INDIRECT ESTIMATORS: DEFINITION, CHARACTERISTICS, AND RECOMMENDATIONS

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1. INTRODUCTION

1.1 The Federal Committee on Statistical Methodology, Subcommittee on Small Area Estimation

The U.S. Federal Committee on Statistical Methodology was organized to investigate methodological issues in the production of federal statistics. The Committee conducts its work through subcommittees organized to study particular issues. In April 1991, a subcommittee was charged with the task of identifying and documenting federal statistical programs that have used small area (indirect) estimators for the production of published estimates. The report that resulted, Indirect Estimators in Federal Programs (U.S. Office of Management and Budget, 1993), documents eight programs and provides discussion on the definition and characteristics of indirect estimators. This paper borrows from the Federal Committee on Statistical Methodology report and from another paper (Schaible, 1993). Very little detail on the eight programs documented in the report is presented here. Instead, emphasis is placed on the definition of direct and indirect estimators and on comparisons of their characteristics. In addition, recommendations and cautions for producers and users of indirect estimates are provided.

1.2 Terminology

Increased interest in non-traditional estimators for domain statistics has occurred somewhat recently among survey statisticians and, even though the term "small area estimator" is commonly used, standard terminology has not yet evolved. Numerous terms have been used to describe indirect estimators, adding to the confusion which seems to be associated with these procedures. The term, small area estimator, has been used because most applications of these estimators have been to produce estimates for geographic areas. However, the word "small" can be misleading. It is the small number of sample observations and the resulting large variance of standard direct estimators that is of concern, rather than the size of the population in the area or the size of the area itself. The word "area" can also be misleading since these methods may be applied to any arbitrary domain, not just those defined by geographic boundaries. Other terms used to describe these estimators include "area breakdowns" (Woodruff 1966), "local area" (Ericksen, 1974), "small domain" (Purcell and Kish, 1979), "subdomain" (Laake, 1979), "small subgroups" (Holt, Smith, and Tomberlin, 1979), "model-dependent" (Särndal, 1984), and "indirect" (Dalenius, 1987). The term "synthetic estimator" has also been used to describe this class of estimators (NIDA, 1979) and, in addition, to describe a specific indirect estimator (NCHS, 1968). Survey practitioners sometimes refer to indirect estimators as "model-based" whereas this term is rarely, if ever, used to describe direct estimators. Of course, direct estimators can be motivated by and justified under models just as readily as indirect estimators.

There is also lack of agreement on what to call the class of direct estimators. In addition, to "direct" (Royall, 1973), authors have used "unbiased" (Gonzalez, 1973), "'standard" (Holt, Smith, and Tomberlin, 1979), "valid" (Gonzalez, 1979), and "sample-based" (Kalton, 1987). In the remainder of this paper, the words "direct" and "indirect" will be used to describe traditional and small area estimators, respectively.

2. DIRECT AND INDIRECT ESTIMATORS

The standard problem in survey sampling is to estimate a finite population quantity (e.g., a mean or total) for a specific variable from observations made on a sample of units drawn from the specified population. The definition of a population usually requires 1) the specification of analytical units with which the variable of interest is associated and 2) a set of restrictions specifying which analytical units are in the population. In practice, analytical units are often people or establishments. Restrictions are usually specified using characteristics of the units themselves, for example, socioeconomic or demographic characteristics when units are people or some measure of size or product when units are establishments. Regardless of what analytical unit is specified, geographic location is a common restriction used to define populations.

Samples are designed to produce estimates, not only for the total population, but also for subpopulations or domains. Domains are defined by partitioning the population using unit characteristic information similar to that needed to define the original population. In federal data systems, the population of primary interest is usually the nation as a whole. However, it is rare that programs do not also find it important to produce estimates for subnational domains. Both national and domain estimates are usually produced at scheduled points in time, often monthly, quarterly, or annually.

A population must be defined at a specific point in time, since both the set of units in a population and the values of the variable of interest associated with units change over time. In this paper, problems associated with the set of population units changing over time will be ignored. However, the fact that values of the variable of interest vary over time is one of two critical facts that underlie much of what will be discussed here. The other fact is that domain population values of interest vary among domains. Whether a unit is located in a particular domain or not is a characteristic associated with the unit. Whether an observation on the variable of interest is made at a particular time or not is a characteristic associated with the observation.

Federal government surveys are generally designed using direct estimators which are unbiased, or approximately unbiased, under finite population sampling theory. When adequate resources are available, the sample design specifies population and domain sample sizes large enough to produce direct estimates which meet reliability requirements for the survey. When a domain sample size is judged to be too small to make a reliable domain estimate using a direct estimator, a decision must be made whether or not an alternative estimator will produce estimates which are adequate. The alternative estimators generally considered are those that increase the effective sample size and decrease the variance by using additional data from other domains and/or time periods through models that assume similarities across domains and/or time periods. These estimators are generally considered to be biased. If the mean square error of the alternative estimator is small compared to the variance of the direct estimator, the selection of the alternative estimator may be justified. In extreme situations, there may be no sample units in the domain of interest and, if an estimate is to be produced, an alternative estimator will be required.

2.1 Definitions

Indirect estimators have been characterized in the empirical Bayes literature as estimators that "borrow strength" by incorporating values of the variable of interest from units in domains other than the domain of interest. This concept can be used to provide a working definition of direct and indirect estimators for a broad class of population quantities including means and totals. A *direct estimator* uses values of the variable of interest only from the time period of interest and from units in the domain of interest. An *indirect estimator* uses values of the variable of interest from a time period other than that of interest and/or from a domain other than that of interest. Three types of indirect estimators can be identified. A d*omain indirect estimator* uses values of the variable of interest from units in another domain but not from another time period. A *time indirect estimator* uses values of the variable of interest from another time period but not from units in another domain. An estimator that is both *domain and time indirect* uses values of the variable of interest from another time period and from units in another domain. Note that these definitions of direct and indirect estimators do not depend on whether or not auxiliary variables from outside the domain or time period of interest are used.

The clear distinction between direct and indirect estimators made in the discussion above reflects the situation during the design stage of a data system. However, when estimators are modified to reflect the realities associated with data system implementation, the distinction becomes less clear. For example, when nonresponse, a common problem in data collection efforts, occurs, even direct estimators must rely on model-based assumptions relating the known information for responders to the unknown information for nonresponders. Even though they will not be discussed here, secondary estimation methods such as nonresponse adjustment, raking, and seasonal adjustment borrow strength and are subject to some of the same concerns as basic indirect estimators.

2.2 Characteristics of Indirect Estimators

Insight into the differences between direct and indirect estimators may be gained by inspecting their underlying models. Notation will be required. Let

$d = 1, 2, \ldots, D$ denote domains,
$t = 1, 2, \ldots, T$ denote time periods,
$i = 1, 2, \ldots, N_{dt}$ denote observations made at time t on units in domain d, and
Y_{dti} denote the variable of interest associated with unit/observation dti.

In addition, within the domain and time period of interest, let s_{dt} denote the set of units that are in the sample and $\tilde{s}_{dt}$, the set of units not in the sample.

The example in Table 1 below illustrates several points that help to better understand characteristics of and the relationship between direct and indirect estimators.

1. A domain and time specific model defines a family of models. For example, associated with the single parameter, domain and time specific model, $E(Y_{dti}) = \mu_{dt}$, are three other models. The domain and time specific model leads to a direct estimator whereas the three remaining models lead to indirect estimators.

2. If the Y's are independent with constant variance, then the best linear unbiased estimators (BLUE's) of the parameters of the four models in this family are: 1) the sample mean in the domain and time period of interest for the model parameter, μ_{dt}, 2) the sample mean for the specified time period across all domains for the model parameter, $\mu_{.t}$, 3) the sample mean for the specified domain across all time periods for the model parameter, $\mu_{d.}$, and 4) the sample mean across all domains and time periods for the model parameter, $\mu_{..}$.

3. Generally, rather than estimate a model parameter, the objective in finite population estimation problems is to estimate the population mean (or total) for a particular domain and time period. The best linear unbiased predictor (BLUP) of the population total in the domain and time of interest is obtained by adding the known sum of the values for sampled units to the predicted sum for the unobserved values associated with nonsampled units. In this example, each unobserved value is predicted by the BLUE for the corresponding model parameter. The BLUP for the population mean is obtained by dividing the predicted population total by the number of units in the population.

Table 1. Direct and Indirect Estimators of Model Parameters and the Finite Population Mean, $\bar{Y}_{dt}$, for the Family of Models Defined by $E(Y_{dti}) = \mu_{dt}$

Expectation Model	BLUE for the Model Parameter	BLUP for $\bar{Y}_{dt}$	Type of Estimator
$E(Y_{dti}) = \mu_{dt}$	$\hat{\bar{Y}}_{dt}$	$(1/N_{dt})\left(\sum_{s_{dt}} Y_{dti} + \sum_{\tilde{s}_{dt}} \hat{\bar{Y}}_{dt}\right)$	Direct
$E(Y_{dti}) = \mu_{.t}$	$\hat{\bar{Y}}_{.t}$	$(1/N_{dt})\left(\sum_{s_{dt}} Y_{dti} + \sum_{\tilde{s}_{dt}} \hat{\bar{Y}}_{.t}\right)$	Domain Indirect
$E(Y_{dti}) = \mu_{d.}$	$\hat{\bar{Y}}_{d.}$	$(1/N_{dt})\left(\sum_{s_{dt}} Y_{dti} + \sum_{\tilde{s}_{dt}} \hat{\bar{Y}}_{d.}\right)$	Time Indirect
$E(Y_{dti}) = \mu_{..}$	$\hat{\bar{Y}}_{..}$	$(1/N_{dt})\left(\sum_{s_{dt}} Y_{dti} + \sum_{\tilde{s}_{dt}} \hat{\bar{Y}}_{..}\right)$	Domain and Time Indirect

4. For the domain and time specific model, the BLUE for the model parameter is algebraically equivalent to the BLUP for the finite population mean. For the remaining models, the BLUE for the model parameter and the BLUP for the finite population mean are not the same.

5. It is straightforward to verify that the direct estimator is robust against model failure in the sense that it is unbiased, not only under the domain and time specific model, but under each of the models in the family. Indirect estimators are not robust in the same sense; each of the indirect estimators is biased under the domain and time specific model. Without evidence to the contrary, the domain and time specific model will be the most plausible in the family, and the bias of indirect estimators under this model will continue to be a major source of concern associated with applications of indirect estimators.

6. This simple example can also be used to help understand the importance of keeping the purpose of the analysis in mind when selecting an indirect estimator. Not all indirect estimators will be equally appropriate for a given analysis. For example, if the purpose of the analysis is to make comparisons across domains for a given time period, it would serve no purpose to use the domain indirect estimator above since this estimator would produce essentially the same estimate for every domain. Even though this is an extreme example, the point is clear. Domain indirect estimators are based on models that assume the expectation of the variable of interest is the same across domains with respect to some model parameter. This inconsistency between the purpose of the analysis and the method used to produce estimates will be avoided if a time indirect estimator is utilized. If, instead of making comparisons across domains, the purpose of the

analysis is to make comparisons across time periods within a given domain, it may be appropriate to select from among the domain indirect estimators. However, it should be stressed that, in practice, the performance of both domain and time indirect estimators depends on the available information and how accurately the model that incorporates this information depicts the actual application of interest.

In addition to the characteristics illustrated in the example above, there are several other, fairly well-known characteristics of indirect estimators that are important to keep in mind.

- Since they not only incorporate observations from the domain and time period of interest, but also from other domains and/or time periods, indirect estimators have smaller variances than the direct estimator in the same family. Holt, Smith, and Tomberlin (1979) discuss estimation of the variance of the modified (best linear unbiased) synthetic estimator and Royall (1979) presents variances of several indirect estimators resulting from various prediction models. Care must be taken since the variance of an indirect estimator may not lead to valid confidence intervals. See, for example, Räbäck and Särndal (1982) and Särndal and Hidiroglou (1989). Confidence intervals for biased estimators is a related issue that has been addressed by Miller (1992).

- Generally, a meaningful measure of error is difficult to produce for an indirect estimator for a specific domain and time. An indirect estimator will be biased if the model assumptions leading to the estimator are not satisfied, and the magnitude of the bias is likely to vary with each application. Estimation of biases is, of course, difficult. Gonzalez and Waksberg (1973) consider the problem of estimating the mean squared error of synthetic estimators, and Prasad and Rao (1990) discuss the estimation of the mean squared error of indirect estimators. Care must be taken when interpreting estimated mean squared errors of indirect estimators; some approaches provide an average measure over all domains rather than an individual measure associated with a specific domain.

- For a given application and estimator, biases in different domains will differ since the model will likely be a better representation of reality in some domains than in others. Many indirect estimators produce domain estimates whose distribution has smaller variance than the corresponding distribution of domain population values being estimated. That is, when domain population values are close to the average population value, indirect estimators have relatively small biases. However, when domain population values are not close to the overall population value, indirect estimators tend to have relatively large biases which act in such a way to make the estimates closer to the average population value. There is considerable evidence illustrating this characteristic (Gonzalez and Hoza 1978; Schaible et al. 1977 and 1979; and Heeringa 1981). Not all indirect estimators display this characteristic to the same extent. Spjøvoll and Thomsen (1987), Lahiri (1990), and Ghosh (1992) have recently addressed this problem and suggest constrained approaches.

3. INDIRECT ESTIMATORS IN U.S. FEDERAL PROGRAMS

As discussed above, indirect estimators can be classified into three types depending on how strength is borrowed. Irrespective of this classification, indirect estimators have different algebraic forms and are often classified as synthetic, regression, or composite estimators.

As with all indirect estimators, synthetic estimators may be domain indirect, time indirect, or domain and time indirect. For example, a domain indirect synthetic estimator for a population total may be written as

$$\hat{T}_{(syn),d,t} = \sum_{h=1}^{H} N_{dth} \hat{\bar{Y}}_{.th} ,$$

where $h = 1, 2, \ldots, H$ denotes poststrata and $\hat{\bar{Y}}_{.th}$ denotes the sample mean across all domains for time period t and poststratum h. Within each poststratum, this estimator simply uses the sample mean across all domains to estimate the y value for each population unit in domain d.

Depending on how the parameters are estimated, regression estimators may be direct or, like the synthetic estimator, domain indirect, time indirect, or domain and time indirect. For example, a domain indirect regression estimator for a population total may be written as

$$\hat{T}_{(reg),d,t} = \sum_{i=1}^{N_{dt}} \mathbf{x}_{dti} \hat{\beta}_{.t} ,$$

where $\mathbf{x}_{dti}$ denotes a row vector of known auxiliary variables and $\hat{\beta}_{.t}$, a column vector of estimators of the corresponding regression coefficients. The regression coefficients are estimated using y values from at least one domain other than d but within the time period t. Although the synthetic estimator is treated separately in this paper, it can be written as a special case of a regression estimator where the auxiliary variables are defined to be variables indicating whether or not each unit is in poststratum h or not.

Another interesting special case of indirect regression estimation arises when there is only one auxiliary variable and the sum of the population Y's is known for the entire population, but the sums for the domain populations are not known. The domain indirect regression (ratio) estimator in this case can be written as,

$$\hat{T}_{(reg),d,t} = \frac{X_{dt.}}{X_{.t.}} Y_{.t.} .$$

This estimator is interesting in that the uncertainty associated with it is not in any way due to sampling. In fact, under design-based theory it can be argued that since there is no sample, this is not an estimator. The Bureau of Economic Analysis uses this approach to produce state and county estimates of annual personal income.

A composite estimator may be written as,

$$\hat{T}_{(com),d,t} = w_{dt}\hat{T}_1 + (1 - w_{dt})\hat{T}_2 ,$$

where w_{dt} is a weight, usually between zero and one, and $\hat{T}_1$ and $\hat{T}_2$ are component estimators. Typically, in small area estimation applications, one component estimator is direct and the other is either domain or time indirect. Note that requiring a component estimator to be direct necessitates that at least one observation be available from the domain of interest. Synthetic and indirect regression estimators can be used even if there are no observations from the domain of interest. There are a variety of approaches to defining the weight for the composite estimator. The three program applications mentioned below are distinguished by different indirect component estimators and different approaches to estimating the composite estimator weight.

The eight indirect estimator programs identified in the Federal Committee on Statistical Methodology report (U.S. Office of Management and Budget, 1993) and briefly described here were initiated in response to a variety of needs and directives. Several are a direct result of legislative requirements to allocate federal funds. (Programs that provided figures allocate over 100 billion dollars annually.) Other programs were created in response to state needs for data and to standardize estimation methods across states. Another is viewed as a research program to develop improved methods. Table 2 provides summary information on these programs. The programs that use indirect estimators to publish estimates are located in five large statistical agencies. Synthetic, regression, and composite estimators that borrow strength over domains, over time, and over both domain and time are found among these programs. The estimation procedures for six of the programs are based on data from sample surveys. There is no sampling involved in the procedures used in the programs that produce estimates of personal income and postcensal populations. In some instances, a program

Table 2. Selected Characteristics of U.S. Federal Programs that Use Indirect Estimators to Publish Estimates

Agency	Estimator	Variables	Domain	Frequency
Bureau of the Census	Domain and time indirect regression	Postcensal populations	Counties	Annually
Bureau of the Census	Domain indirect composite	Median income for 4-person families	States	Annually
Bureau of Economic Analysis	Domain indirect regression (ratio)	Personal income, annual income, gross product	States and counties	Annually (Quarterly)
Bureau of Labor Statistics	Time indirect regression	Employment and unemployment	States	Monthly
National Agricultural Statistics Service	Domain indirect regression	Cotton, rice, and soybean acreage	Counties	Annually
National Agricultural Statistics Service	Time indirect composite	Livestock inventories, crop production and acreage	Counties	Annually
National Center for Health Statistics	Domain indirect synthetic	Infant and maternal health characteristics	States	Periodically
National Center for Health Statistics	Domain indirect composite	Disabilities, hospital utilization, physician and dental visits	States	Periodically

produces estimates for a single variable; in other instances, estimates are produced for numerous variables. At present, states and counties are the only domains for which indirect estimates are published. Four of the programs publish estimates for states, three for counties, and one for both states and counties. There is considerable variability in the frequency with which estimates are published. Two programs publish estimates only periodically, every few years. The remainder publish indirect estimates on a fixed schedule: four publish annually, one publishes annually with selected estimates on a quarterly schedule, and one publishes monthly.

4. RECOMMENDATIONS AND CAUTIONS

4.1 Recommendations

Indirect estimators rarely, if ever, are considered for federal statistical programs when sufficient resources to produce direct estimates of adequate precision are available. However, when direct estimation is judged to be inadequate, indirect estimation may, in some cases, prove to be a valuable alternative. There are reasons that direct estimators are preferable to indirect ones and, if federal statistical agencies are to improve the usefulness of indirect estimates, a number of important issues should receive additional attention. The brief recommendations that follow are discussed more fully in Indirect Estimators in Federal Programs (U.S. Office of Management and Budget, 1993).

- Generally, statistical programs are designed to produce direct estimates for specified large domains, and indirect estimates for other domains are considered only after the data have been collected. Planning for both direct and indirect estimators at the design stage should lead to improved data systems.

• Selection of an appropriate indirect estimation method should take into account the purpose for which estimates are to be used.

• More coordination and cooperation among Federal agencies would allow expanded access to the auxiliary information on which indirect estimators depend.

• Additional evaluations are needed to help determine whether indirect estimators are adequate for the intended purposes.

• Additional research on errors associated with indirect estimators is necessary. Not only should estimation of variances receive additional attention, but also estimation of biases, mean square errors, and confidence intervals.

• Indirect estimators should be distinguished from direct estimators. When indirect estimates are published, they should be accompanied by appropriate cautions and clear explanations of the model assumptions.

4.2 Cautions for Producers and Users of Indirect Estimates

As evidenced by the large and growing literature on indirect estimation methods, numerous researchers have been working on the challenging problems facing those who must produce estimates with inadequate resources. Many authors suggest new approaches or variations of existing approaches. However, only a few caution about the dangers associated with the use of indirect estimation methods.

"The synthetic estimator is a dangerous tool, but with careful further development, it has an attractive potential." (Simmons 1979)

"A workshop of this sort, focused on a specific technique, can spur development, but it can also be dangerous. The danger is that, from hearing many people speak many words about synthetic estimation we become comfortable with the technique. The idea and the jargon become familiar, and it is easy to accept that 'Since all these people are studying synthetic estimation, it must be okay.' We must remain skeptical and not allow familiarity to dull our healthy skepticism. There is reason for some optimism, but it must be guarded optimism." (Royall 1979)

" . . . a cautious approach should be adopted to the use of small area estimates, and especially to their publication by government statistical agencies. When government statistical agencies do produce model-dependent small area estimates, they need to distinguish them clearly from conventional sample-based estimates. . . . Before small area estimates can be considered fully credible, carefully conducted evaluation studies are needed to check on the adequacy of the model being used. Sometimes model-dependent small area estimators turn out to be of superior quality to sample-based estimators, and this may make them seem attractive. However, the proper criterion for assessing their quality is whether they are sufficiently accurate for the purposes for which they are to be used. In many cases, even though they are better than sample-based estimators, they are subject to too high a level of error to make them acceptable as the basis for policy decisions." (Kalton 1987)

Indirect estimators should only be considered when other, more robust alternatives are not available, and then, only with appropriate caution and in conjunction with substantial research and evaluation efforts. Both producers and users must not forget that, even after such efforts, indirect estimates may not be adequate for the intended purpose.

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REFERENCES

Dalenius, T. (1987), "Panel Discussion" in *Small Area Statistics*, New York: John Wiley and Sons.

Ericksen, E.P. (1974), "A Regression Method for Estimation Population Changes for Local Areas," *Journal of the American Statistical Association*, 69, 867-875.

Ghosh, M. (1992), "Constrained Bayes Estimation With Applications," *Journal of the American Statistical Association*, 87, 533-540.

Gonzalez, M.E. (1973), "Use and Evaluation of Synthetic Estimates," *Proceedings of the Social Statistics Section,* American Statistical Association, 33-36.

Gonzalez, M.E. (1979), "Case Studies on the Use and Accuracy of Synthetic Estimates: Unemployment and Housing Applications" in *Synthetic Estimates for Small Areas* (National Institute on Drug Abuse, Research Monograph 24), Washington, D.C.: U.S. Government Printing Office.

Gonzalez, M.E. and Hoza, C. (1978), "Small-Area Estimation with Application to Unemployment and Housing Estimates," *Journal of the American Statistical Association*, 73, 7-15.

Gonzalez, M.E. and Waksberg, J (1973), "Estimation of the Error of Synthetic Estimates," International Association of Survey Statisticians, Vienna, Austria, 18-25 August, 1973.

Heeringa, S.G. (1981), "Small Area Estimation Prospects for the Survey of Income and Program Participation," *Proceedings of the Section on Survey Research Methods*, American Statistical Association, 133-138.

Holt, D., Smith, T.M.F., and Tomberlin, T.J. (1979), "A Model Based Approach to Estimation for Small Subgroups of a Population," *Journal of the American Statistical Association*, 74, 405-410.

Kalton, G. (1987), "Panel Discussion" in *Small Area Statistics*, New York: John Wiley and Sons.

Laake, P. (1979), "A Prediction Approach to Subdomain Estimation in Finite Populations," *Journal of the American Statistical Association*, 74, 355-358.

Lahiri, P. (1990), "Adjusted Bayes and Empirical Bayes Estimation in Finite Population Sampling," *Sankhya B*, 52, 50-66.

Miller, S.M. (1992), "Confidence Interval Coverage for Biased Normal Estimators," *Proceedings of the Section on Survey Research Methods*, American Statistical Association.

National Center for Health Statistics (1968), *Synthetic State Estimates of Disability* (PHS Publication No. 1759), Washington, D.C.: U.S. Government Printing Office.

National Institute on Drug Abuse (1979), *Synthetic Estimates for Small Areas* (Research Monograph 24), Washington, D.C.: U.S. Government Printing Office.

Prasad, N.G.N. and Rao, J.N.K. (1990), "The Estimation of the Mean Squared Error of Small-Area Estimators," *Journal of the American Statistical Association*, 85, 163-171.

Purcell, N.J. and Kish, L. (1979), "Estimation for Small Domains," *Biometrics*, 35, 365-384.

Räbäck, G. and Särndal, C.E. (1982), "Variance Reduction and Unbiasedness for Small Area Estimators," *Proceedings of the Social Statistics Section,* American Statistical Association, 541-544.

Royall, R.A. (1973), "Discussion of papers by Gonzalez and Ericksen," *Proceedings of the Social Statistics Section,* American Statistical Association, 42-43.

Royall, R.A. (1979), "Prediction Models in Small Area Estimation," in *Synthetic Estimates for Small Areas* (National Institute on Drug Abuse, Research Monograph 24), Washington, D.C.: U.S. Government Printing Office.

Särndal, C.E. (1984), "Design-Consistent versus Model-Dependent Estimation for Small Domains," *Journal of the American Statistical Association*, 79, 624-631.

Särndal, C.E. and Hidiroglou, M.A. (1989), "Small Domain Estimation: A Conditional Analysis," *Journal of the American Statistical Association*, 84, 266-275.

Schaible, W.L. (1993), "Use of Small Area Estimators in U.S. Federal Programs," in *Small Area Statistics and Survey Designs*, Vol. 1, Central Statistical Office, Warsaw, Poland.

Schaible, W.L., Brock, D.B., Casady, R.J., and Schnack, G.A. (1979), "Small Area Estimation: An Empirical Comparison of Conventional and Synthetic Estimators for States," PHS Publication No. 80-1356, Washington, D.C.: U.S. Government Printing Office.

Schaible, W.L., Brock, D.B., and Schnack, G.A. (1977), "An Empirical Comparison of the Simple Inflation, Synthetic and Composite Estimators for Small Area Statistics," *Proceedings of the Social Statistics Section*, American Statistical Association, 1017-1021.

Simmons, W.R. (1979), "Discussion of a paper by Levy," in *Synthetic Estimates for Small Areas* (National Institute on Drug Abuse, Research Monograph 24), Washington, D.C.: U.S. Government Printing Office.

Spjøvoll, E. and Thomsen, I. (1987), "Application of Some Empirical Bayes Methods to Small Area Statistics," *Proceedings of the International Statistical Institute,* Vol. 2, 435-449.

U.S. Office of Management and Budget (1993), "Indirect Estimators in Federal Programs," *Statistical Policy Working Paper 21*, National Technical Information Service, Springfield, Virginia.

Woodruff, R.S., (1966), "Use of a Regression Technique to Produce Area Breakdowns of the Monthly National Estimates of Retail Trade," *Journal of the American Statistical Association*, 79, 496-504.