka (Nour, 1983, Table 2, p.320). The hypothetical populations are considered for studying properties of the five existing probability models and the proposed procedure whereas the empirical data are used for illustrating the steps involving in the estimation process.

Let $N_i$, $M_i$, and $L_i$ denote the population values of $n_i$, $m_i$, and $l_i$, respectively. Suppose that these population values are given, it is possible to obtain the marginal distributions of $P(Y=i)$
according to the five existing models as well as
the means, standard deviations, and relevant
confidence intervals, for Y and ε under the
proposed procedure. In Table 1, five hypothetical
populations under consideration are characterized
by the values of the preference implementation
index (ε = .00, .25, .50, .75 and 1.00) and μγ is
set at 1.85 in all configurations.

In all cases, the procedures of Nour(1983) and
the proposed method successfully reproduce the
assumed values of μγ and ε. Except when ε = 0,
values of μγ are under-reported under the first
three probability models and over-reported by
Model 4 (Rodriguez & Trussell). The
confidence intervals for μγ in the proposed
method contain the true value of μγ as well as
those values obtained under Model 4 (Rodriguez
& Trussell). As another observation, Cov(μγ, ε)increase with the magnitude of ε.

For the Sri Lanka data set, first of all, an
estimate of K has to be selected because values
of n, l, and m are quite small for large parity
levels. Since p(X=k) = .0137, .0326 and .0690
for k = 10, 9 and 8, respectively, the estimate of
K can be set at k = 10 for α = .01 or k = 8 for
α = .05. To illustrate how different values of k
can influence the estimates of μγ and ε, two
values of k (14 and 8) have been chosen. For k
= 14, estimates of P(Y=i), μγ, σγ, ε, σγ and the
relevant confidence intervals are reported in
Table 2.

Comparing to the more recent methods, the
first three estimation methods (Udry et al.,
Pullum and Lightboume) yield smaller values of
p(Y=i) for i ≥ 3 and larger values of p(Y = i) for
smaller parity levels. The estimates of P(Y=i) are
virtually equal to zero for i > 10 in all cases and
for i ≥ 6 under the first three estimation models.
The estimates derived by Model 4 (Rodriguez &
Trussell) and Model 5 (Nour) are quite similar.

In most cases, they are identical up to two
decimal points.

Estimates for the mean and standard deviation
of Y are also reported in Table 2. Under the
proposed procedure, it is possible to compute the
standard deviation for ε (by (5.4)) as well as the
confidence intervals for μγ and ε. As expected,
values of ̂μγ under the first three estimation
methods (Udry et al., Pullum and Lightboume)
are biased downwards. Comparing to Nour’s
(1983) results, the estimates of μγ and ε under
the proposed procedure are larger and the
standard deviation for Y is substantially smaller.

In all estimation methods, p(Y=i) = 0 for i >
8. Hence, values of p(Y=i) are recomputed in
Table 3 by setting k = 8+. This change of parity
range does not affect values of p(Y=i) and μγ
under the first three estimation models (Udry et
al., Pullum and Lightboume) at all. For other
models, there are small changes in all values
across parity levels. For i > 5, p(Y=i) increases
in Model 4 (Rodriguez & Trussell) and Model 5
(Nour). Values of ̂μγ and ̂ε under the proposed
model are smaller than those under Model 4
(Rodriguez & Trussell) and Model 5 (Nour). In
both Tables 2 and 3, the confidence intervals for
μγ and ε obtained in the proposed model do not
contain the relevant estimates derived under the
existing estimation models.

It has been shown in Theorem 4 that the
estimates of μγ and ε under the proposed
procedure are consistent. By means of
hypothetical data, both Nour’s (1983) and our
methods can reproduce the assumed values of μγ
and ε in all configurations under consideration.
Whereas sample properties of Nour’s (1983)
estimates have not been investigated, limiting
distributions of our estimates are derived. Since
the size of most national surveys is substantially
large, the marginal and joint asymptotic distribu-
tions of the estimates of μγ and ε are relevant
and practical. These distributions facilitate
statistical inferences involving desired family
REFERENCES

APPENDIX I: UPPER BOUNDS OF $\varepsilon$

In the proposed procedure, the expression of $\varepsilon$ in (4.1) can be rewritten as,

$$\varepsilon = \frac{\sum_{i=0}^{k} (l_i - m_i) - (1/(k+1)) \sum_{i=0}^{k} m_i}{\sum_{i=0}^{k} (l_i - m_i) - (1/(k+1)) \sum_{i=0}^{k} m_i}.$$  \hspace{1cm} \text{(1.1)}$$

The upper bound of $\varepsilon$ is $\leq 1$ since $\sum_{i=0}^{k} (l_i - m_i) \leq \sum_{i=0}^{k} (l_i - m_i).$

Figure 1. Functional Relationships Between X and Y

Panel a. Grouping of Women in the ith Parity Level (X=i) under Model 5 (Nour)

Panel b. Grouping of Women in the Fertile Population under the Proposed Model

Table 1. Predetermined and Derived Parameters of Five Synthetic Fertility Populations (500 Women)

<table>
<thead>
<tr>
<th>Assumed Parameters</th>
<th>Values of $P(Y=i)$ as determined by the probability models of $\varepsilon$</th>
<th>$\mu_Y$, $\sigma_Y$, $\mu_\varepsilon$, $\sigma_\varepsilon$, $B_{12}$, $Cov(\mu_Y, \epsilon)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Existing procedures:}$</td>
<td>$\mu_Y = 1.66$, $\sigma_Y = 0.10$, $\mu_\varepsilon = 1.85$, $\sigma_\varepsilon = 0.012$, $B_{12} = 0.002$, $Cov(\mu_Y, \epsilon) = 0.0001$</td>
<td></td>
</tr>
<tr>
<td>$\text{Proposed procedure:}$</td>
<td>$\mu_Y = 1.85$, $\sigma_Y = 0.012$, $\mu_\varepsilon = 1.638$, $\sigma_\varepsilon = 2.061$, $B_{12} = 0.002$, $Cov(\mu_Y, \epsilon) = 0.0001$</td>
<td></td>
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</tbody>
</table>

Table 2. Parameter Values for Five Synthetic Fertility Populations (500 Women)

<table>
<thead>
<tr>
<th>Assumed Parameters</th>
<th>Values of $P(Y=i)$ as determined by the probability models of $\varepsilon$</th>
<th>$\mu_Y$, $\sigma_Y$, $\mu_\varepsilon$, $\sigma_\varepsilon$, $B_{12}$, $Cov(\mu_Y, \epsilon)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Existing procedures:}$</td>
<td>$\mu_Y = 1.42$, $\sigma_Y = 0.80$, $\mu_\varepsilon = 1.50$, $\sigma_\varepsilon = 0.94$, $B_{12} = 0.684$, $Cov(\mu_Y, \epsilon) = 0.031$</td>
<td></td>
</tr>
<tr>
<td>$\text{Proposed procedure:}$</td>
<td>$\mu_Y = 1.85$, $\sigma_Y = 0.108$, $\mu_\varepsilon = 1.645$, $\sigma_\varepsilon = 2.070$, $B_{12} = 0.684$, $Cov(\mu_Y, \epsilon) = 0.031$</td>
<td></td>
</tr>
</tbody>
</table>

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Assumed Parameters: Values of $P(Y=i)$ as determined by the probability models of:

\[ P(Y=i) \]

Existing procedure: $\mu_Y = 1.24, \sigma^2_Y = 0.82, 0.90, 1.34, 0.83, 0.93$.

Proposed procedure:

\[ P(Y=i) \]

Existing procedure: $\mu_Y = 1.85, \sigma^2_Y = 0.108, CI_{95}$ for $\mu_Y = (1.642, 2.066)$

\[ \epsilon = 0.50, \sigma_\epsilon = 0.083, CI_{95} \text{ for } \epsilon = (0.338, 0.663) \]

$B_{12} = 1.95, Cov(\mu_Y, \epsilon) = 0.061$

Notes:
1. The results for $\epsilon = 0$ and $\epsilon = 1$ are similar to those reported in Rodriguez & Trussell (1981) whereas the results for other cases are the same as those reported in Nour (1983).
2. Under the five existing models, the mean and standard deviation of $Y$ are computed as $\mu_Y = \sum P(Y=i)$ and $\sigma^2_Y = \sum P(Y=i) \cdot (\mu_Y)^2$.
3. Under the proposed procedure, the estimates are computed according to results of Theorems 2, 3 and 4.

Table 2. Marginal Distribution of Desired Family Size

Based on Data for Sri Lanka (15 parity levels)

<table>
<thead>
<tr>
<th>Par. $n_i$</th>
<th>$\mu_Y$</th>
<th>$\sigma^2_Y$</th>
<th>$\mu_Y$</th>
<th>$\sigma^2_Y$</th>
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<td>0</td>
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<td>1</td>
<td>0</td>
<td>.054</td>
<td>.0001</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>0</td>
<td>.054</td>
<td>.0001</td>
</tr>
</tbody>
</table>

Existing procedures: $\mu_Y = 2.173, 2.375, 3.823, 4.003, 4.059$

$\sigma^2_Y = 1.194, 1.427, 2.457, 2.152, 2.221$

$\epsilon = .360$

Proposed procedure:

$\mu_Y = 5.793, \sigma_Y = 0.100, CI_{95}$ for $\mu_Y = (5.597, 5.989)$

$\epsilon = 0.436, \sigma_\epsilon = 0.026, CI_{95}$ for $\epsilon = (0.385, 0.488)$

$B_{12} = 2.835, Cov(\mu_Y, \epsilon) = 0.039$

Source:
Data for $n_i$, $l_i$, and $m_i$ are reported in Table 2, Nour (1983)
Table 3. Marginal Distribution of Desired Family Size
Based on Data for Sri Lanka (9 parity levels)

<table>
<thead>
<tr>
<th>Parity</th>
<th>Estimates of P(Y=i)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Udry Pul.</td>
</tr>
<tr>
<td></td>
<td>lum.</td>
</tr>
<tr>
<td>0</td>
<td>536</td>
</tr>
<tr>
<td>1</td>
<td>894</td>
</tr>
<tr>
<td>2</td>
<td>883</td>
</tr>
<tr>
<td>3</td>
<td>811</td>
</tr>
<tr>
<td>4</td>
<td>648</td>
</tr>
<tr>
<td>5</td>
<td>535</td>
</tr>
<tr>
<td>6</td>
<td>359</td>
</tr>
<tr>
<td>7</td>
<td>290</td>
</tr>
<tr>
<td>8+</td>
<td>196</td>
</tr>
</tbody>
</table>

Existing procedures:
- $\mu_Y = 2.173, \sigma_Y = 1.050, \varepsilon = 0.428$
- for $\mu_Y = (3.358, 3.594)$
- $\sigma_Y = 1.069, \varepsilon = 2.290, \sigma = 2.079$
- $\varepsilon = 0.368, \sigma = 0.021, CI_{95} for \varepsilon = (0.326, 0.410)$
- $\sigma_{\mu_Y} = 1.564, Cov(\mu_Y, \varepsilon) = 0.021$

Proposed procedure:
- $\mu_Y = 3.476, \sigma_Y = 0.060, CI_{95} for \mu_Y = (3.358, 3.594)$
- $\varepsilon = 0.368, \sigma = 0.021, CI_{95} for \varepsilon = (0.326, 0.410)$
- $\sigma_{\mu_Y} = 1.564, Cov(\mu_Y, \varepsilon) = 0.021$