VARIANCE ESTIMATION FOR PRICE INDEXES FROM A TWO–STAGE SAMPLE WITH ROTATING PANELS

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1. INTRODUCTION

An important application of panel survey design in many government statistical programs is the estimation of price indexes. In the United States, Laspeyres indexes calculated by the Bureau of Labor Statistics (BLS) for consumer prices, industry input and output prices, and export and import prices are derived from panel surveys of establishments. Prices are collected monthly or quarterly and continuing series of indexes are published for various classes of commodities and industries. Key statistical questions are how to efficiently combine data from the panels to estimate an index and, having done that, how to estimate the variance of the result.

A number of sources of statistical variation exist that may be reasonably accounted for in index estimators. Balk and Kersten (1986) and Biggeri and Giommi (1987) studied the effect on CPI's of using estimated weights derived from a household expenditure survey. Andersson, Forsman, and Wretman (1987a, b) examined sources of error in a CPI due sampling of points of purchase. Earlier studies of sampling error were done by Banerjee (1960), Wilkerson (1967), and Kish (1968). As an illustration that the study of precision of index numbers is not a recent phenomenon, the collected papers of Edgeworth (1925) contain work published in 1889 on the subject.

This paper examines the problem of index estimation under a rotating panel survey design in a simplified case where the universe of items is stable. Statistical calculations are conditional on the set of weights used to aggregate commodity groups with the sampling of items being the source of variation studied. Section 2 introduces notation and a superpopulation model. Sections 3 and 4 discuss classes of index estimators. Section 5 derives linearization variance estimators for long and short–term indexes. Section 6 summarizes results of a simulation study and Section 7 concludes with some additional discussion.

2. NOTATION AND THE MODEL

The population of items is divided into H strata with stratum h containing N_h establishments. Establishment (hi) contains M_{hi} items and the total number of items in all establishments in stratum h is $M_h = \sum_{i=1}^{N_h} M_{hi}$. At time t the price of item j in establishment (hi) is p^t_{hij} and the price relative between times t and 0, the base period, is $r^t_{hij} = p^t_{hij}/p^0_{hij}$. The values of t are assumed to be integers, t=1,...,T. The quantity of item (hij) purchased in the base period is q^t_{hij}. The finite population value of the long–term fixed–base Laspeyres price index for comparing period t to period 0 is then defined as

$$I^{t,0} = \sum_{h=1}^{H}\sum_{i=1}^{N_h}\sum_{j=1}^{M_{hi}} p^t_{hij}\,q^0_{hij} \Big/ \sum_{h=1}^{H}\sum_{i=1}^{N_h}\sum_{j=1}^{M_{hi}} p^0_{hij}\,q^0_{hij}$$
$$= \sum_{h=1}^{H}\sum_{i=1}^{N_h}\sum_{j=1}^{M_{hi}} W^0_{hij}\,r^t_{hij} \qquad (1)$$

where $W^0_{hij} = p^0_{hij}\,q^0_{hij} \Big/ \sum_{h=1}^{H}\sum_{i=1}^{N_h}\sum_{j=1}^{M_{hi}} p^0_{hij}\,q^0_{hij}$. Based on the long–term index, the population short–term index for comparing periods t and s (s < t) is defined as $I^{t,s} = I^{t,0}/I^{s,0}$. Short–term changes that are often of economic interest are monthly, quarterly, semiannual, and annual changes. For simplicity, we consider a static universe of items, i.e. one in which there are no births, deaths or quality changes, although the problems created by a changing universe are certainly some of the most important and difficult ones in price index estimation.

We will consider both design and model–based properties of various index and variance estimators. The model to be used is a simple autoregressive one given by

$$\begin{aligned} r^t_{hij} &= \alpha_{th} + \omega_{thi} + \varepsilon_{thij} \\ \varepsilon_{thij} &= \rho_h \varepsilon_{t-1,hij} + u_{thij} \end{aligned} \qquad (2)$$

where $E(\omega_{thi}) = E(\omega_{t_1h_1i_1}\omega_{t_2h_2i_2}) = 0$ (for all t_1, t_2, h_1, h_2, and $i_1 \neq i_2$), $E(\omega^2_{thi}) = \sigma^2_{bh}$; $E(u_{thij}) = E(u_{t_1h_1i_1j_1}u_{t_2h_2i_2j_2}) = 0$ (for all t_1, t_2, h_1, h_2, i_1, i_2 and $j_1 \neq j_2$), $E(u^2_{thij}) = \sigma^2_{uh}$, and $-1 < \rho_h < 1$. We also define $\alpha_{0h} \equiv 1$ and $\varepsilon_{0hij} \equiv 0$. Considering times only back to the base period, (2) implies $\varepsilon_{thij} = \sum_{k=0}^{t-1} \rho_h^k u_{t-k,hij}$. From this identity and the properties of u_{thij}, it follows that the covariance structure implied by model (2) is

$$\mathrm{cov}(r^t_{hij}, r^s_{h'i'j'}) = \begin{cases} \sigma^2_{bh} + (1-\rho_h^{2t})\Delta^2_h & s=t,\ h=h',\ i=i',\ j=j' \\ \rho_h^{t-s}(1-\rho_h^{2s})\Delta^2_h & s<t,\ h=h',\ i=i',\ j=j' \\ \sigma^2_{bh} & s=t,\ h=h',\ i=i',\ j\neq j' \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$

where $\Delta^2_h = \sigma^2_{uh}/(1-\rho^2_h)$. Expression (3) states that, at a given time period, the relatives for different items within the same establishment are correlated. Relatives at different time periods for the same item are also correlated while all other items are not. In practice ρ_h will be positive and often large.

The type of sample design to be considered here is a rotating panel survey in which establishments are sampled as the first stage units. Items are then sampled within each sample establishment. At each time t (t=1,...,T) we have a sample s_{th} of n_h establishments from stratum h and a sample s_{thi} of m_{hi} items from sample establishment (thi). A two–stage sampling plan, often used in practice, is one in which the stratum establishment samples are selected with probabilities proportional to size (*pps*) with the measure of size for each establishment being the weight

$W^0_{hi} = \sum_{j=1}^{M_{hi}} W^0_{hij}$. Items within establishments might then be selected with probabilities proportional to W^0_{hij}. At each time period the total establishment sample size is $n = \sum_h n_h$ and the total number of sample items in stratum h is $m_h = \sum_{i\in s_{th}} m_{hi}$. At each time period a proportion δ_h of the sample establishments is rotated out and an equal number rotated in. The size of the overlap, $s_{tuh} = s_{th} \cap s_{uh}$, between samples from times t and u $(t \geq u)$ is $n_h[1-(t-u)\delta_h]$.

3. ESTIMATORS THAT USE ONLY CURRENT PERIOD DATA

First, consider estimators that use only the sample data from period t. The within stratum component of $I^{t,0}$ is $R^t_h = \sum_{i=1}^{N_h}\sum_{j=1}^{M_{hij}} W^0_{hij} r^t_{hij}$. A class of estimators of this component is

$$\bar{z}^t_{th} = \sum_{i\in s_{th}} \lambda_{hi}\bar{r}^t_{thi}$$

where $\bar{r}^t_{thi} = \sum_{j\in s_{thi}} r^t_{hij}/m_{hi}$. An estimator of the overall index $I^{t,0}$ is then $\hat{I}^{t,0} = \sum_h \bar{z}^t_{th}$. In order for $\bar{z}^t_{th}$ to be a model unbiased estimator of R^t_h, i.e. for $E(\bar{z}^t_{th} - R^t_h) = 0$, we must have

$$\sum_{i\in s_{th}} \lambda_{hi} = W^0_h . \quad (4)$$

Some examples are given below of estimators $\hat{I}^{t,0}$, whose implied values of λ_{hi} satisfy (4).

Example 1. Best linear unbiased (BLU) predictor.
By analogy to the BLU given in Royall (1976, expression 3.4), the BLU in this case is

$$\hat{I}^{t,0} = \sum_h\Big[\sum_{i\in s_{th}} m_{hi}\bar{r}^t_{thi} + \sum_{i\in s_{th}} (W^0_{hi} - m_{hi})\{w_{thi}\bar{r}^t_{thi} + (1-w_{thi})\dot{\alpha}_{th}\} + \sum_{i\in s^c_{th}} W^0_{hi}\dot{\alpha}_{th}\Big]$$

where s^c_{th} is the complement of s_{th} in stratum h,

$$w_{thi} = \frac{g_{th} m_{hi}}{1 + (m_{hi}-1)g_{th}},$$

$$g_{th} = \frac{\sigma^2_{bh}}{\sigma^2_{bh} + (1-\rho^{2t})\Delta^2_h},$$

$$\dot{\alpha}_{th} = \sum_{s_{th}} c_{thi}\bar{r}^t_{thi}, \text{ and}$$

$$c_{thi} = \frac{m_{hi}}{1 + (m_{hi}-1)g_{th}} \Big/ \sum_{i\in s_{th}} \left[\frac{m_{hi}}{1 + (m_{hi}-1)g_{th}}\right].$$

The term g_{th} is the model correlation between different relatives in the same establishment at time t. The implied value of λ_{hi} in the BLU is, after some rearranging,

$$\lambda_{hi} = c_{thi}W^0_h + (m_{hi} - c_{thi}m_h) - (m_{hi}w_{thi} - c_{thi}\sum_{j\in s_{th}} m_{hj}w_{thj}) + (W^0_{hi}w_{thi} - c_{thi}\sum_{j\in s_{th}} W^0_{hj}w_{thj}) . \quad (5)$$

Some special cases of the BLU are given in *Examples 2* and *3*.

Example 2. Simple expansion estimator.
If $\sigma^2_{bh} = 0$ (i.e. relatives for different items are uncorrelated), then $g_{th} = w_{thi} = 0$, $c_{thi} = m_{hi}/m_h$ and $\lambda_{hi} = \frac{W^0_h}{m_h} m_{hi}$, leading to

$$\hat{I}^{t,0} = \sum_h \frac{W^0_h}{m_h} \sum_{s_{th}}\sum_{s_{thi}} r^t_{hij}$$

where $W^0_h = \sum_{i=1}^{N_h} W^0_{hi}$. This is also the Horvitz–Thompson estimator if a single–stage sample of items were selected with probabilities equal to $m_h W^0_{hij}/W^0_h$ and the estimator constructed by weighting $W^0_{hij}r^t_{hij}$ inversely by the item's selection probability.

Example 3. The estimator when the within–establishment correlation is 1.
If $\Delta^2_h = 0$, then $g_{th} = w_{thi} = 1$, $c_{thi} = 1/n_h$ and $\lambda_{hi} = W^0_h/n_h + (W^0_{hi} - \bar{W}^0_{ths})$ where $\bar{W}^0_{ths} = \sum_{s_{th}} W^0_{hi}/n_h$, producing

$$\hat{I}^{t,0} = \sum_h \left[\frac{W^0_h}{n_h}\sum_{s_{th}} \bar{r}^t_{thi} + \sum_{s_{th}}(W^0_{hi} - \bar{W}^0_{ths})\bar{r}^t_{thi}\right]$$

Example 4. A weighted mean estimator.
The model expectation of the second term in brackets in *Example 3* above is 0 which suggests the estimator

$$\hat{I}^{t,0} = \sum_h W^0_h \bar{r}^t_{th\cdot}.$$

where $\bar{r}^t_{th\cdot} = \sum_{s_{th}} \bar{r}^t_{thi}/n_h$. This estimator is also design–unbiased when establishments and items are selected in such a way that the selection probability of an item is $\pi_{hij} = n_h m_{hi} W^0_{hij}/W^0_h$.

4. ESTIMATORS WHICH USE DATA FROM ALL TIME PERIODS

A class of estimators of the long–term index, which use data from times u=1,..., t, is defined by

$$\hat{I}^{t,0} = \sum_h \bar{z}^t_{th} \prod_{u=1}^{t-1} \left[\frac{\bar{z}^u_{uh}}{\bar{z}^u_{u+1,h}}\right]^{\gamma^{tu}_h} \quad (6)$$

where $\bar{z}^k_{jh} = \sum_{s_{jh}} \lambda_{hi}\bar{r}^k_{jhi}/n_h$ for j=k, or k+1 (k=1,...,t−1) and γ^{tu}_h is a real number. Expression (6) is the two–stage version of the class of estimators introduced in Valliant and Miller (1989). Being based on the general $\bar{z}$ means, class (6) is also an extension of the class considered in Valliant and Miller (1989) where only the choice $\bar{z}^k_{jh} = \bar{r}^k_{jh\cdot}$ was studied. The term in brackets in (6) is a ratio of estimators based on the samples from adjacent time periods. Under the model specified by (2) and the condition in (4), each ratio estimates the constant 1. Thus, within a stratum, estimator (6) consists of the current period estimator $\bar{z}_{th}^{t}$ times an adjustment based on previous periods' data. The estimator $\hat{I}^{t,0}$ is approximately model–unbiased under (2). A similar design–based explanation also holds under any sample design for which $\bar{z}_{jh}^{k}$ is an exactly or approximately design–unbiased estimator of $\sum_{i=1}^{N_h}\sum_{j=1}^{M_{hi}} W_{hij}^{0} r_{hij}^{k}$. Estimators of short–term price change are derived from (6) by simply taking ratios of long–term estimators: $\hat{I}^{t,s} = \hat{I}^{t,0}/\hat{I}^{s,0}$.

Three members of the class defined by (6) are notable. If $\gamma_h^{tu} \equiv 1$, then (6) is the product estimator, which can be written as

$$\hat{I}_1^{t,0} = \Sigma_h \prod_{u=1}^{t} \left[\frac{\bar{z}_{uh}^{u}}{\bar{z}_{uh}^{u-1}}\right]$$

with $\bar{z}_{1h}^{0} \equiv 1$. This is a sum of a chain of estimated 1–period price changes and is similar to the estimator used in a number of U.S. government index programs. If $\gamma_h^{tu} \equiv 0$, then (6) reduces to the class of simple index estimators,

$$\hat{I}_2^{t,0} = \Sigma_h \bar{z}_{th}^{t} ,$$

which were covered in section 2.

A third choice of γ_h^{tu} is the value that minimizes the approximate variance of $\hat{I}^{t,0}$ under model (2). For the general estimator in class (6) the optimum depends on the particular sets of units in the samples from different time periods. However, if a constant number of sample items $\bar{m}_h$ is selected per establishment, and λ_{hi} is a constant for each sample establishment in stratum h, as in *Examples 2* and *4* in section 2 when $m_{hi} = \bar{m}_h$, then the approximately optimum γ_h^{tu} simplifies to

$$\overset{*}{\gamma}{}_h^{tu} = \frac{1}{2}\frac{\alpha_{uh}}{\alpha_{th}} \rho_h^{t-u} \left[1 + \bar{m}_h \frac{g_{uh}}{1 - g_{uh}}\right]^{-1} \text{for } 1 \le u \le t-1.$$

If $\rho_h > 0$ and $\alpha_{uh} \le \alpha_{u+1,h}$ as would be the case in times of inflation, the effect of data prior to the current period is rapidly damped out in $\hat{I}^{t,0}$.

5. VARIANCES OF THE INDEX ESTIMATORS

The approach used here for variance estimation is a standard one for nonlinear estimators. A first–order Taylor series approximation to an index estimator is made and the variance of that approximation is estimated.

5.1 Long–term Index Estimators

A linear approximation to $\hat{I}^{t,0}$ is

$$\hat{I}^{t,0} \approx \sum_h \sum_{u=1}^{t} d_{uh\cdot}^{t} . \tag{7}$$

where $d_{uh\cdot}^{t} = \sum_{i\in s_{uh}} \lambda_{hi} d_{uhi}^{t}$ with d_{uhi}^{t} defined as

$$d_{uhi}^{t} = \beta_{uh}^{t}\bar{r}_{uhi}^{u} - \beta_{u-1,h}^{t}\bar{r}_{uhi}^{u-1}$$

and $\beta_{uh}^{t} = \gamma_h^{tu}\alpha_{th}/\alpha_{uh}$. As a convention, we define $\beta_{0h}^{t} \equiv 0$ and $\beta_{th}^{t} \equiv 1$. From (7), the long–term index is approximately a sum of sample sums, and the approximate model variance of $\hat{I}^{t,0}$ is

$$\mathrm{var}(\hat{I}^{t,0}) \approx \sum_h \sum_{u=1}^{t} \sum_{v=1}^{t} \mathrm{cov}(d_{uh\cdot}^{t} , d_{vh\cdot}^{t}) . \tag{8}$$

Although this variance calculation is made with respect to the superpopulation model, the same general variance expression (8) can be obtained from a design–based calculation if the first–order approximation (7) is valid with respect to the design and α_{uh} is replaced by the design expectation of $\bar{r}_{uh}^{u}$ (and $\bar{r}_{u+1,h}^{u}$). Under model (2), an unbiased estimator of $\mathrm{cov}(d_{uh\cdot}^{t} , d_{vh\cdot}^{t})$ is

$$v_{uvh}^{t} = \frac{n_h}{n_h - 1} \sum_{s_{uvh}} \lambda_{hi}^{2}(d_{uhi}^{t} - \bar{d}_{uvh\cdot}^{tu})(d_{vhi}^{t} - \bar{d}_{uvh\cdot}^{tv})$$

where $\bar{d}_{uvh\cdot}^{tk} = \sum_{s_{uvh}} d_{khi}^{t}/n_{uvh}$ (k=u or v) and n_{uvh} is the number of items in s_{uvh}. A model–unbiased estimator of the approximate model–variance of $\hat{I}^{t,0}$ is then

$$v(\hat{I}^{t,0}) = \sum_h \sum_{u=1}^{t} \sum_{v=1}^{t} v_{uvh}^{t} . \tag{9}$$

In evaluating (9), note that it is necessary to estimate terms such as α_{th}/α_{uh} from the samples for the different time periods. This will also be true for the variance estimators discussed subsequently. For the sample design used in the simulation described in section 6, (9) can also be regarded as a design–based "with replacement" estimator of the type presented in Hansen, Hurwitz, and Madow (1953, pp. 418–419) or Andersson, Forsman, and Wretman (1987a).

5.2 Short–term Index Estimators

The short–term price index estimator from time s to time t (t > s) is defined as $\hat{I}^{t,s} = \hat{I}^{t,0}/\hat{I}^{s,0}$. The development for the short–term index given below parallels that in section 5.1 for the long–term index. After considerable computation, the first–order approximation to $\hat{I}^{t,s}$ can be written as

$$\hat{I}^{t,s} \approx \dot{I}^{t,s} + \frac{1}{\dot{I}^{s,0}} \Sigma_h \sum_{u=1}^{t} e_{uh\cdot}^{ts} . \tag{10}$$

where $\dot{I}^{t,s}$ and $\dot{I}^{s,0}$ are the values of $\hat{I}^{t,s}$ and $\hat{I}^{s,0}$ evaluated at $E(\bar{z}_{uh}^{u})$ and $E(\bar{z}_{u+1,h}^{u})$, $e_{uh\cdot}^{ts} = \sum_{s_{uh}} \lambda_{hi} e_{uhi}^{ts}$, and

$$e_{uhi}^{ts} = \psi_{uh}^{ts}\bar{r}_{uhi}^{u} - \psi_{u-1,h}^{ts}\bar{r}_{uhi}^{u-1} ,$$

with ψ_{uh}^{ts} defined as

$$\psi_{uh}^{ts} = \begin{cases} \gamma_h^{tu} \dfrac{\alpha_{th}}{\alpha_{uh}} - \hat{I}^{t,s} \gamma_h^{su} \dfrac{\alpha_{sh}}{\alpha_{uh}} & u=1,\ldots,s \\ \gamma_h^{tu} \dfrac{\alpha_{th}}{\alpha_{uh}} & u=s+1,\ldots,t \end{cases}$$

and $\gamma_h^{ss} = \gamma_h^{tt} \equiv 1$.

The term e_{uhi}^{ts} can be also be written in terms of quantities already used for the long–term estimator. In particular,

$$e_{uhi}^{ts} = \begin{cases} d_{uhi}^{t} - \hat{I}^{t,s} d_{uhi}^{s} & u=1,\ldots,s \\ d_{uhi}^{t} & u=s+1,\ldots,t\,. \end{cases}$$

An estimator of the variance of the righthand side of (10) is then

$$v(\hat{I}^{t,s}) = \frac{1}{(\hat{I}^{s,0})^2} \Sigma_h \left\{ \sum_{u=1}^{t} \sum_{v=1}^{t} v_{uvh}^{t} + (\hat{I}^{t,s})^2 \sum_{u=1}^{s} \sum_{v=1}^{s} v_{uvh}^{s} - 2\hat{I}^{t,s} \sum_{u=1}^{t} \sum_{v=1}^{s} v_{uvh}^{ts} \right\} \tag{11}$$

where v_{uvh}^{t} and v_{uvh}^{s} were defined in section 5.1 and

$$v_{uvh}^{ts} = \frac{n_h}{n_h - 1} \sum_{s_{uvh}} \lambda_{hi}^2 (d_{uhi}^{t} - \bar{d}_{uvh}^{tu})(d_{vhi}^{s} - \bar{d}_{uvh}^{tv}).$$

Expression (11) is recognizable as one of the forms of the Taylor series variance estimator for a ratio.

6. AN EMPIRICAL STUDY

To test the various estimators, we conducted a simulation study using a population derived from data collected by the Bureau of Labor Statistics for the CPI program. Because the CPI samples few items per sample establishment, an existing CPI dataset was augmented, as described in Valliant (1990), in order to create a finite population of establishments and items that could be used for two–stage sampling. Five strata of items — beef, eggs, milk and other dairy products, fresh vegetables, and sugar — from the population used in Valliant (1989) were selected as a starting set for creating the new population. Each establishment in each stratum of the starting dataset had one sample item which was priced monthly during the 3½ year period from January 1980 through June 1982 . The items were part of a national probability sample used for the CPI during this period. The starting dataset included only items for which prices were obtained in all 42 months during the period. The prices used in computing item relatives were those used in the actual CPI after all quality change and other economic adjustments had been made. The base period, time 0, varied among the items but in all cases was one of the months in the latter half of 1977. A sampling weight was available for each item which was designed to be, under ideal circumstances, proportional to the base period trade values $p_{hi}^0 q_{hi}^0$. These CPI sampling weights were used, after some modification to avoid certainty selections of establishments and items, to compute the base period index weights W_{hij}^0 for the study population.

Table 1 gives various population and sample allocation numbers and Figure 1 graphs the means $\bar{r}_h^t$ for each stratum versus time. Two sets of 500 stratified samples were selected with the number of sample establishments allocated to a stratum being roughly proportional to W_h^0 . The total establishment sample sizes in the two sets of samples were n=50 and 100. Samples were selected in such a way that 20% of the sample establishments were rotated in a 12 month period. To implement this plan, a large systematic random start sample of establishments was selected in each stratum with probabilities proportional to W_{hi}^0 . This initial stratum sample, which was large enough to accommodate all 42 months accounting for the amount of establishment rotation, was then sorted in a random order. The stratum establishment sample for a particular time period t consisted of establishments $1 + (t-1)\delta_h n_h$,..., $n_h + (t-1)\delta_h n_h$ where δ_h is the proportion of establishments rotated in a month. For the samples of total size 50 the initial, large sample size was 84 while for the samples of size 100 the initial size was 168. In both cases $\delta_h = 1/60$ which produces an annual turnover of $12(n_h/60) = n_h/5$ units or 20%. The number of sample items selected from each sample establishment was $\bar{m}_h = 2$ with items being selected with probabilities proportional to W_{hij}^0 .

From each of the 500 samples we computed the long–term estimators $\hat{I}_j^{t,0}$ (j=1, 2, 3 ; t=1,...,42), and the short–term estimators of 1, 6, and 12 month change, $\hat{I}_j^{t,s}$; (j=1, 2, 3 ; s=t–1, t–6, t–12 for s ≥ 1). For each estimator we used the special case $\lambda_{hi} = W_h^0 / n_h$ to evaluate (6). In computing $\hat{I}_3^{t,0}$, we approximated the optimum value $\overset{*}{\gamma}_h^{tu}$ by using a common value of $\rho_h = .8$, estimating $\alpha_{uh} / \alpha_{th}$ by $\bar{r}_{u+1,h}^{u} / \bar{r}_{th}^{t}$, and using the set of values $\{g_{uh}\}_{h=1}^{5} = \{.1,.2,.3,.4,.5\}$, for all u, which was the set used in generating the population. Across the samples we then computed biases, defined as $\Sigma\,(\hat{I} - I)/500$ where the summation is over the 500 samples, $\hat{I}$ is one of the long or short–term estimators, and I is the corresponding population index defined in section 2. Similarly square roots of the empirical mean squared errors were computed as $[\Sigma(\hat{I} - I)^2/500]^{1/2}$. The linearization variance estimators given by (9) and (15) were modified by an *ad hoc* inclusion of a finite population correction, $1 - f_h$ ($f_h = n_h / N_h$), for each stratum. All computations were made in Turbo Pascal 5.5 on an IBM PS/2 Model 80 microcomputer.

Figure 2 gives plots of root mean squared errors (*rmse's*) versus time for the product, simple, and optimal estimators for long–term, 1–month, 6–month, and 12–month price changes. Biases were negligible for each of the three estimators and are not shown. The upper group of curves in each panel of Figure 2 is for the samples of size n=50 while the lower group is for n=100. For long–term change the product estimator is the worst of the three while the optimal is best, although the simple estimator is quite competitive with the optimal. The increasing *rmse* of the product estimator over time was also seen empirically by Wilkerson (1967) and Leaver (1990) in studies of the U.S. CPI and was predicted theoretically by Valliant and Miller (1989). For the short–term changes, on the other hand, the product estimator is generally the best, particularly for 12–month change.

Figures 3–5 graph the *rmse* and the square root of the average linearization variance estimate at each time for each of the three index estimators. The linearization estimates for long–term, 6–month, and 12–month changes based on the product, simple, and optimal index estimates are all nearly unbiased for either sample size.

7. DISCUSSION

The linearization or Taylor series method of variance estimation, studied here, is sometimes criticized as requiring a separate derivation for each type of estimator used in a survey. In price index programs and other continuing surveys this seems less of a concern since the derivation and computer programming are one time tasks, the results of which can be used for some time thereafter. Survey redesign will naturally require revamping of existing systems, but this would be true even if one of the general purpose replication estimators were used. A further, and possibly more important, criticism is the difficulty of accounting for the effect of nonresponse and other missing data adjustments when using the linearization estimator.

Commonly used competitors to the linearization estimator in complex surveys are different versions of replication estimators obtained by random groups, balanced half sampling, or some other means. Nonresponse and other adjustments can be made separately for each replicate thereby incorporating their effect into variance estimates. The replication methods are often implemented by collapsing together primary sampling units or strata before forming replicates. Such collapsing can be inefficient compared to the linearization estimator, which is one reason that the latter should be seriously considered despite shortcomings noted above. A difficulty with replication estimators occurs in a panel survey when the primary sampling units (*psu*'s) are rotated. With a stable sample of *psu*'s, as is the case in many area probability samples, the first stage units can be assigned to replicates and the same replicate definitions maintained over time. When *psu*'s are rotated, which was the situation studied here, and an estimator involves data from many prior time periods, how to appropriately form replicates becomes a more difficult question.

REFERENCES

Andersson, C., Forsman, G., and Wretman, J. (1987a), "Estimating the Variance of a Complex Statistic: A Monte Carlo Study of Some Approximate Techniques," *Journal of Official Statistics*, 3, 251–265.

——— (1987b), "On the Measurement of Errors in the Swedish Consumer Price Index, "*Bulletin of the International Statistical Institute*, 52, Book 3, 155–171.

Banerjee, K. (1960), "Calculation of Sampling Errors for Index Numbers," *Sankhyā*,22,19–130.

Balk, B. and Kersten, H.M.P. (1987), "On the Precision of Consumer Price Indices Caused by the Sampling Variability of Budget Surveys," *Journal of Economic and Social Measurement*, 14, 19–35.

Biggeri, L., and Giommi, A. (1987), "On the Accuracy and Precision of the Consumer Price Indices. Methods and Applications to Evaluate the Influence of the Sampling of Households," *Bulletin of the International Statistical Institute*, 52, Book 3, 137–154.

Edgeworth, F.Y. (1925), *Papers Relating to Political Economy*, Vol. 1, The Royal Economic Society: London.

Hansen, M., Hurwitz, W., Madow, W. (1953), *Sample Survey Methods and Theory*, Vol. I, John Wiley: New York.

Kish, L. (1968), "Standard Errors for Indexes from Complex Samples," *Journal of the American Statistical Association*, 63, 512–529.

Leaver, S. (1990), "Estimating Variances for the U.S. Consumer Price Index for 1978–1986, "*Proceedings of the Section on Survey Research Methods*, American Statistical Association, to be published.

Royall, R.M. (1976), "The Linear Least Squares Prediction Approach to Two–Stage Sampling," *Journal of the American Statistical Association*, 71, 657–664.

——— (1986), "The Prediction Approach to Robust Variance Estimation in Two–Stage Cluster Sampling," *Journal of the American Statistical Association*, 81, 119–123.

Valliant. R. (1990), "Variance Estimation for Price Indexes from a Two–stage Sample with Roatating Panels," Bureau of Labor Statistics Technical Report, submitted for publication.

——— (1989), "Price Index Estimation from Panel Survey Data," *Bulletin of the International Statistical Institute*, 53, Book 2, 445–461.

Valliant, R., and Miller, S. (1989), "A Class of Multiplicative Estimators of Laspeyres Price Indexes," *Journal of Business and Economic Statistics*, 7, 387–394.

Wilkerson, M. (1967), "Sampling Error in the Consumer Price Index," *Journal of the American Statistical Association*, 62, 899–914.

Table 1. Universe and sample characteristics for the study population .

Stratum	W_h^0	N_h	M_h	n_h/n	f_h n=50	f_h n=100
1 Beef	.32	154	1800	.32	.10	.20
2 Eggs	.13	57	653	.12	.10	.21
3 Milk, other dairy	.33	155	1800	.32	.10	.20
4 Fresh vegetable	.10	193	1013	.12	.03	.06
5 Sugar	.12	100	1175	.12	.06	.12
Total	1.00	659	6441	1.00		

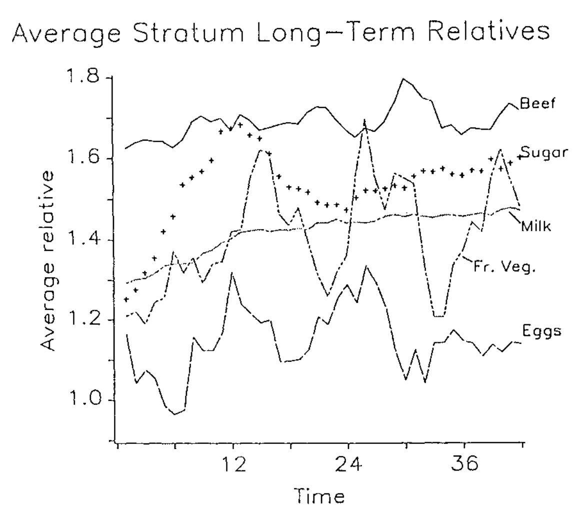

Figure 1. Average stratum long-term relatives for the five strata in the study population.

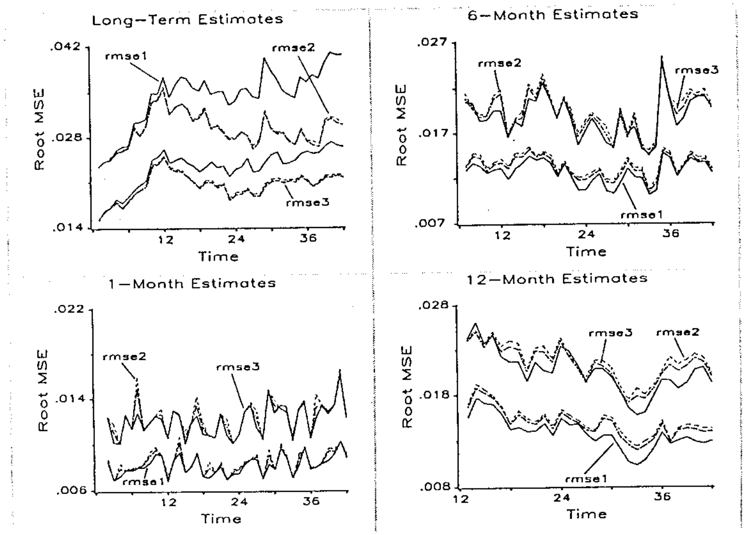

Figure 2. Root mean squared errors (*rmse*'s) of the product (*rmse1*), the simple (*rmse2*), and the optimal (*rmse3*) estimators of long-term, 1-month, 6-month, and 12-month price change plotted versus time. The upper set of curves in each panel is for samples of size n=50. The lower set is for samples of size n=100.

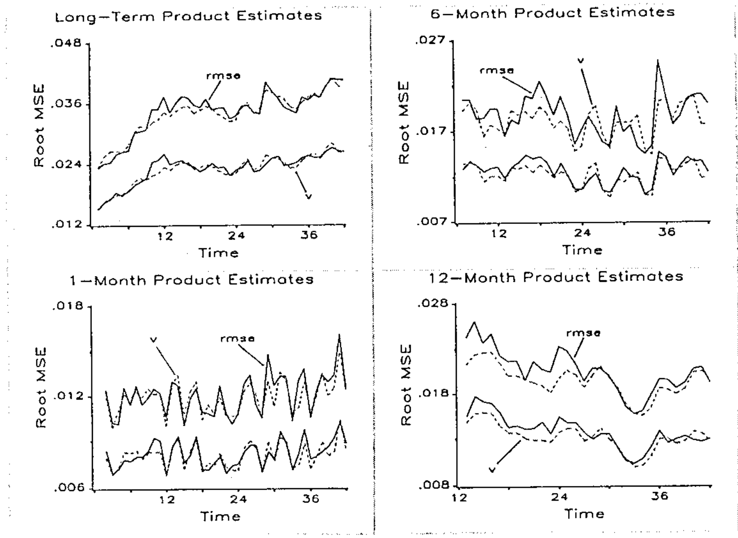

Figure 3. Root mean squared errors (*rmse*'s) and square roots of average linearization variance estimators (*v*'s) for the product estimators of long-term, 1-month, 6-month, and 12-month change plotted versus time. The upper set of curves in each panel is for samples of size n=50. The lower set is for samples of size n=100.

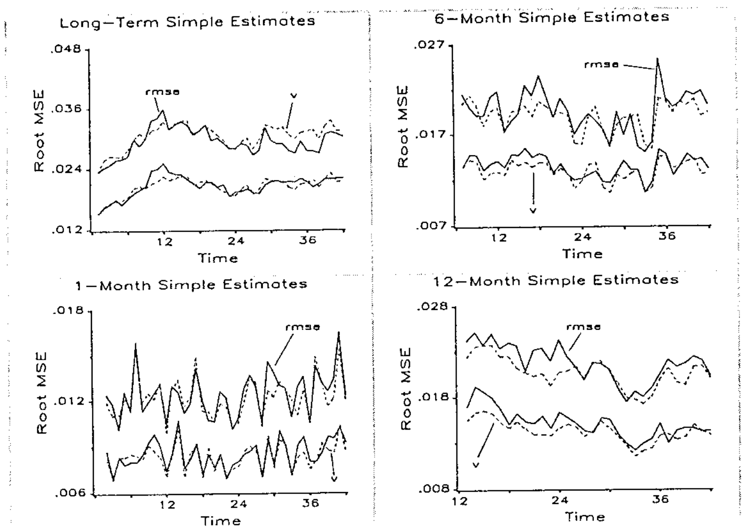

Figure 4. Root mean squared errors (*rmse*'s) and square roots of average linearization variance estimators (*v*'s) for the simple estimators of long-term, 1-month, 6-month, and 12-month change plotted versus time. The upper set of curves in each panel is for samples of size n=50. The lower set is for samples of size n=100.

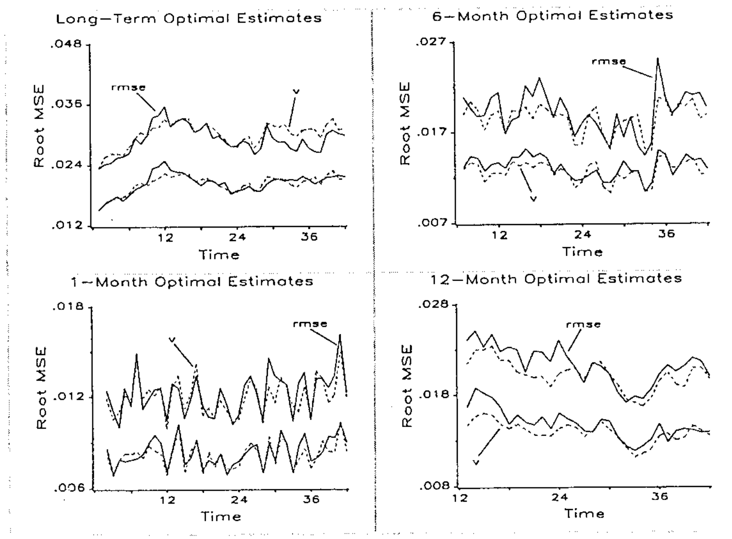

Figure 5. Root mean squared errors (*rmse*'s) and square roots of average linearization variance estimators (*v*'s) for the optimal estimators of long-term, 1-month, 6-month, and 12-month change plotted versus time. The upper set of curves in each panel is for samples of size n=50. The lower set is for samples of size n=100.