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I. INTRODUCTION

In survey sampling practice, unequal sampling weights (the inverse of the selection probabilities) can be both beneficial and deleterious. Extreme variation in the sampling weights can result in excessively large sampling variances, especially when the data and the selection probabilities are not positively correlated. In such situations, a few extreme weights can offset the precision gained from an otherwise well-designed and executed survey. On the other hand, if a positive correlation exists between the data and the selection probability for the sampling units, the extreme weights can result in reductions in the sampling variances, so that the extreme weights will be beneficial. Unplanned extreme variation in the sampling weights may result from the sample selection procedure, inaccuracies or errors in frame data, the nonresponse compensation procedures, or other sources.

In practice, several post-design procedures are used to limit or reduce the number and size of extreme sampling weights (Potter 1988). The practices and procedures fall into two categories:

1. procedures used to avoid or minimize the number and size of extreme weights by trimming or limiting components of the weights during the weight computation process; and
2. procedures used to identify, trim, and explicitly compensate for extreme sampling weights are implemented after the weights are fully computed.

The most notable uses of procedures to avoid or minimize the number and size of extreme weights are in the Census Bureau's Current Population Survey (CPS) and the Consumer Expenditure Survey (CES). In the CPS, the Census Bureau limits the size of the noninterview adjustment factor and the first-stage ratio adjustment factor so that extreme weights are less likely to occur (Hanson 1978, Bailar, et al. 1978). In the CES, the Census Bureau also sets a limit on an intermediate weighting factor (Alexander 1986). Any weight trimming here is compensated for implicitly by post-stratification.

In most survey situations, the final adjusted sampling weights are analyzed for extremely large sampling weights. In some of these situations, the survey statistician may impose a trimming strategy for excessively large weights. A trimming strategy will generally include a procedure to determine excessive weights and a method to distribute the trimmed portion of the weights among the untrimmed weights. Because of the weight trimming, the survey statistician will usually expect an increased potential for bias in the estimate and a decrease in the sampling variance. Hence, a trimming strategy may reduce the sampling variance for an estimate but increase the mean square error (the sampling variance plus the bias squared). The ultimate goal of weight trimming is to reduce the sampling variance more than enough to compensate

for the possible increase in bias and, thereby, reduce the mean square error (MSE).

In this research, I investigated current procedures and proposed new procedures that utilize the final adjusted sampling weights. In the empirical study of this research, the current and proposed procedures are demonstrated in a setting where the population can be fully enumerated. The specific empirical goal is to evaluate the two current procedures and two proposed procedures in terms of bias, sampling variance reduction, and mean square error as well as the consistency and variability of trimming levels using a data base containing data that are correlated or uncorrelated to the sampling weights.

II. PROPOSED AND CURRENT TRIMMING PROCEDURES

A. Overview

Four weight trimming procedures are discussed in this paper: two proposed weight trimming procedures and the two current procedures. The proposed procedures are: (1) Taylor Series procedure and (2) the Weight Distribution procedure. The two current procedures are: (1) the estimated MSE procedure and (2) the NAEP procedure. The Taylor series procedure and the estimated MSE procedure use data in the trimming procedure; the weight distribution procedure and the NAEP procedure, as described, do not use data. An alternative version of the NAEP procedure, which uses data, is described by Johnson et al. (1987).

For the procedures that use survey data, the analyst needs to select the key data items that represent the possible relationships between the weights and the data expected in the full set of data items. Most or all of these key data items should be non-zero because a zero value may mask an extreme weight. A description of how these four procedures are implemented in the empirical study is given below.

B. Proposed Procedures

For these procedures, assume a sampling frame of N units. Define the following:

Y_k = the observed data for the k th unit.

p_k = the single draw selection probability for the k th unit.

π_k = the expected number of selections for the k th unit when a sample of size n is selected (π_k is assumed less than 1 for all k) that is,

$$\pi_k = n p_k.$$

w_k = the untrimmed sampling weight for the k th unit in a sample of size n ; that is,

$$w_k = 1 / \pi_k.$$

w_{kt} = the sampling weight for the k th unit when a weight trimming strategy is used.

1. The Taylor Series Procedure

The Taylor series procedure uses the estimated MSE and the estimated relative bias computed for each data item at candidate trimming levels. The "optimal" trimming level is the trimming

level (from among the candidate trimming levels) that results in a minimum composite score for the estimated MSE and relative bias across the data items. The estimated MSE is computed using the derived forms for the bias and Taylor series linearized variate. Therefore, this procedure permits the assessment of both the potential for bias and the variance reduction introduced by the weight trimming.

Assume a sample of size n is selected with unequal probabilities and with replacement. The following derivations are conditional on a fixed weight trimming value of w_0 for all possible samples of size n . All weights below this value are adjusted by a factor A_s so that the original weight sum (W_s) for the sample is preserved. That is, the trimmed weight w_{kt} is defined as

$$w_{kt} = \tau_k w_0 + (1 - \tau_k) A_s w_k \quad (1)$$

where $\tau_k = 1$ if $w_k \geq w_0$,
 $= 0$ if $w_k < w_0$;

$$A_s = (S(1 - \tau_k w_0 \pi_k)) / (S(1 - \tau_k) w_k) \\ = A_{s1} / A_{s2}; \quad (2)$$

and S denotes the summation over the sample.

Using the definition of the trimmed weights in equations (1) and (2), the usual estimator can be written as a function of weighted totals. That is,

$$\hat{y}_t = S w_{kt} Y_k / S w_{kt} \\ = \{ S [\tau_k w_0 \pi_k + (1 - \tau_k) A_s w_k] Y_k \} / W_s \\ = C_s / W_s + (A_{s1} / A_{s2}) (B_s / W_s) \quad (3)$$

where $B_s = S(1 - \tau_s) w_k Y_k$

$$C_s = S \tau_k w_0 \pi_k w_k Y_k.$$

Because the estimator \hat{y}_t is a nonlinear function (equation (3)), the variance of the estimator can be approximated using the Taylor series linearization method. The linearized variate for variance estimation uses the sample estimates for the population values and is as follows:

$$\hat{z}_k = (1/\hat{N}) \{ w_{kt} (Y_k - \hat{y}_{NT}) \\ - w_k [S \tau_k w_0 (Y_k - \hat{y}_{NT}) / \hat{N}] \}. \quad (4)$$

where

$$\hat{y}_{NT} = S(1 - \tau_k) w_k Y_k / S(1 - \tau_k) w_k$$

From equation (4), we note that \hat{y}_{NT} becomes the pivotal quantity for variance estimates. The variance is estimated using the usual variance estimator for an unequal probability with replacement sample design when a Taylor series linearized variate is being used. The estimator of the bias is

$$\text{Bias}(\hat{y}_{t1}) = - S \tau_k (w_k - w_0) (Y_k - \hat{y}_{NT}) / \hat{N}.$$

For the Taylor series trimming procedure, the estimated MSE and the relative bias are computed

for each key data item l ($l=1, \dots, m$):

1. an estimated mean square error measure:

$$\text{MSE}_l = \text{Var}(\hat{y}_{t1}) + \text{Bias}(\hat{y}_{t1})^2, \text{ and}$$

2. the relative bias:

$$\text{RelBias}_l = \text{Bias}(\hat{y}_{t1}) / \hat{y}_{t1}.$$

The approximate MSE provides a measure of the mean square error resulting from the variance reduction and the potential bias caused by weight trimming. The relative bias (RelBias_l) provides information on the estimated magnitude of the bias.

For the empirical study, these two measures were used to identify a trimming level among a set of candidate trimming levels that, jointly for multiple data items, has the smallest estimated MSE and absolute value of the relative bias. Because multiple data items were used, the "optimal" trimming level may not be the smallest estimated MSE and bias for all or any of the data items.

In the empirical study, 20 candidate trimming levels were identified and 20 sets of weights were computed for each sample. Using these candidate weights, the procedure implemented is the following:

1. For each data item, the estimated MSE and relative bias are computed for each set of weights and are assigned a rank (1 to smallest value and 20 to the largest value) for each data item.
2. An average rank is computed for the estimated MSEs and for the square of the relative biases across the data items for each set of weights (defined by a trimming level).
3. The two average ranks are then combined and the trimming level with the lowest combined rank is defined as "optimal".

This procedure seeks the joint minimum of the estimated MSE and absolute value of the relative bias.

2. Weight Distribution

This trimming procedure is based on an assumed distribution for the sampling weights, and no survey data are used. If the selection probabilities are assumed to follow a Beta distribution, the sampling weight distribution can be shown to be of a form that is essentially an inverse of a beta variate.

In this procedure, the parameters for the sampling weight distribution are estimated using the sampling weights and a trimming level is computed that has a prespecified probability of occurrence, based on the distribution model. Sampling weights in excess of this trimming level are trimmed to this level and the excess is distributed among the untrimmed weights. The parameters for the sampling weight distribution are then estimated using the trimmed adjusted sampling weights and a revised trimming level is computed that has the prespecified probability of occurrence. The trimmed adjusted sampling weights are then compared to the revised trimming levels. If any weights are in excess of this trimming level, they are trimmed to this level and the excess is distributed among the

untrimmed weights. The comparison of the sampling weights to the trimming level is performed 10 times. This weight trimming procedure identifies and trims sampling weights with a small probability of occurrence, based on the model.

The key result is that, when a standard beta distribution is assumed for the single draw selection probabilities, the distribution model for the sampling weights (w) is in a form of a beta distribution. The density function for the distribution is

$$f_{w_k}(w) = n(1/nw)^{\alpha+1} (1 - 1/nw)^{\beta-1} / B(\alpha, \beta)$$

for $1/n \leq w \leq \infty$

where

$$B(\alpha, \beta) = \Gamma(\alpha) \Gamma(\beta) / \Gamma(\alpha + \beta)$$

Estimates for alpha and beta can be computed from the sample size, the mean weight, and the variance of the weights. That is,

$$\hat{\alpha} = [\bar{w} (n\bar{w} - 1) / ns_w^2] + 2 \quad (5)$$

$$\hat{\beta} = (n\bar{w} - 1) [\bar{w} (n\bar{w} - 1) / ns_w^2 + 1] \quad (6)$$

where

$$\bar{w} = S w_i / n$$

$$s_w^2 = S (w_i - \bar{w})^2 / n$$

The percentiles for the cumulative distribution function ($F_w(w)$) for the distribution can be computed using the standard Beta distribution ($Beta(x, \alpha, \beta)$) where

$$Beta(x, \alpha, \beta) = \int_0^x (1 - u)^{\beta-1} u^{\alpha-1} du / B(\alpha, \beta)$$

The values for the cumulative distribution function of the weight distribution $F_w(w)$ is

$$F_w(w_0) = 1 - \int_0^{1/nw_0} (1 - u)^{\beta-1} u^{\alpha-1} du / B(\alpha, \beta)$$

The weight distribution trimming procedure compares the distribution of the weights relative to the theoretical distribution. The probability of weights as large or larger than an observed weight (w_k) is given by

$$1 - F_w(w_k) = Beta(1/n w_k, \alpha, \beta).$$

A weight value with an extremely low probability of occurring can be trimmed to a specific probability of occurrence.

For the empirical study, the probability of occurrence criterion was set at 0.01; that is, a weight with a value in excess of w_{op} where $1 - F(w_{op}) = 0.01$ was trimmed to w_{op} . For the first iteration, the original weights were used to estimate α and β using equations (5) and (6), respectively. For the second to the tenth iteration, α and β was estimated using the weights from the prior iteration.

C. Currently Used Procedures

1. Estimated Mean Square Error (MSE) Trimming

In this procedure, an estimate of the mean square error for selected data items is evaluated at various trimming levels to determine empirically the trimming level (Cox & McGrath 1981). The assumption underlying this procedure is that, for a set of weights and data, a point exists at which the reduction in the sampling variance resulting from the trimming is offset by the increase in the square of the bias introduced into the estimate. In

this procedure, the $MSE(\hat{Y}_t)$ is estimated by

$$\begin{aligned} MSE(\hat{Y}_t) &\cong (\hat{Y}_t - \hat{Y})^2 - \text{Var}(\hat{Y}) \\ &\quad + 2[\text{Var}(\hat{Y}_t) \text{Var}(\hat{Y})]^{1/2} \end{aligned} \quad (7)$$

where

\hat{Y} = the estimate of the mean using the untrimmed weights;

\hat{Y}_t = the estimate of the mean using trimmed weights;

$\text{Var}(\hat{Y})$ = the estimated variance of \hat{Y} ; and

$\text{Var}(\hat{Y}_t)$ = the estimated variance of \hat{Y}_t .

The procedure is implemented by repeatedly computing the estimate of the MSE (equation (7)) for selected data items at differing levels of weight truncation. The 'optimal' level of truncation is the point that minimizes estimated MSE (i.e., minimizes sampling variance and estimated squared bias) for the set of key data items.

The estimated MSE is used to identify a trimming level among a set of candidate trimming levels that, jointly for multiple data items, has the smallest estimated MSE. In the empirical study, 20 candidate trimming levels were used and 20 sets of weights were computed for each sample. Using these candidate weights, the procedure implemented is the following:

1. For each data item, the estimated MSE is computed for each set of weights and assigned a rank (1 to the smallest value and 20 to the largest value).
2. An average rank is computed for the estimated MSEs across the data items for each set of weights and the lowest average rank is defined as the trimming level.

2. The NAEP Procedure

The procedure, referred to as the NAEP procedure in this paper, uses the comparison of the contribution of each weight to the sampling variance of an estimate by systematically comparing all weights to a value computed from the sum of the squared weights for the sample. If a weight is above the computed value, the weight is assigned this value and the other weights are adjusted to have the new weights sum to the original weight total. The sum of the squared adjusted weights is computed again and used in a second comparison of each individual adjusted weight. The procedure is repeated

until all adjusted weights are below or equal the value based on the sum of the adjusted squared weights. The procedure has been reported in conjunction with the National Assessment of Educational Progress (NAEP).

In this NAEP procedure, the relative contribution is limited to a specific value by comparing the square of each weight to a multiple of the the sum of the squared weights. That is,

$$w_k^2 \leq c \sum w_k^2 / n \text{ or}$$

$$w_k \leq K_n \cdot \quad (8)$$

where $K_n = (c \sum w_k^2 / n)^{1/2}$.

The value for c is arbitrary and can be chosen empirically by looking at the distribution of the square root of the values of

$$n w_k^2 / \sum w_k^2.$$

In the NAEP algorithm, each weight in excess of K_n is given this value and the other weights are adjusted to reproduce the original weight sum. The sum of square adjusted weights is computed and each weight is again compared using equation (8). The procedure is performed repeatedly until none of the weights exceed this criterion.

For the empirical study, the NAEP procedure was allowed to go through 10 iterations. The use of this NAEP procedure is documented in a methodological report of the NAEP study (Benrud et al. 1978) and, in this report, c is assigned a value of 10. In the NAEP 1983-84 Technical Report (Johnson et al. 1987), an analogous weight trimming procedure is described that uses data (estimated student counts). In this report, an empirical method is described to determine a value for c ; c was assigned a value of 10. Smaller or larger values of c will generate different trimming levels.

III. EMPIRICAL INVESTIGATION

A. Overview

The goals of the empirical study are to investigate and evaluate weight trimming procedures using multiple data items from a population that can be fully enumerated. The performance measures used in the empirical study include the change in the estimated variance of the estimate (that is, how much variance reduction is achieved), the extent of bias introduced and the change in the mean square error of the estimate (that is, whether the bias introduced by these procedures offsets the variance reduction), and the average and variance of the trimming levels (that is, whether these procedures result in consistent trimming levels over repeated samples).

B. Empirical Study Design

For the empirical study, county-level data on medical resources and demographic characteristics of the county population were obtained from the Area Resource File (ARF) data base developed by the Health Resources and Services Administration (U.S. Department of Health and Human Services 1987). For each

county, the number of households in 1980 was used as a size measure (for probability proportional to size (pps) sample selection) and four data items (both correlated and uncorrelated to the size measure) were used to assess the impact of the weight trimming. A total of 2,989 of the 3,080 county units in the ARF data base was used. County units excluded either had a very large (greater than 200,000 households) or a very small (less than 250 households) count of households, or were likely to have a zero value for one or more of the data items (county units in Hawaii and Alaska). Two hundred (200) samples of 100 units each were selected using the probability minimal replacement sampling procedure developed by Chromy (1979) for the pps selection.

The four variables were chosen because of the varying levels of estimated correlation between the data items and the sampling weight across the 200 samples: (1) median family income (negative), (2) birth rate among teenagers (positive), (3) percentage of 5 to 17 year old population that are white (zero), and the average temperature in July (zero).

The Taylor series procedure and the estimated MSE procedure evaluate statistics for predetermined candidate trimming levels. For the empirical study, 20 candidate trimming levels were computed for each sample. The candidate trimming levels were computed as follows:

- a. Trimming level 1 is the next to largest weight;
- b. Trimming level 2 is the average of the second and third largest weights;
- c. Trimming level 3 is the average of the second, third and fourth largest weights;
- d. Trimming levels 4 to 20 were computed as similar averages of the largest weights (excluding the largest weight).

For each trimming candidate level, a set of trimmed adjusted weights were computed. For the other procedures (as described in Section II), the trimming levels were generated within the procedure.

C. Summary of Results

The findings of the empirical study show that, of the four procedures, the Taylor series and the estimated mean square error procedure tended to perform similarly. Also, the NAEP procedure and the weight distribution procedure operated almost identically.

The average maximum sampling weight before trimming across the 200 replicated samples was 346.3 (Table 1). The average trimming level less than 50 percent of the maximum weight for all procedures. The average reduction in the maximum weight was the greatest for the estimated MSE procedure (an average trimming level of 125.4). The Taylor series procedure, the NAEP procedure, and the weight distribution procedure resulted in approximately the same reduction in the maximum weight. The design effects attributable to unequal weights also follow this pattern. The NAEP procedure and the weight distribution procedure exhibit substantially less variation (standard deviation between 8.3 and 8.5) in the trimming level over the samples than either the estimated MSE

procedure or the Taylor series procedure (standard deviations of approximately 42.0).

In comparing the Taylor series and the estimated MSE procedure, the estimated MSE procedure resulted in the larger average reduction (25.3 percent reduction) in the variance over the 200 replicated samples than the Taylor series procedure (22.2 percent) for the four variables (Table 2.). For the one variable (median family income) where a negative bias was expected, the estimated MSE procedure resulted in a larger average MSE than the Taylor series procedure (32.7 percent average increase (relative to the variance of the untrimmed weights) for the estimated MSE procedure versus a 15.3 percent average increase for the Taylor series procedure). Therefore, although the average variance reduction was larger for the estimated MSE procedure, the average reduction in the MSE was slightly larger for the Taylor series procedure (3.4 percent reduction for the Taylor series procedure versus 2.8 percent for the estimated MSE procedure).

Both the NAEP procedure and the weight distribution procedure produced almost identical trimming levels over the 200 samples. The NAEP procedure and the weight distribution procedure resulted in an average variance reduction of 16.7 and 16.0 percent (Table 2.) and average MSE reduction of 3.4 and 3.5 percent, respectively. Although the NAEP procedure and the weight distribution procedure provide the same results, the trimming level established for the weight distribution procedure can be specified by a probability statement (for example, extremely large weights with probability of less than 1 percent are trimmed) whereas the trimming level for the NAEP procedure is established empirically for each sample.

For each variable, interval estimates were investigated by computing the relative standard errors and assessing the proportion of 95 and 99 percent confidence intervals (computed using the individual sample estimates) that contained the true value (Table 4). Based on the change in the relative standard errors, the weight trimming procedures reduced the width of the confidence intervals by 15 to 20 percent (Table 3). Therefore, the effect of trimming on interval estimates was of keen interest.

For the first variable (median family income) that was expected to have a negative bias from weight trimming, the proportions of intervals containing the true value were higher for the NAEP and the weight distribution procedure than the proportions for the other two procedures and, for the 99 percent confidence interval, larger than the proportion for the untrimmed weights (Table 4). For the other three variables, the proportion of intervals containing the true value was generally larger for the trimmed weights than for the untrimmed weights. The reason for this improvement could be that the extreme weights in the samples increased the variation of the estimated means over the repeated sample for these three variables. Because the weight trimming introduced relatively little bias in the estimated means for these variables in the repeated samples and reduced the variation of the means over the repeated samples, this resulted in improved interval estimates.

In summary, the NAEP procedure and the weight distribution procedure are direct competitors in the sense that both procedures use only the sampling weights for weight trimming. These two procedures resulted in the least trimming and almost the same average variance and average MSE reductions. However, the weight distribution procedure permits a statistical basis, albeit model-based, for the choice of a trimming level whereas the criterion for the NAEP procedure is empirically based. The Taylor series and the estimated MSE procedures are also direct competitors in the sense that both use the observed survey data and an estimator of the mean square error in the trimming procedure. The Taylor series procedure also incorporates an estimator of the bias introduced by the trimming into the weight trimming procedure.

IV. CONCLUSIONS

In survey sampling practice, an analyst may encounter a survey with substantial variation in the sampling weights and a few extreme weights. Before implementing a weight trimming procedure, the analyst should evaluate whether the sampling weight variation has beneficial or deleterious effects on the sampling variances. When observed survey data are negatively correlated with the sampling weights and extremely large weights are associated with very small data values, the sampling variances computed using the original weights can be smaller than the sampling variances computed using equal or trimmed sampling weights. Therefore, weight trimming is not needed and may result in increased sampling variance as well as biased estimates. However, if the extremely large weights are determined to have adverse effects, weight trimming is a reasonable strategy to reduce the estimated sampling variances.

In terms of the four weight trimming procedures as evaluated in the empirical study, the estimated MSE procedure and the Taylor series procedure utilize the data and an estimate of the MSE, and, hence, these procedures are deemed preferable to the other two procedures. However, since the MSE estimate can mask the true extent of the bias introduced by weight trimming, the use of a measure of the relative bias, in addition to the estimated MSE, is preferred. The Taylor series procedure also utilizes a more correct variance estimator because the linearized variate took into account the weight trimming.

When no data or only categorical data are available, the weight distribution procedure is preferred over the NAEP procedure described here because the trimming level for the weight distribution procedure can be established on a theoretical basis. The trimming level for the NAEP procedure is based on an empirically evaluated subjective criterion.

As a general protocol for weight trimming, it is recommended that, if data are available for weight trimming, the analyst should evaluate the data used in the weight trimming procedure to ensure that they are representative of the data to be analyzed. That is, if some of the data to be analyzed are expected to be correlated (either positively or negatively) with the sampling weights, data with similar correlations should be used in the weight trimming. If no

data are available for weight trimming, the analyst should evaluate the weight trimming to assess the bias that may be introduced by evaluating the bias after data are available.

The four weight trimming procedures were evaluated in the empirical study as separate procedures. In practice, components of these procedures can be combined to form a single approach. For example, the sampling weight distribution can be used to determine candidate trimming levels for the Taylor series procedure.

The primary conclusion based on the empirical study results is that weight trimming can have both positive and negative effects. The positive effects (for example, the improvement in the interval estimates) occurred for some variables when little or no bias is introduced. However, for some data, the estimates using trimmed weights may be biased and the weight trimming can result in misleading point and interval estimates. All of the procedures resulted in reductions in the estimated sampling variance. However, all procedures also resulted in an increase in the estimated sampling variance for at least some of the 200 replicated samples. Therefore, the survey analyst needs to be cautious when trimming sampling weights because, unless weight trimming is conducted carefully and evaluated for various data items, larger sampling variance or substantial bias can result for some survey estimates.

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Table 1. The Effect of Trimming on Weights (200 Replicated Samples)

Item	Average	Minimum Value	Maximum Value	Standard Deviation	C.V.
Untrimmed Weights					
Maximum Weight	346.3	222.0	1,375.1	165.0	47.6
Design Effect	3.57	2.81	12.79	1.26	35.3
Maximum Trimmed Weight					
Estimated MSE	125.4	79.6	222.0	42.0	33.5
Taylor Series	142.2	79.6	222.0	42.2	29.6
NAEP Procedure	147.3	138.7	199.6	8.3	5.7
Weight Dist'n	151.0	142.3	204.7	8.5	5.7
Design Effect Due to Unequal Weights After Trimming					
Estimated MSE	2.23	1.67	2.95	0.341	15.3
Taylor Series	2.37	1.72	2.95	0.321	13.5
NAEP Procedure	2.46	2.42	2.50	0.015	0.6
Weight Dist'n	2.48	2.44	2.53	0.017	0.7

Table 2. Average Percentage Change in Variance and Mean Square Error Based on Variances Estimates from 200 Replicated Samples

Variable	Procedure			
	Estimated MSE	Taylor Series	NAEP	Weight Distribution
Estimated Sampling Variances				
1*	-21.7	-19.9	-15.0	-14.5
2	-31.7	-28.0	-22.1	-21.4
3	-22.8	-19.1	-12.4	-11.8
4	-25.1	-21.7	-17.2	-16.5
Average	-25.3	-22.2	-16.7	-16.0
Estimated Mean Square Error				
1	32.7	15.3	2.5	1.8
2	-19.8	-14.4	-9.2	-9.0
3	-10.7	-6.0	-1.4	-1.2
4	-13.5	-8.5	-5.6	-5.5
Average	-2.8	-3.4	-3.4	-3.5

Percentage Change = $100 * (\text{Trimmed} - \text{Untrimmed}) / \text{Untrimmed}$.

* Variables: 1 = median family income; 2 = birth rate among teenagers; 3 = percentage white age 10-17; 4 = temperature in July.

Table 3. Percentage Relative Standard Errors Based on Estimates from 200 Replicated Samples

Variable	Untrimmed Weights	Estimated MSE	Taylor Series	NAEP	Weight Distribution
1	3.35	2.74	2.79	2.90	2.92
2	6.97	5.36	5.55	5.77	5.81
3	4.04	3.10	3.25	3.40	3.42
4	1.17	0.96	0.98	1.02	1.02
Average	3.88	3.04	3.14	3.27	3.29
Percentage Change		-21.7%	-19.1%	-15.8%	-15.2%

Percentage Relative Error = $100 * \text{standard error} / \text{estimate}$.

Percentage Change = $100 * (\text{Trimmed} - \text{Untrimmed}) / \text{Untrimmed}$.

* Variables: 1 = median family income; 2 = birth rate among teenagers; 3 = percentage white age 10-17; 4 = temperature in July.

Table 4. Percentage of Confidence Intervals Covering True Value Based on 200 Replicated Samples

	Variable ¹			
	1	2	3	4
95 Percent Confidence Interval²				
Untrimmed Weights	94.5	90.5	84.5	90.5
Estimated MSE Procedure	83.5	91.0	85.5	90.5
Taylor Series Procedure	88.0	90.5	85.5	89.5
NAEP Procedure	91.0	91.5	88.0	92.0
Weight Distribution Procedure	91.5	91.5	87.5	92.5
99 Percent Confidence Interval³				
Untrimmed Weights	96.5	95.5	91.0	98.0
Estimated MSE Procedure	93.5	97.0	90.5	98.5
Taylor Series Procedure	95.0	97.5	91.5	98.5
NAEP Procedure	97.5	97.5	92.0	98.5
Weight Distribution Procedure	97.5	97.5	92.0	98.5

¹ Variables: 1 = median family income; 2 = birth rate among teenagers; 3 = percentage white age 10-17; 4 = temperature in July.

² 95 Percent Confidence Interval = $\hat{\theta}_g \pm 1.96 * \text{SQRT}(\text{Var}(\hat{\theta}_g))$

³ 99 Percent Confidence Interval = $\hat{\theta}_g \pm 2.58 * \text{SQRT}(\text{Var}(\hat{\theta}_g))$