

EMPIRICAL BAYES ESTIMATION FOR QUARTERLY HOG SERIES

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Abstract. The National Agriculture Statistics Service of the U.S. Department of Agriculture uses quarterly survey data to produce multiple frame total hog inventory indications (estimates). This paper presents an application of the parametric empirical Bayes estimation method with regression. By incorporating three years to six years of the quarterly hog multiple frame indications, the empirical Bayes estimators reduce the variances of the current indications for two large hog producing states. Various tables and graphs illustrate how well the empirical Bayes estimates perform compared to the multiple frame estimates and to the official Agricultural Statistics Board estimates.

1. Introduction

The National Agricultural Statistics Service (NASS) of the U.S. Department of Agriculture (USDA) conducts quarterly agricultural surveys in March, June, September, and December. Although the June survey serves as the base for the survey cycle with four summary estimates, only the multiple frame estimates (indications) and the adjusted multiple frame estimates (indications) are available during all four quarters. The adjusted multiple frame indication uses an alternative imputation method for nonrespondents. The multiple frame indications are available at the state and U.S. levels for hog and pig items such as total hogs, breeding hogs, hogs under 60 pounds, etc.

The Agricultural Statistics Board, a committee of senior NASS statisticians from headquarters and major state statistical offices, uses the multiple frame and the adjusted multiple frame indications during September, December and March as well as census data and other non-survey information to set official hog inventory estimates. Perry et al., (1989) have shown the benefit of using three additional indications during the June quarters to make an inverse variance composite indication. However, this method can help only for the June quarters. Consequently, this paper explores a methodology to incorporate information from past surveys to improve upon the official hog inventory estimates for all quarters.

Empirical Bayes (EB) methodology reduces the sum of the mean squared errors for all the estimators present and past by incorporating information from similar sources. The data collected from the quarterly hog series can be treated as observations from repeated experiments of the same type. The EB method has the potential of incorporating information from past surveys. Instead of applying EB to all the current and past data, we propose applying an EB method to the summary statistics, that is, the estimated inventory and its variance estimate for each quarter over a period of time. A parametric EB method using the regression model is studied here for the quarterly hog series. This regression model is more general than the one with stationary means because of the assumptions of either a linear or quadratic trend for the hog inventories.

Maritz and Lwin (1989) discuss many aspects of EB methods. The EB method with regression model studied here is similar to the one applied by Fay and Herriot (1979) to estimate income for small domains. The method is also discussed in Morris (1983) and Berger (1985, pp. 169-190) in more detail. The EB method with stationary means for a finite population has been developed by Ghosh and Meeden (1986), and Ghosh and Lahiri (1987).

2. Empirical Bayes Estimation

We shall apply the EB method to the quarterly hog series. Let X_i denote the estimated inventory for a particular item (for example, total hogs and pigs of a state) for the i^{th} period. In our analysis, let X_i be the multiple frame indication. Another possibility is the adjusted multiple frame indication. Let θ_i denote the true inventory to be estimated for the i^{th} period. It is assumed that given the θ_i , the X_i are independently $N(\theta_i, \sigma_i^2)$. To simplify our analysis, we assume the σ_i^2 are known. In practice, we replace the σ_i^2 by s_i^2 , the estimated variance of X_i . Instead of assuming that the θ_i are i.i.d. $N(\mu_\pi, \sigma_\pi^2)$, we assume the θ_i satisfy a regression model

$$\theta_i = y_i^t \beta + \varepsilon_i,$$

where $\beta = (\beta_1, \dots, \beta_\ell)^t$ is a vector of unknown regression coefficients ($\ell < n-2$), $y_i^t = (y_{i1}, \dots, y_{i\ell})$ is a known set of regressors for each i , and the ε_i are independently $N(0, \sigma_\pi^2)$, where σ_π^2 is unknown.

In particular, we consider two specific regression models:

- Model 1: $\theta_i = \beta_1 + \beta_2 i + \varepsilon_i$, corresponding to $y_i^t = (1, i)$;
- Model 2: $\theta_i = \beta_1 + \beta_2 i + \beta_3 i^2 + \varepsilon_i$, corresponding to $y_i^t = (1, i, i^2)$.

These models do not incorporate the cyclic behavior of the hog inventories. However, Thomas et al, (1990) have examined a model with seasonality effects. Additional analysis will be needed to examine the relative importance of the trend effects compared to the seasonality effects.

To elaborate on the previous discussion, we have made the following assumptions:

- (1) A heteroscedastic model on the data

The multiple frame indication X_i is assumed to be model unbiased for the unobserved parameter of interest θ_i . (The multiple frame estimator is design unbiased for the hog inventory totals. However, nonsampling errors may cause bias.) The model unbiased assumption refers to the superpopulation model $N(\theta_i, \sigma_i^2)$. Given the θ s, the X_i s have independent normal distributions with different variances σ_i^2 , where s_i^2 estimates σ_i^2 .

- (2) A homoscedastic model on the unknown parameters

By treating the θ s as random variables, we incorporate a second stochastic process having an unknown, but restricted, class of distributions, namely, $N(y_i^t \beta, \sigma_\pi^2)$, where the mean is assumed to be a function of time.

Modeling the prior information of the θ s produces appropriate estimates for β and σ_π^2 by using all the data.

Consequently, a better estimate of θ_i than X_i is produced. This EB estimate depends not only on X_i , but on all the data (X_1, \dots, X_n) and (s_1^2, \dots, s_n^2) as well.

To estimate β and σ_π^2 , we need to derive the marginal density $m(x)$ of X_1, \dots, X_n . Observe that the X_i are

independent $N(y_i^t \beta, \sigma_i^2 + \sigma_\pi^2)$. Therefore, the marginal density is given by

$$m(x|\beta, \sigma_\pi^2) = \left\{ \prod_{i=1}^n \left[2\pi(\sigma_i^2 + \sigma_\pi^2) \right]^{-\frac{1}{2}} \right\} \exp \left\{ -\frac{1}{2} \sum_{i=1}^n \left(x_i - y_i^t \beta \right)^2 / \left(\sigma_i^2 + \sigma_\pi^2 \right) \right\} \quad (2.1)$$

We seek estimates β and σ_π^2 that maximize (2.1) by differentiating the log function of (2.1) with respect to the β_i and σ_π^2 , and setting the equations equal to zero. The equations obtained can be written

$$\hat{\beta} = (y'v^{-1}y)^{-1}(y'v^{-1}x), \quad (2.2)$$

where y is the $n \times \ell$ matrix with rows y_i^t , v is the $n \times n$ diagonal matrix with the diagonal elements $v_{ii} = \sigma_i^2 + \hat{\sigma}_\pi^2$, $x = (x_1, \dots, x_n)^t$, and

$$\hat{\sigma}_\pi^2 = \frac{\sum_{i=1}^n \left[n / (n - \ell) (x_i - y_i^t \hat{\beta})^2 - \sigma_i^2 \right] / \left[\sigma_i^2 + \hat{\sigma}_\pi^2 \right]^2}{\sum_{i=1}^n \left(\sigma_i^2 + \hat{\sigma}_\pi^2 \right)^{-2}}. \quad (2.3)$$

We can obtain solutions $\hat{\beta}$ and $\hat{\sigma}_\pi^2$ to (2.2) and (2.3) by an iterative scheme. Starting out with an initial guess of $\hat{\sigma}_\pi^2$ (for example, a sample variance of the s_i), we use this guess to calculate $\hat{\beta}$ from (2.2). Then substitute this $\hat{\beta}$ and $\hat{\sigma}_\pi^2$ into the right hand side of (2.3) to obtain a new guess of $\hat{\sigma}_\pi^2$. Repeat this process until $\hat{\beta}$ and $\hat{\sigma}_\pi^2$ stabilize. If the iteration yields a negative value for $\hat{\sigma}_\pi^2$, then set $\hat{\sigma}_\pi^2 = 0$.

The empirical Bayes estimate for the i^{th} quarter is given by

$$\hat{\theta}_i = (1 - \hat{B}_i) x_i + \hat{B}_i y_i^t \hat{\beta}, \quad (2.4)$$

where the shrinkage factor is $\hat{B}_i = \frac{n - \ell - 2}{n - \ell} \frac{\sigma_i^2}{\sigma_i^2 + \hat{\sigma}_\pi^2}$.

Observe that the empirical Bayes estimate shrinks the i^{th} quarter current estimate to the regression mean $y_i^t \hat{\beta}$. As the σ_i^2 increases, the shrinkage factor \hat{B}_i becomes larger and pulls the current estimate closer to the regression mean. The regression mean is either a linear function (model 1) or a quadratic function (model 2) of time. The coefficients of this regression mean are computed from the weighted regression model with weights inversely proportional to the sum of two variances, one for the data in the i^{th} period, σ_i^2 , and one for the prior estimated by $\hat{\sigma}_\pi^2$.

The variance of the EB estimator in (2.4) is given in Berger (1985, p. 174).

$$V(\hat{\theta}_i) = \sigma_i^2 \left[1 - \frac{n - \hat{\ell}_i}{n} \hat{B}_i \right] + \frac{1}{n - \ell - 2} \hat{B}_i^2 \left(\frac{\bar{\sigma}^2 + \hat{\sigma}_\pi^2}{\sigma_i^2 + \hat{\sigma}_\pi^2} \right) (x_i - y_i^t \hat{\beta})^2 \quad (2.5)$$

where $\hat{\ell}_i = n \left[y(y^t v^{-1} y)^{-1} y^t \right]_{ii} / (\sigma_i^2 + \hat{\sigma}_\pi^2)$, and $\bar{\sigma}^2 = \left[\sum_{i=1}^n \sigma_i^2 / (\sigma_i^2 + \hat{\sigma}_\pi^2) \right] / \sum_{i=1}^n 1 / (\sigma_i^2 + \hat{\sigma}_\pi^2)$.

3. Numerical Example

The EB method has been applied to eight major hog producing states to estimate the total inventory of hogs per state. This report gives EB estimates and analyses for Indiana and Iowa. Analyses for all the eight states are in Cook and Kuo (1990). Both the linear regression model and quadratic model (Models 1 and 2) are applied to the summary statistics for the 12 quarters starting from June 1986 through March 1989, and for the 24 quarters starting from June 1983 through March 1989 for the two states. Tables 1 and 2 show the EB estimates, shrinkage factors, variance estimates, and variance reduction in percentage.

Graphs 1.2 and 2.2 (A.1–B.2) display the biases of the multiple frame and the empirical Bayes indications compared to the board estimates. The purpose of including the graphs of the differences between the Board and the EB and the Board and the Multiple Frame indications is to show how closely the magnitudes of these differences compare on a state by state basis. In Cook and Kuo (1990), Friedman (Hollander and Wolfe, 1983, page 139) and ANOVA procedures test the biases to show that there are no significant differences among the EB indications and the Multiple Frame indications for the two states in the study.

Tables 1 and 2 present the quarterly total Hog and Pig Multiple Frame indications with standard deviations, the Empirical Bayes linear and quadratic indications with standard deviations for 12 and 24 quarters respectively, the Board estimate, the variance reduction provided by the Empirical Bayes indication $(V(\text{MF}) - V(\text{EB}))/V(\text{MF})$, and

the Shrinkage Factor (equation 2.4) used in the Empirical Bayes indication for the two states of Indiana and Iowa.

The tables disclose that the quarterly total hog and pig estimates for two states are clearly different. For Indiana, considering either the 12 quarters or the 24 quarters, the linear EB indications have slightly greater variance reduction than the quadratic indications. The quadratic indication as well as the linear indication based on 24 quarters have a greater variance reduction than those using 12 quarters of indications. When we compare the biases for the four EB indications for the final quarter (March, 1989), the quadratic EB with 24 quarters has the smallest bias. The remaining three biases are ordered from smallest to largest as follows: the linear EB with 24 quarters, the linear EB with 12 quarters, and the quadratic EB with 12 quarters.

For Iowa, the tables show that both the linear EB indications and the quadratic EB indications give moderately good variance reductions. The variance reduction for the quadratic model is larger than that of the linear model for both 12 and 24 quarters, respectively. Although we expect the variance reductions based on 24 quarters to be larger than those based on 12 quarters, tables 1 and 2 do not support this possibility. Perhaps the large variances of the Multiple Frame indications from the first 12 quarters have diminished the variance reduction for the second 12 quarters. When considering only the last quarter, the linear EB with 12 quarters produces the smallest bias of the four EB indications under study. The remaining three EB indications have the following order for biases: quadratic EB with 24 quarters has the next smallest, the quadratic EB with 12 quarters is next, and finally, linear EB with 24 quarters.

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Table 1

Empirical Bayes Estimates Using 12 Quarters of Multiple Frame Indications for Total Hogs with Comparisons of Variances and \hat{A} 's for Two States

A. Indiana

1. Linear Empirical Bayes Estimates

$$\sigma_r^2 = 223,781$$

$$\hat{A}_1 = 4,304,770 \quad \hat{A}_2 = -8,618$$

$$\hat{\sigma}_r^2 = 260,608$$

YEAR QTR	MF	MF SD	ZB1	ZB1 SD	BD	VAR RED.	B(1)
			000's				
86-2	4294	421	4295	304	4150	48.0%	0.58
86-3	4443	421	4353	298	4250	49.7%	0.58
86-4	4146	158	4175	145	4150	14.9%	0.21
87-1	4007	176	4073	161	3950	16.6%	0.25
87-2	4107	150	4137	137	4200	15.6%	0.20
87-3	4925	297	4621	260	4600	23.4%	0.45
87-4	4615	198	4507	178	4600	18.8%	0.29
88-1	3873	157	3950	147	4100	11.6%	0.21
88-2	4421	182	4370	162	4500	20.8%	0.26
88-3	4597	193	4489	178	4600	15.6%	0.28
88-4	4168	178	4179	160	4300	19.5%	0.25
89-1	3857	155	3929	150	4050	7.3%	0.21

2. Quadratic Empirical Bayes Estimates

$$\sigma_r^2 = 223,781$$

$$\hat{A}_1 = 3,934,603 \quad \hat{A}_2 = 128,534 \quad \hat{A}_3 = -9,903$$

$$\hat{\sigma}_r^2 = 259,874$$

YEAR QTR	MF	MF SD	ZB2	ZB2 SD	BD	VAR RED.	B(2)
			000's				
86-2	4294	421	4159	349	4150	31.5%	0.51
86-3	4443	421	4279	319	4250	42.7%	0.51
86-4	4146	158	4164	146	4150	14.5%	0.19
87-1	4007	176	4076	162	3950	15.0%	0.22
87-2	4107	150	4150	140	4200	12.6%	0.17
87-3	4925	297	4671	260	4600	23.5%	0.40
87-4	4615	198	4539	178	4600	18.6%	0.26
88-1	3873	157	3968	154	4100	3.8%	0.19
88-2	4421	182	4387	163	4500	20.3%	0.23
88-3	4597	193	4495	178	4600	14.8%	0.25
88-4	4168	178	4164	161	4300	18.2%	0.22
89-1	3857	155	3897	151	4050	6.1%	0.18

B. Iowa

1. Linear Empirical Bayes Estimates

$$\sigma_r^2 = 549,712$$

$$\hat{A}_1 = 11,388,4934 \quad \hat{A}_2 = 186,332$$

$$\hat{\sigma}_r^2 = 349,486$$

YEAR QTR	MF	MF SD	ZB1	ZB1 SD	BD	VAR RED.	B(1)
			000's				
86-2	11490	489	11798	424	12200	24.9%	0.53
86-3	12389	547	12314	407	12700	44.7%	0.57
86-4	12223	458	12335	359	12600	38.7%	0.51
87-1	12282	528	12476	391	12300	45.2%	0.56
87-2	13207	535	12988	392	13500	46.3%	0.56
87-3	14100	615	13436	508	14300	31.9%	0.60
87-4	13493	610	13310	414	14100	53.9%	0.60
88-1	13011	638	13235	432	13500	54.1%	0.62
88-2	14197	530	13843	422	14300	36.5%	0.56
88-3	14409	554	14031	443	14500	36.3%	0.57
88-4	13709	595	13843	435	14000	46.5%	0.59
89-1	13052	498	13625	503	13600	-1.7%	0.54

2. Quadratic Empirical Bayes Estimates

$$\sigma_r^2 = 549,712$$

$$\hat{A}_1 = 10,738,714 \quad \hat{A}_2 = 686,784 \quad \hat{A}_3 = -38,503$$

$$\hat{\sigma}_r^2 = 0.0$$

YEAR QTR	MF	MF SD	ZB2	ZB2 SD	BD	VAR RED.	B(2)
			000's				
86-2	11490	489	11410	367	12200	43.6%	0.70
86-3	12389	547	12054	371	12700	54.1%	0.70
86-4	12223	458	12402	291	12600	59.5%	0.70
87-1	12282	528	12739	388	12300	45.8%	0.70
87-2	13207	535	13209	308	13500	66.8%	0.70
87-3	14100	615	13612	416	14300	54.4%	0.70
87-4	13493	610	13623	351	14100	66.9%	0.70
88-1	13011	638	13600	442	13500	51.9%	0.70
88-2	14197	530	13889	347	14300	57.2%	0.70
88-3	14409	554	13901	412	14500	44.9%	0.70
88-4	13709	595	13651	354	14000	64.5%	0.70
89-1	13052	498	13350	417	13600	30.0%	0.70

Table 2
Empirical Bayes Estimates Using 24 Quarters of Multiple
Frame Indications for Total Hogs with Comparisons of
Variances and \hat{A} 's for Two States

A. Indiana

1. Linear Empirical Bayes Estimates

$$\sigma_v^2 = 220,092$$

$$\hat{A}_1 = 4,244,371 \quad \hat{A}_2 = -2,285$$

$$\hat{\sigma}_v = 206,504$$

YEAR QTR	MF	MF SD	EB1	EB1 SD	BD	VAR RED.	B(1)
000's							
83-2	4592	205	4434	168	4650	32.6%	0.45
83-3	4759	237	4491	191	4800	35.1%	0.52
83-4	4046	174	4119	145	4200	30.8%	0.38
84-1	3957	188	4072	154	4000	32.9%	0.41
84-2	4331	178	4293	144	4350	34.5%	0.39
84-3	4251	170	4243	139	4350	33.4%	0.37
84-4	4257	152	4248	128	4300	29.1%	0.32
85-1	3825	150	3951	134	3950	20.0%	0.31
85-2	4054	131	4099	115	4150	22.6%	0.26
85-3	4153	175	4179	140	4250	35.7%	0.38
85-4	4286	402	4238	218	4150	70.6%	0.72
86-1	4147	436	4199	226	3950	73.1%	0.74
86-2	4294	421	4236	223	4150	72.0%	0.73
86-3	4443	421	4274	225	4250	71.4%	0.73
86-4	4146	158	4168	131	4150	31.3%	0.33
87-1	4007	176	4084	143	3950	33.9%	0.38
87-2	4107	150	4138	127	4200	28.4%	0.31
87-3	4925	297	4483	221	4600	44.6%	0.61
87-4	4615	198	4435	163	4600	31.7%	0.44
88-1	3873	157	3981	137	4100	23.8%	0.33
88-2	4421	182	4332	149	4500	32.8%	0.40
88-3	4597	193	4426	163	4600	29.3%	0.42
88-4	4168	178	4177	146	4300	33.2%	0.39
89-1	3857	155	3966	138	4050	20.6%	0.33

B. Iowa

1. Linear Empirical Bayes Estimates

$$\sigma_v^2 = 567,258$$

$$\hat{A}_1 = 13,815,410 \quad \hat{A}_2 = -35,174$$

$$\hat{\sigma}_v = 815,105$$

YEAR QTR	MF	MF SD	EB1	EB1 SD	BD	VAR RED.	B(1)
000's							
83-2	14736	505	14495	456	15200	18.4%	0.25
83-3	15405	635	14834	559	15600	22.5%	0.34
83-4	14589	731	14233	592	15000	34.4%	0.40
84-1	12661	496	12910	446	13250	19.0%	0.25
84-2	14149	663	13965	545	14000	32.5%	0.36
84-3	14722	772	14242	611	14900	37.2%	0.43
84-4	13612	509	13601	445	14200	23.6%	0.26
85-1	12325	516	12639	460	13200	20.4%	0.26
85-2	13709	592	13643	496	13900	29.6%	0.31
85-3	13432	548	13441	468	13900	26.9%	0.28
85-4	13070	521	13165	452	13500	24.8%	0.26
86-1	12166	531	12498	469	12600	21.8%	0.27
86-2	11490	489	11940	454	12200	13.8%	0.24
86-3	12389	547	12653	475	12700	24.6%	0.28
86-4	12223	458	12456	415	12600	17.8%	0.22
87-1	12282	528	12543	463	12300	22.9%	0.27
87-2	13207	535	13210	461	13500	25.7%	0.27
87-3	14100	615	13797	520	14300	28.6%	0.33
87-4	13493	610	13380	511	14100	29.9%	0.33
88-1	13011	638	13046	527	13500	31.6%	0.35
88-2	14197	530	13895	472	14300	20.7%	0.27
88-3	14409	554	14016	495	14500	20.2%	0.29
88-4	13709	595	13407	511	14000	26.1%	0.32
89-1	13052	498	13032	444	13600	20.6%	0.25

A. Indiana (continued)

2. Quadratic Empirical Bayes Estimates

$$\sigma_v^2 = 220,092$$

$$\hat{A}_1 = 4,372,241 \quad \hat{A}_2 = -31,969 \quad \hat{A}_3 = 1,169$$

$$\hat{\sigma}_v = 208,430$$

YEAR QTR	MF	MF SD	EB2	EB2 SD	BD	VAR RED.	B(2)
000's							
83-2	4592	205	4481	175	4650	27.2%	0.42
83-3	4759	237	4532	194	4800	33.1%	0.49
83-4	4046	174	4136	148	4200	27.3%	0.35
84-1	3957	188	4082	157	4000	30.9%	0.39
84-2	4331	178	4297	145	4350	34.0%	0.36
84-3	4251	170	4241	139	4350	32.8%	0.34
84-4	4257	152	4241	129	4300	28.2%	0.30
85-1	3825	150	3938	134	3950	20.1%	0.29
85-2	4054	131	4086	116	4150	21.7%	0.24
85-3	4153	175	4159	143	4250	33.3%	0.36
85-4	4286	402	4198	227	4150	68.1%	0.68
86-1	4147	436	4154	235	3950	70.8%	0.70
86-2	4294	421	4193	233	4150	69.5%	0.69
86-3	4443	421	4233	235	4250	68.8%	0.69
86-4	4146	158	4150	133	4150	28.9%	0.31
87-1	4007	176	4064	145	3950	32.5%	0.36
87-2	4107	150	4125	128	4200	27.2%	0.29
87-3	4925	297	4471	227	4600	41.7%	0.58
87-4	4615	198	4432	165	4600	30.2%	0.41
88-1	3873	157	3980	137	4100	23.1%	0.31
88-2	4421	182	4341	150	4500	32.4%	0.37
88-3	4597	193	4445	163	4600	28.9%	0.40
88-4	4168	178	4201	150	4300	29.3%	0.36
89-1	3857	155	3993	147	4050	10.7%	0.31

B. Iowa (continued)

2. Quadratic Empirical Bayes Estimates

$$\sigma_v^2 = 567,258$$

$$\hat{A}_1 = 15,329,013 \quad \hat{A}_2 = -387,360 \quad \hat{A}_3 = 14,095$$

$$\hat{\sigma}_v = 536,492$$

YEAR QTR	MF	MF SD	EB2	EB2 SD	BD	VAR RED.	B(2)
000's							
83-2	14736	505	14829	433	15200	26.6%	0.41
83-3	15405	635	14985	500	15600	38.0%	0.50
83-4	14589	731	14415	511	15000	51.0%	0.56
84-1	12661	496	13222	442	13250	20.7%	0.40
84-2	14149	663	13928	472	14000	49.3%	0.52
84-3	14722	772	13984	540	14900	51.0%	0.58
84-4	13612	509	13482	399	14200	38.5%	0.41
85-1	12325	516	12676	417	13200	34.6%	0.41
85-2	13709	592	13349	450	13900	42.3%	0.47
85-3	13432	548	13170	425	13900	39.8%	0.44
85-4	13070	521	12940	407	13500	39.0%	0.42
86-1	12166	531	12410	417	12600	38.2%	0.43
86-2	11490	489	11977	424	12200	25.0%	0.39
86-3	12389	547	12518	419	12700	41.4%	0.44
86-4	12223	458	12401	377	12600	32.4%	0.36
87-1	12282	528	12486	412	12300	39.1%	0.42
87-2	13207	535	13031	413	13500	40.4%	0.43
87-3	14100	615	13495	481	14300	38.8%	0.49
87-4	13493	610	13271	448	14100	46.1%	0.49
88-1	13011	638	13122	458	13500	48.5%	0.51
88-2	14197	530	13846	430	14300	34.2%	0.43
88-3	14409	554	14045	448	14500	34.6%	0.45
88-4	13709	595	13793	463	14000	39.3%	0.48
89-1	13052	498	13512	454	13600	17.1%	0.40

MF = Multiple Frame
SD = Standard Deviation
EB1 = Linear Empirical Bayes
EB2 = Quadratic Empirical Bayes
BD = Agricultural Statistics Board Final Estimate
VAR RED. = Variance Reduction = (V(MF) - V(EB))/V(MF)
B(1) = Linear Shrinkage Factor
B(2) = Quadratic Shrinkage Factor

