APPORTIONMENT METHODS FOR THE HOUSE OF REPRESENTATIVES AND THE COURT CHALLENGES

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1. INTRODUCTION

Article I, Section 2 of the U.S. Constitution requires that the House of Representatives "shall be apportioned among the several States---according to their respective Numbers," and that "each State shall have at least one Representative." That section also includes the requirement that an enumeration of the population for the purpose of apportioning the House be conducted every ten years. The quoted words obviously do not explicitly state what method should be used for apportionment, and for over 200 years the issues of which is the "best" method and which methods are constitutional have been debated.

The "best" method issue is, in this author's opinion, unresolvable, since it depends on the criteria employed. However, the constitutional question was at least partially resolved on March 31, 1992, when Justice Stevens delivered an opinion for a unanimous Supreme Court upholding the constitutionality of the currently used apportionment method, equal proportions (EP), also known as the Hill or Huntington method.

The path to this resolution began in 1991 when the states of Montana and Massachusetts initiated separate lawsuits in federal court (*Montana v. United States Department of Commerce* 1991; *Massachusetts v. Mosbacher* 1992) challenging, for the first time in U.S. history, the constitutionality of the current method. Montana proposed two methods as alternatives to EP. Their preferred methods are the method of harmonic means (HM), also known as the Dean method, and the method of smallest divisors (SD), also known as the Adams method, both of which would have given Montana two seats instead of the single seat allocated by EP, but would have not increased Massachusetts' EP allocation of ten seats. Massachusetts proposed, using different arguments, the use of the method of major fractions (MF), also known as the Webster method, which would have allocated eleven seats to Massachusetts, and one to Montana.

The two cases were considered by separate three-judge panels. The panel in the Montana case, by a two-to-one vote, declared EP unconstitutional, while the judges in the Massachusetts case unanimously upheld the constitutionality of EP. The ruling in the Montana case was appealed to the Supreme Court (*United States Department of Commerce v. Montana* 1992), with Massachusetts filing a friend-of-the-court brief before the Supreme Court in order to present their position in favor of MF. On March 4, 1992, the Supreme Court heard the case and 27 days later unanimously overruled the decision of the three-judge panel in the Montana case.

This paper discusses the mathematical and statistical issues in these cases. This author wrote the declarations that served as a basis for many of the technical arguments used by the defense in these cases, and this paper is in part an outgrowth of that work. Section 2 of the paper provides an historical background on the apportionment issue and a discussion of the properties of the major apportionment methods. Balinski and Young (1982), the major source of the material in that section, provides a more detailed treatment of these matters. Sections 3 and 4 discuss the issues debated before the three-judge panels in the Montana and Massachusetts cases, respectively. Finally, the Supreme Court appeal is discussed in Section 5.

Due to space limitations, approximately two-thirds of the original paper, including the proofs of all theorems and the list of references, are omitted here. The complete paper is available from the author.

2. HISTORICAL BACKGROUND AND PROPERTIES OF METHODS

Six apportionment methods are considered here. They are the four methods mentioned in the Introduction, the method of greatest divisors (GD), also known as the Jefferson method, and the method of greatest remainders (GR), also known as the Hamilton or Vinton method.

All of these methods except GR are members of a class of apportionment methods known as divisor methods. Although there are an infinite number of possible divisor methods, only the five considered here have had any significant role in apportionment history. They will be referred to as the historical divisor methods. With a divisor method, the number of seats assigned to a state is a function of its population, p, and a divisor, λ, which can be thought of as a target district size. The same value of λ must be used for each state. If $\lfloor p/\lambda \rfloor = b$ (where $\lfloor x \rfloor$ denotes the integer portion of x), then the state receives either b or $b+1$ seats. It receives $b+1$ seats if and only if $p/\lambda > \delta(b)$, where δ, the function that determines the rounding, depends on the particular method. δ is a strictly increasing function of b satisfying $b \le \delta(b) \le b+1$ for all nonnegative integers b. $\delta(b)$ equals: b for SD, 2b(b+1)/(2b+1) for HM, $\sqrt{b(b+1)}$ for EP, b+.5 for MF, and b+1 for GD.

A GR apportionment is obtained slightly differently. Begin with a fixed house size n, and a set of N states with populations p_i, i=1,...,n. Let $d = \sum_{i=1}^{N} p_i/n$, the national average district size; $q_i = p_i/d$, the exact quota for state i; and a_i denote the number of seats allocated to state i under any method. Then for GR, either $a_i = \lfloor q_i \rfloor$ or $a_i = \lfloor q_i \rfloor + 1$,

with $a_i = \lfloor q_i \rfloor + 1$ for the $n - \sum_{i=1}^{N} \lfloor q_i \rfloor$ states with largest fractional remainders, $q_i - \lfloor q_i \rfloor$.

State i is said to satisfy quota if $\lfloor q_i \rfloor \le a_i < \lfloor q_i \rfloor + 1$. GR, can never violate quota. Furthermore, although all five historical divisor methods can violate quota in theory, EP, HM and MF would never have violated quota for any of the 21 censuses through 1990, while SD and GD would have violated quota for at least one state for each census since 1820. For example, for California for 1990, $a_i = 50$ for SD and $a_i = 54$ for GD, while $q_i = 52.124$.

GD was used to apportion the House for the first five censuses through 1830. Eventually, Congress became dissatisfied with this method because it appeared to favor large states. SD, MF and HM were developed as alternatives. MF was used in 1840. GR was the specified method from 1850-1900. However, Congress became disenchanted with GR, because under this method, unlike any divisor method, it is possible, with a fixed set of state populations, for a state to lose seats if the House size is increased, an anomaly known as the "Alabama paradox."

Congress returned to MF for the 1910 census. About the time of the 1920 census, Professor Edward Huntington of Harvard refined and became the principal champion of EP, which had first been developed by Joseph Hill of the Census Bureau in 1911. The case for EP rested primarily on the pairwise optimality tests. An apportionment is said to be pairwise optimal with respect to a particular measure of inequity if no transfer of representatives between any pair of states can decrease the amount of inequity between these states. HM is pairwise optimal with respect to absolute difference in average district sizes, that is with respect to the measure $|p_i/a_i - p_j/a_j|$ between states i and j. MF is pairwise optimal with respect to the absolute difference in per capita shares of a representative, that is $|a_i/p_i - a_j/p_j|$. However, EP is pairwise optimal with respect to relative differences in both district sizes and shares of a representative, which became the key argument for EP. (The relative difference between two positive numbers x,y is $|x-y|/\min\{x,y\}$.)

The opposition to the views of Huntington was led by Professor Walter Wilcox of Cornell, who supported MF. He was of the opinion that EP is biased in favor of small states, while MF is mathematically neutral between small and large states. Huntington disagreed, contending that it is actually EP that is mathematically neutral in this respect.

Congress failed to reapportion the House at all after the 1920 census, but in an attempt to resolve the technical dispute, the Speaker of the House requested that the National Academy of Sciences (NAS) review the mathematical aspects of the problem of reapportionment. A NAS committee issued a report in 1929 (Bliss et al.). The report considered the five divisor methods discussed in this paper and focused on the pairwise comparison tests described above. The committee adopted Huntington's reasoning that EP is preferred on the basis of the pairwise tests for which it is optimal.

The 1930 allocations for EP and MF were identical, so Congress took no further action after that census. Under the applicable law, the House was automatically apportioned under the method last used, MF.

In 1940, however, EP and MF differed, with Arkansas allocated 7 seats by EP and 6 by MF, while Michigan was allocated 17 by EP and 18 by MF. In 1941, legislation was enacted apportioning the House by EP. This method has been used ever since and, under the 1941 law, its continued use is automatic until superseding legislation is enacted.

In 1948, a new NAS committee revisited the apportionment issue and also endorsed EP (Morse et al.). Their report included the new argument that among the four pairwise comparison tests previously mentioned for which either EP, HM or MF are optimal, EP is always superior to each of the other four divisor methods for at least three of them. For example, it can be shown that EP is superior to MF for each of these tests except absolute difference in shares of a representative and EP is superior to HM except for absolute difference in district sizes.

Much of the interest in the apportionment issue since the mid 1970s is a result of the work of Michael Balinski and H. Peyton Young. After first supporting a method known as the quota method, not described here, they eventually became proponents of MF. Their main argument for MF was, like Wilcox's decades earlier, their belief that MF is the only historical divisor method that is not biased in favor of either large or small states. Their work, culminating in the book *Fair Representation* (Balinski and Young 1982), presented a number of new theoretical and empirical results to support their view.

For example, corresponding to a divisor λ and a divisor method based on δ, they considered intervals

$$[\delta(b-1)\lambda,\ \delta(b)\lambda], \qquad b = 1,2,3..., \qquad (2.1)$$

(where $[\alpha,\beta]$ denotes $\{x: \alpha \le x \le \beta\}$), that is populations for which b seats are assigned, and established that MF is pairwise unbiased in the sense that if states 1 and 2 have independent populations p_1 and p_2, respectively, uniformly distributed in intervals $[\delta(b_1-1)\lambda,\ \delta(b_1)\lambda]$, $[\delta(b_2-1)\lambda,\ \delta(b_2)\lambda]$, respectively, for positive integers $b_2 > b_1$, then the probability is .5 that state 2 is favored over state 1 in the sense that $b_2/p_2 > b_1/p_1$. They also established that MF is the only one of the five historical divisor methods with this property.

Their empirical results included comparisons of the historical divisor methods for the "bias ratio" and "percentage bias," two measures of apportionment method bias developed by these authors. For both measures they excluded states with exact quotas below .5 as their means of compensating for the constitutional requirement of at least one representative per state, a provision which in effect creates a constitutionally mandated bias in favor of the small states. The bias ratio was obtained by first computing for each census the number of pairs of non-excluded states i,j with $p_i < p_j$, where state i, the smaller

state, is favored in the sense that $a_i/p_i > a_j/p_j$. The total of the number of pairs for which the smaller state was favored, summed over the 19 censuses through 1970, was then divided by the total number of pairs of non-excluded states in these 19 censuses to obtain the bias ratio. Balinski and Young's results for the five historical divisor methods showed MF at 51.5% was closest to the ideal value of 50%, and EP next closest at 54.6%.

They computed percentage bias for each census by first dividing the non-excluded states into approximately equal classes of large (L), middle, and small states (S). The percentage bias for each census is then $(\sum_S a_i/\sum_S p_i)/(\sum_L a_j/\sum_L p_j) - 1$ expressed as a percentage. Balinski and Young's (1982) results, averaged over the 19 censuses through 1970, showed MF at 0.3% to be closest among the five historical divisor methods to the ideal value of 0, with EP second at 3.4%.

A final set of properties of apportionment methods are measures of total error of an apportionment. Let $d_i=p_i/a_i$, $d=(\sum p_j)/n$, $s_i=1/d_i$, and $s=1/d$. Three classes of error measures are, for $\rho \geq 1$,

$$\sum_{i=1}^{N} |a_i - q_i|^\rho, \tag{2.2}$$

$$\sum_{i=1}^{N} a_i |d_i - d|^\rho, \tag{2.3}$$

$$\sum_{i=1}^{N} p_i |s_i - s|^\rho. \tag{2.4}$$

(2.2), (2.3) and (2.4) are, respectively, the sum of the ρ-th power of each state's absolute deviation from its exact quota, each district's absolute deviation from the national average district size, and each person's absolute deviation from the national average share of a representative.

GR minimizes (2.2) for all $\rho \geq 1$ (Birkoff 1976), while for $\rho=2$, EP minimizes (2.3) (Huntington 1928) and MF minimizes (2.4) (Owen 1921). As observed by Gilford (1981), for $\rho=1$, (2.3) and (2.4) are minimized by GR.

3. THE MONTANA DISTRICT COURT CASE

The Montana lawsuit was primarily based on the following legal reasoning (Racicot et al. 1991). In *Wesberry v. Sanders* (1964), the U.S. Supreme Court had declared that the *intrastate* redistricting of congressional districts must be accomplished to provide "equal representation for equal numbers of people," that is the "one person, one vote" principle. That case did not set any test for meeting this principle but, citing subsequent intrastate decisions, the plaintiffs concluded that the courts required this principle be met in the intrastate context by minimizing "absolute population variances between districts" and that this requirement also applied to interstate apportionment. Of course, no court had previously ruled that the "one person, one vote" principle applied to interstate apportionment, much less that a certain test was superior to another for interstate apportionment. In fact, even for intrastate redistricting, no court had specifically ruled that a test based on district sizes is a better test than one based on shares of a representative, or that absolute difference is a better measure than relative difference. Furthermore, this issue of the best test would not even be relevant for intrastate redistricting, since differences, at least in theory, can be made as close to zero as desired for any of these methods of measurement. Finally, the plaintiffs never offered any specific reasons why absolute difference in district sizes is the only appropriate test beyond citing these redistricting cases.

After using these prior cases as their rationale for their view that absolute difference between district sizes is the only appropriate test, the plaintiffs noted that the pairwise test for which HM is optimal is a criterion that they considered consistent with the cited cases. In addition, in the affidavits of the plaintiffs' experts, Hill (1991) and Tiahrt (1991), it was observed that for the 1990 census, among EP, HM and SD, HM produces the smallest variance while SD produces the smallest range, and the plaintiffs declared that either of these are appropriate tests of inequity among district sizes. Furthermore, the Hill affidavit included the formula used in computing the variances, namely, using the notation of Section 2,

$$\sum_{i=1}^{50} (d_i - d)^2/49. \tag{3.1}$$

The defendants' reply to the plaintiffs' assertions (Gerson, Poppler et al. 1991) contained a number of legal arguments, including the argument that apportionment of the House is a political question to be decided by Congress, and that it should not be considered by the courts. It was also argued that in carrying out its constitutionally mandated duty to apportion the House, Congress should be allowed broad discretion by the courts even if the issue is considered justiciable. In addition, it was observed that interstate apportionment is very different from intrastate redistricting, since large differences in district sizes between states are inevitable because districts cannot cross state lines and each state must have at least one representative. Consequently, the defendants claimed that the redistricting cases cited by the plaintiffs are not applicable to interstate apportionments.

The (U.S.) Government used these arguments before each of the courts that considered the two apportionment cases. In addition to the above arguments, substantive arguments were presented to demonstrate the advantages of EP, based primarily on the declaration of this author (Ernst 1991a), which will be the focus in this paper. There was no attempt to demonstrate that EP is clearly superior to all other apportionment methods, or the only constitutional method, but instead that neither of these claims is true for any other apportionment method or set of apportionment methods which exclude EP.

After first reviewing the apportionment history, including the 1929 NAS report, we responded to the general argument that absolute differences in district sizes is the only proper

criterion for evaluating an apportionment. We pointed out that it can be argued that a test involving differences in shares of a representative is a better test of "the one person, one vote" principle for interstate apportionment than a test involving differences in district sizes, since share of a representative measures the portion of a vote to which a person is entitled in the House. It was also observed that intrastate redistricting and interstate apportionment are conceptually very different, since in the former case, the people in each state are allocated to a fixed number of districts, while in the latter case, districts are allocated to the fixed number of people in the various states.

In addition to increasing Montana's EP allocation, HM would decrease Washington's allocation from nine seats to eight. It was noted that for 1990, as guaranteed by the optimality results for the pairwise difference tests, the relative difference between Washington's and Montana's average district sizes and average shares of a representative under EP (48.0%) is smaller than under HM (52.1%). It was also observed that the relative difference between Montana's average district size and the national average district size is 40.4% under EP and 42.5% under HM, while Washington's is 5.4% under EP and 6.7% under HM.

Although the plaintiffs declared the proper measure of inequity in an apportionment is absolute population variance among all districts and claimed that HM results in the smallest such variance, the defendants observed that it is actually EP that always minimizes this measure, since it minimizes (2.3) with $\rho=2$. The reason for the discrepancy in the claims is that the formula used by the plaintiffs, (3.1), did not take into account the number of districts in each state. Their formula measures variability among the mean district sizes of the 50 states, not the variance of the sizes of the 435 districts which is the criterion actually stated in the plaintiffs' briefs.

As for SD, it is indeed true that SD minimizes the range of district sizes for the 1990 census among the three methods considered by the plaintiffs. The defense case against SD focused on its tendency to violate quota. We noted that while, for 1990, California's exact quota is 52.124 seats, SD only allocates it 50 seats and also results in quota violations for Illinois, New York and Ohio. It was also noted that if SD had been employed for all 21 censuses, quota violations would have resulted for every census since 1820, with a total of 47 violations.

By a two-to-one majority, the three-judge panel in the Montana case upheld Montana's position that equal proportions was unconstitutional (Lovell 1991). Judges Lovell and Battin, both from Montana, constituted the majority. They agreed with the plaintiffs' argument that the "one person, one vote" principle applies to interstate apportionment and, citing prior intrastate redistricting cases, that absolute difference in district sizes is the only proper standard for testing this principle. The judges provided their rationale for rejecting tests involving representatives per person or using relative differences, in two footnotes, stating:

> By arguing that proportions and percentages are the proper criteria, rather than absolute numbers, Defendants ignore the fact that each number represents a person whose voting rights are potentially impacted by the population disparities.
>
> The Constitution decreed that one house should be chosen on the basis of population (persons per representative) and Congress cannot ignore that mandate by choosing a method which considers each person's share of a representative.

This author has been unable to understand the meaning of either of these quotes.

The majority never made clear what specific tests involving absolute differences in district sizes should be used, but they considered that HM comes closer than EP to satisfying the "one person, one vote" principle, and concluded that the use of EP is unconstitutional. They did, however, reject SD from consideration based on the quota violations.

Circuit Court Judge O'Scannlain of Oregon, while agreeing with the majority on the justiciability of the case, dissented on the merits, noting several points (O'Scannlain 1991). He first found, as did the majority, that SD is inconsistent with the constitutional requirement of allocating House seats by population, since it results in quota violations for four states in 1990. He cited the fact, from the defense declaration, that the relative difference between Montana's and Washington's average district sizes is larger under HM than under EP. Judge O'Scannlain also stated that range of district sizes is not the best test of disparity, noting that range only considers the largest and smallest of the 435 congressional districts.

Judge O'Scannlain's opinion focused on measures of total error. Citing the defense declaration, he observed that (2.3) with $\rho=2$, not (3.1), measures variance among all districts, and that EP, not HM, minimizes the appropriate variance. Judge O'Scannlain interpreted one of the statements in the majority opinion to require the test (2.3) with $\rho=1$. He calculated that EP for 1990 produces a lower value for this measure than HM. The Judge concluded: "In sum, neither of the formulae proposed by the State lead to less population variance than the Hill equal proportions formula in use for the last fifty years. The State, in my view, has failed to demonstrate that a better formula exists than the one chosen by Congress."

4. MASSACHUSETTS DISTRICT COURT CASE

The Massachusetts case was much more complex than the Montana case in terms of the technical issues involved. Massachusetts would have received eleven seats for 1990 if either MF, GD or GR had been used. The plaintiffs chose only to claim that MF is constitutionally superior to EP (Harshbarger et al. 1991). MF, in addition to increasing Massachusetts' EP allocation, would reduce Oklahoma's EP allocation of six seats to five seats, but would produce the same apportionment for the remaining 48 states as EP.

The plaintiffs claimed that EP is unconstitutional for three separate reasons, the first two of which were based on

the work of Balinski and Young (1982). Their major claim was that EP is unconstitutionally biased on favor of small states. They also found EP lacking because it, unlike MF, can yield apportionments which violate the "near the quota" principle. (This issue is discussed only in the full paper, where the "near the quota" concept is defined.) Finally, the plaintiffs claimed that the "one person, one vote" principle in interstate apportionment is best met by the pairwise test for which MF is optimal, absolute difference in shares of a representative.

The plaintiffs retained as their expert, H. Peyton Young, who wrote three affidavits in support of Massachusetts' claims (Young 1991), which formed the heart of their case.

Young's key points on the bias issue included the following. He described the percentage bias test, mentioned in Section 2, and presented the percentage bias figures averaged over all 21 censuses for EP and MF, which are virtually the same as those in Section 2 for the first 19 censuses. He stated that the percentage bias (in absolute value) for MF was less than or equal to the percentage bias for EP for each of the 21 censuses. Young also referred to his theoretical result on the unbiasedness of MF mentioned in Section 2.

To support their claim that absolute difference in shares of a representative is the best pairwise test, the plaintiffs essentially used the same reasoning that the defendants had used in the Montana district court to argue the superiority of share of a representative over district size as a test of the "one person, one vote" principle. However, they had no real argument to support the claim that absolute difference is a better measure of inequity than relative difference for pairwise comparisons. The plaintiffs were only able to argue that their preferred pairwise test is best since MF is optimal for it and MF is, in their opinion, unbiased.

The technical arguments used by the defense (Gerson, Budd et al. 1991) were based primarily on three declarations written by this author (Ernst 1991b). The key point of contention on the bias issue was the plaintiffs' assumption that only states with exact quotas less than .5 should be excluded in bias measures. The defense argued that it would be more appropriate to exclude all states with exact quotas less than 1, since even though all such states are overrepresented, this is an overrepresentation mandated by the Constitution.

The following are some of the changes in the empirical results that we noted occurred with this change in the set of excluded states. While the bias ratio presented in Section 2 is 54.6% for EP and 51.5% for MF, this ratio for the same 19 censuses with all states with exact quotas below 1, instead of only those below .5, excluded is 50.8% for EP and 47.4% for MF. Similarly, the average percentage bias presented in Section 2 is 3.4% for EP and .3% for MF, while with all states with exact quotas below 1 excluded, it is 1.8% for EP and -.9% for MF. Furthermore, with all states with exact quotas less than 1 excluded, Young's assertion that the percentage bias for MF never exceeded the percentage bias for EP for each of the 21 censuses does not hold. In fact, with these states excluded, the percentage bias for the 1990 census is -.6% for EP and -1.0% for MF. That is, by this measure, the 1990 EP apportionment favors the large states and substitution of MF would simply increase the magnitude of the favoritism.

Two new theoretical results were obtained by the defendants. First it was observed that Balinski and Young's (1982) result that MF is pairwise unbiased is dependent on use of the partition (2.1), which for MF reduces to

$$[.5\lambda,\ 1.5\lambda],\ [1.5\lambda,\ 2.5\lambda],\ [2.5\lambda,\ 3.5\lambda],\ \ldots\ . \quad (4.1)$$

However, we argued that the alternate partition

$$[\lambda,\ 2\lambda],\ [2\lambda,\ 3\lambda],\ [3\lambda,\ 4\lambda],\ \ldots \quad (4.2)$$

would be more consistent with the exclusion of all states with exact quotas less than 1. Partition (4.2) leads to the following very different result than partition (4.1) on the pairwise bias of MF.

Theorem 4.1. For a divisor λ, if states 1 and 2 have independent populations p_1, p_2 uniformly distributed in intervals $[b_1\lambda,\ (b_1+1)\lambda]$ and $[b_2\lambda,\ (b_2+1)\lambda]$, respectively, for positive integers $b_2 > b_1$, and the states have allocations a_1 and a_2, respectively, then for an MF apportionment the probability is greater than .5 that $a_2/p_2 > a_1/p_1$.

Thus, in the sense of Theorem 4.1, MF is pairwise biased in favor of large states.

The defendants' second theoretical result on bias is:

Theorem 4.2. With the assumptions and notation of Theorem 4.1, $E(d_2)=E(d_1)$ for EP and $E(d_2)<E(d_1)$ for MF.

Thus, in the sense of Theorem 4.2, EP is unbiased and MF is biased in favor of large states.

The defendants responded in several ways to the plaintiffs' claim that absolute difference in average shares of a representative is the best test of the "one person, one vote" principle. We noted that for 1990, as guaranteed by the theory, the relative difference between Oklahoma's and Massachusetts' average district sizes and average shares of a representative, and the absolute difference between the two states' average district sizes are smaller under EP than under MF.

The defendants noted the symmetry in the fact that among the four pairwise tests for which either EP, MF and HM are optimal, the plaintiffs in this case consider the one test for which MF is superior to EP to be the only appropriate test, just as the plaintiffs in the Montana case consider absolute difference in average district sizes to be the only appropriate test since it the only one of these four tests for which HM is superior to EP. The defendants expressed concurrence with Balinski and Young's (1975, p. 709) rhetorical question: "Why choose.....one divisor criterion [rather] then another?"

The three-judge panel in this case, in a unanimous decision, written by Judge Woodlock (1992) of Massachusetts, upheld the constitutionality of EP. Although the judges agreed with the plaintiffs that the "one person, one vote" principle applies to interstate apportionment, they

rejected each of the three major substantive issues raised by the plaintiffs.

On the bias issue, the court observed that with states with exact quotas below 1 excluded, "the historical bias showing made by plaintiffs all but evaporates," and that for the 1990 census, EP yields an apportionment with a percentage bias closer to 0 than MF.

The opinion explicitly addressed not only Massachusetts' claim that the pairwise test that best meets the "one person, one vote" principle in interstate apportionment is absolute difference in average shares of a representative, but also the claim in the Montana case that absolute difference in average district sizes is the only constitutional test. Judge Woodlock stated simply that "we can find nothing in the Constitution mandating a particular mathematical formula be employed to the exclusion of others."

The court, summarizing their views, stated: "The Constitution does not prescribe a particular formula, a specific methodology or a set standard to embody the 'one person, one vote' principle in this complex setting." The judges concluded that EP does satisfy this principle and hence the courts have no authority to interpose a different method than the one adopted by Congress. It is clear from the opinion that their ruling would have been the same if they had the Montana case before them.

Massachusetts did not appeal this decision to the Supreme Court. However, Massachusetts did present their views on apportionment methods to the Supreme Court through a friend-of-the-court brief in the Montana case.

5. THE SUPREME COURT CASE

The (U.S.) Government appealed the decision of the three-judge district court in the Montana case to the Supreme Court, which granted an expedited review.

Generally, new factual information is not introduced on appeal, and for the most part, both sides did adhere to this rule. No important new issues were raised. The few new points that were brought out are discussed in the full paper.

On March 31, 1992, the Supreme Court unanimously upheld the constitutionality of EP, in an opinion written by Justice Stevens.

Justice Stevens observed that while the same principle of equality that the Supreme Court requires in intrastate districting might apply to interstate apportionment, he did not find that the facts constituted a violation of the Wesberry standard. He noted that there is no incompatibility within a state in minimizing both absolute and relative differences, and that all districts within a state can be brought closer to the ideal simultaneously. However, for 1990, HM, while bringing Montana's average district size closer to the ideal district size as measured by absolute difference, brings Washington's average district further away from the ideal district size with respect to absolute difference, and moves both states further from this ideal with respect to relative difference.

Justice Stevens also noted that it can be argued, as in Judge O'Scannlain's dissent in the Montana district court case, that a measure of deviation from the ideal district size should take into account the number of districts in each state.

Justice Stevens then made the critical observation that "neither mathematical nor constitutional interpretation provides a conclusive answer" to the question of the best measure of inequality among the four measures obtained by pairing either absolute or relative difference with either district size or share of a representative. As had Judge Woodlock in the Massachusetts district court case, he concluded: "The polestar of equal representation does not provide sufficient guidance to allow us to discern a single constitutionally permissible course."

The opinion further observed that the goal of mathematical equality, while appropriate in the intrastate context, is illusory for interstate apportionment, since each state must have at least one representative and districts cannot cross state lines. In addition, since the Constitution expressly authorizes Congress to enact legislation to carry out its delegated responsibilities, its choice of a method that apportions representatives "according to their respective Numbers" commands far more deference than a state redistricting decision that can be required to meet a rigid mathematical standard.

The bias issue raised by Massachusetts was discussed in a footnote, which first described Balinski and Young's (1982) views and then simply noted, citing the opinion of the Massachusetts district court, that this contention has been disputed. Later in the opinion, Justice Stevens returned to this issue, stating that a fair apportionment required some compromise between the interests of the smaller and larger states, and indicating that Congress had been delegated the authority in the Constitution to reach this compromise.

Justice Stevens concluded his answer to this 200 year old constitutional question, stating:

> The decision to adopt the method of equal proportions was made by Congress after decades of experience, experimentation, and debate about the substance of the constitutional requirement. Independent scholars supported both the basic decision to adopt a regular procedure to be followed after each census, and the particular decision to use the method of equal proportions. For a half century the results of that method have been accepted by the States and the Nation. That history supports our conclusion that Congress had ample power to enact the statutory procedure in 1941 and to apply the method of equal proportions after the 1990 census.

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*This paper reports the general results of research undertaken by Census Bureau staff. The views expressed are attributable to the author and do not necessarily reflect those of the Census Bureau.