COMPARISON OF VARIANCE ESTIMATORS FOR PRODUCER PRICE INDEX DATA

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1. INTRODUCTION

In an effort to measure sample variability in the Producer Price Index (PPI) the Bureau of Labor Statistics is evaluating different variance estimation methodologies. A simulation study has been conducted to evaluate how balanced half-sample replication methods of variance estimation behave on PPI data and to determine which form of the estimator is most appropriate for PPI data. This paper presents the results of the simulation study. An overview of the study is given in Section 2. The sampling procedures, index estimation, and data chosen for the simulation are described in Section 3. Statistics computed for comparison of the variance estimators are also described. The results of the comparison of the variance estimators for the long term index are presented in Section 4. Section 5 is a summary of the results on the comparison of the variance estimators for one month index change. Conclusions are in Section 6.

2. OVERVIEW OF THE STUDY

Seven estimators of variance of the industry level PPI using balanced half-sample replication methodology are computed and evaluated in this simulation study. The estimators are compared on the basis of relative bias, relative mean squared error, and confidence interval coverage rates.

An exact estimator of the variance of the PPI is not available because the estimator of the PPI is nonlinear and the price data are obtained through a complex sample design. Therefore, approximate techniques of variance estimation must be evaluated using PPI data and the most accurate and appropriate method should be chosen. The computation of the index is described below in Section 3.B. Hill (1987) provides more detailed information on the PPI index estimation methodology.

The variance estimators evaluated in this study are those for nonlinear estimators given in Wolter (1985). The seven estimators of variance of the industry level PPI, or $\hat{I}$, are based on k balanced replicates or half-samples. $\hat{I}_\alpha$ is the index estimate computed from the α-th replicate and $\hat{I}^c_\alpha$ is the index estimate computed from the half-sample which is complementary to α, or the complement of α. The estimators considered are:

$$V_1(\hat{I}) = \sum_{\alpha=1}^{k} (\hat{I}_\alpha - \hat{I})^2 / k$$

$$V_2(\hat{I}) = \sum_{\alpha=1}^{k} (\hat{I}^c_\alpha - \hat{I})^2 / k$$

$$V_3(\hat{I}) = [\, V_1(\hat{I}) + V_2(\hat{I}) \,] / 2$$

$$V_4(\hat{I}) = \sum_{\alpha=1}^{k} (\hat{I}_\alpha - \hat{I}^c_\alpha)^2 / 4k$$

$$V_5(\hat{I}) = \sum_{\alpha=1}^{k} (\hat{I}_\alpha - \hat{\hat{I}})^2 / k$$

$$V_6(\hat{I}) = \sum_{\alpha=1}^{k} (\hat{I}^c_\alpha - \hat{\hat{I}}^c)^2 / k$$

$$V_7(\hat{I}) = [\, V_5(\hat{I}) + V_6(\hat{I}) \,] / 2$$

where $\hat{\hat{I}} = \sum_{\alpha=1}^{k} \hat{I}_\alpha / k$, and

$$\hat{\hat{I}}^c = \sum_{\alpha=1}^{k} \hat{I}^c_\alpha / k.$$

Since $\hat{I}$ is nonlinear these estimators are unequal and biased for Var{$\hat{I}$}. Also, as stated in Wolter (1985), all of the estimators except $V_4(\hat{I})$ are sometimes regarded as estimators of the mean squared error of $\hat{I}$, while $V_4(\hat{I})$ is considered an estimator of the variance of $\hat{I}$.

3. SIMULATION STUDY

A. Sample Design

The simulation study was designed to follow PPI sampling and estimation procedures as closely as feasible. Actual price data from three manufacturing industries were used as finite populations for sampling.

For each of the three industries in the study, the finite population consisted of all companies that responded to the Bureau of Labor Statistics (BLS) survey. The BLS survey was a two-stage sample with probability proportional to size systematic sampling conducted in each stage. In the first stage companies were selected with probability proportional to employment. In the second stage items or products were selected within each company with probability proportional to revenue generated by the product within the company. The sample design for the simulation study was essentially a two-stage stratified design with probability proportional to size sampling in each stage. The companies were first ordered by descending employment and then seven variance strata were formed so that the total employments of the strata were approximately equal. Within each variance stratum two independent systematic samples were drawn and designated the A and B samples. Units within each stratum that had employment greater than the sampling interval were selected with certainty. From the remaining companies in the stratum, the A and B samples were drawn with probability proportional to employment. The items that were selected in the second stage of sampling in the original BLS survey were taken as the second stage units in the simulation study.

For each of the industries and for each of two sample sizes, 1000 samples were drawn from the finite population. The two sample sizes of 15 and 20 were chosen with the hope that some difference in the results due to sample size would be detected. Using probability proportional to size sampling and sample sizes larger than 20, most of the additional units selected would have been certainty units. These units would have been selected in every sample and would have decreased the variability between the samples. With an increase of only five sample units, it was uncertain whether any significant differences would be detected, and in fact, no significant differences were found in the results. Only the results for sample size 15 are presented in this paper.

B. Index Calculation

Eight balanced half-samples and their complements, as described in McCarthy (1966), were formed using the items of the companies selected in a sample. Once the items were assigned to the appropriate replicate or complement, half-sample indexes were computed. In addition, an overall sample index, $\hat{I}_s$, s = 1,...,1000, was computed in the same manner, except that all the items of the companies selected in the sample were used in the computation.

An item is defined as a product, unique with respect to price determining characteristics, which is selected by a systematic sampling method from the universe of products manufactured by a company. The values for the items used in calculation of the indexes are the product of the long term price relative of the item and the weight of the item. This product, W_{ij}^t is represented in this paper, for an item j belonging to company i at time t, by

$$W_{ij}^{t} = \omega_{ij} \times r_{ij}^{t,b} \quad \text{where}$$

$r_{ij}^{t,b}$ is the long term price relative from the base period b to time t for item j in company i, and

ω_{ij} is the item weight which is derived from the probability of selection of company i, and the revenue of company i represented by item j.

The index for each industry has a defined aggregation structure. This structure includes the detailed product cells to which items are assigned and the order of aggregation, or combination, of the lowest level cells to form higher level aggregate cells.

In this study the indexes were computed only at the industry level. These indexes, which are long term indexes representing change from the base period b to time t, were computed using the following equation:

$$\hat{I}^{t,b} = \hat{Z}^{t} \times \hat{I}^{t-1,b} \times 100\,, \quad \text{where}$$

$$\hat{Z}^{t} = \frac{\displaystyle\sum_{n} \frac{\sum_{ij\in n} W_{ij}^{t}}{\sum_{ij\in n} W_{ij}^{t-1}} \times CLW_n^{t-1}}{\displaystyle\sum_{n} \frac{\sum_{ij\in n} W_{ij}^{t-1}}{\sum_{ij\in n} W_{ij}^{t-2}} \times CLW_n^{t-2}}\,,$$

n is a detailed product cell, and

$$CLW_n^{t-1} = r_n^{t-1,b} \times (\text{Census Weight})_n^{b}$$

is the cell n link weight for time period t-1, which is represented in this paper by the product of the long term relative for cell n in period t-1 and the Census weight at the base period representing made-in-industry value of shipments (VOS) from the Census of Manufactures.

In the simulation study the lowest level cells were not aggregated to the next higher level cell as in the actual PPI computations, but were combined in one step to form the industry level index. This affected how the current period link weights were updated when a cell had no items in a particular month. In the simulation study, the link weight for a cell that had no items was adjusted by the relative price change of the industry level index. In the PPI calculation, the link weight for the cell would have been adjusted using the relative price change of the next higher level cell.

C. Simulation Study Data

The data chosen for the study are price data from three manufacturing industries. The industries were chosen from different sectors of manufacturing -- food products, printing and publishing, and primary metal industries. The industries were sampled early in the current cycle of industry resampling and nineteen months of data were available. Also, these industries had a large number of items in repricing compared to other industries sampled concurrently. Characteristics of each of the industries, referenced by their Standard Industrial Classification (SIC) four-digit code and title, are shown in Table 1. The number of items in repricing varies over the nineteen months because of companies discontinuing repricing some or all of their items.

Table 1
SIMULATION STUDY POPULATION CHARACTERISTICS

Industry	Finite Population Size (No. of comp.)	No. of Items over 19 mos. Min	Max
SIC 2051 Bread and other bakery products, except cookies and crackers	104	356	373
SIC 2711 Newspapers, publishing or publishing and printing	143	368	394
SIC 3321 Gray and ductile iron foundries	106	341	386

D. Statistics Computed

The following calculations were performed for each of the three industries. For each of the 1000 samples and each of the 19 months of data the following were computed:

a) the half-sample indexes, $\hat{I}_\alpha, \alpha = 1,\ldots,k$, and complementary half-sample indexes, $\hat{I}^c_\alpha$, $\alpha = 1,\ldots,k$, $k = 8$
b) the overall sample index, $\hat{I}$
c) the seven estimates of variance using $V_1(\hat{I})$ through $V_7(\hat{I})$ defined in Section 2
d) the relative variance of $\hat{I}$ using $V_3(\hat{I})$, $V_4(\hat{I})$, and $V_7(\hat{I})$ defined by the following using $V_3(\hat{I})$ as an example

$$\text{Relative variance} = \frac{V_3(\hat{I})}{(\hat{I})^2}$$

e) 95% confidence intervals of $\hat{I}$, using $V_3(\hat{I})$, $V_4(\hat{I})$, and $V_7(\hat{I})$, using the t distribution and 8 degrees of freedom, computed using

$$\text{Conf. Interval} = \hat{I}_s \pm 2.306[V_3(\hat{I}_s)]^{1/2}$$

The following summary statistics were computed for each of the 19 months. The statistics involving the variance estimators were computed only for $V_3(\hat{I})$, $V_4(\hat{I})$, and $V_7(\hat{I})$. $V_3(\hat{I})$ is used in the equations as an example.

a) Average relative bias of index =

$$(1/1000) \sum_{s=1}^{1000} \frac{(\hat{I}_s - I)}{I}, \text{ where}$$

I is the population index computed using all the items of the companies in the finite population using the equation in Section 3.B.

b) Average relative bias of sample variances =

$$(1/1000) \sum_{s=1}^{1000} \frac{V_3(\hat{I}_s) - V}{V}, \text{ where}$$

V is an approximation of the true variance of $\hat{I}$ computed as

$$V = (1/999) \sum_{s=1}^{1000} (\hat{I}_s - \hat{\hat{I}})^2, \text{ where}$$

$$\hat{\hat{I}} = (1/1000) \sum_{s=1}^{1000} \hat{I}_s$$

c) Average relative MSE of sample variances =

$$(1/1000) \sum_{s=1}^{1000} \frac{[V_3(\hat{I}_s) - V]^2}{V^2}$$

d) Variance of the variance estimators =

$$(1/999) \sum_{s=1}^{1000} [V_3(\hat{I}_s) - \overline{V_3(\hat{I})}]^2$$

e) Confidence interval coverage rates of the variance estimators computed as the proportion of intervals that contain the population index.

In the results presented in Section 4.C., the variances were computed with an approximate finite population correction factor (fpc) of [1 - (n/N)]. Sampling fractions ranged from 0.10 to 0.19. An alternate fpc for probability proportional to size sampling given in Wolter (1985) was also computed. This fpc would reduce the variance estimates by 0.50 to 0.65 depending on the industry and sample size. The effect on the statistics computed of applying the pps fpc to the variance estimates is included at the end of Section 4.C.

4. RESULTS

A. General

The results presented here are for sample size 15. The sample size 20 results are similar in that the variance estimators give the same results in relationship to one another. Significant differences in the magnitude of the results were not obtained based on sample size.

The evaluation was conducted on $V_3(\hat{I})$, $V_4(\hat{I})$, and $V_7(\hat{I})$. $V_1(\hat{I})$, $V_2(\hat{I})$, and $V_3(\hat{I})$ are identical for linear estimators and the same is true for $V_5(\hat{I})$, $V_6(\hat{I})$, and $V_7(\hat{I})$. In our case, with the nonlinear estimator $\hat{I}$, these estimators should be fairly close. In fact, $V_1(\hat{I})$ and $V_2(\hat{I})$ were found to differ by no more than 0.09 in any month. The difference between $V_5(\hat{I})$ and $V_6(\hat{I})$ was also no more than 0.09. In order to simplify the analysis, statistics were computed for $V_4(\hat{I})$ along with $V_3(\hat{I})$, which is the average of $V_1(\hat{I})$ and $V_2(\hat{I})$, and $V_7(\hat{I})$, which is the average of $V_5(\hat{I})$ and $V_6(\hat{I})$.

B. Sample Indexes

The average relative bias of the sample indexes is small and almost always positive. The largest relative bias is 0.52% of the population index.

The distributions of the 1000 sample indexes computed are not normally distributed in most months. Of the 114 months of 1000 sample indexes computed for the three SICs and two sample sizes, the distributions of only seven months could not be rejected as having a normal distribution. When the distributions of the sample indexes for an SIC are skewed, they are skewed to the same direction in all of the months. Skewness is pronounced when the underlying item data contain an extreme value as in the data of SIC 2051 for MAY87. The skewness persists in the months after JUN87 and becomes slightly less prominent after NOV87.

C. Variances

For all three SICs over the 19 months the variance estimates increase with time. The estimator $V_4(\hat{I})$ produces the smallest variances in all cases and $V_3(\hat{I})$ produces the largest variances in all cases. The result of $V_4(\hat{I})$ producing the smallest variances of these estimators was expected because, as mentioned previously, it is regarded as an estimator of the variance of $\hat{I}$, while the remaining estimators are considered estimators of the mean squared error of $\hat{I}$. In Table 2, the observed variance estimates of $\hat{I}$ and the MSE($\hat{I}$) for one of the industries, SIC 2711, are shown. The relative variance of $\hat{I}$ of the three industries in this study also shows an increasing trend over time. Plots of the relative variance of $\hat{I}$ for the three SICs using $V_4(\hat{I})$ are shown in Exhibit 1.

A distinct increase in the variance and relative variance was observed in MAY87 of SIC 2051 and in JUL87 of SIC 3321. The increase in SIC 2051 can be attributed to a legitimate increase in the price relative of one item of 67%. After the increase the weight of the item was 21% of its cell weight and the cell represented 14% of the total industry. When this item was selected in a sample the variance was large (i.e., anywhere from 10 to 40 depending on what other establishments were selected in the sample) for MAY87 and remained large in the following months. Samples without the item showed a change no greater than 1.0 in MAY87. The approximation of the true variance, V, increased by 3.8 in MAY87, while the three observed variance estimates, which increased by 1.6 to 2.2, are seriously underestimating V from MAY87 on.

The increase in JUL87 in SIC 3321, on the other hand, cannot be attributed to only one item. This increase in variance is caused by, in most samples, approximately 10 items, some with increasing price relatives and some with decreasing price relatives. V increased by 0.4 in JUL87 and the observed variance estimates increased by larger amounts of 0.5 to 0.6. The variance estimators are in most cases overestimating V in the months after the jump.

The relative biases of the variance estimators are shown in Table 3 below. In most of the months the variance estimators, excluding $V_3(\hat{I})$ in SIC 2711 and SIC 3321, are underestimating V. For $V_4(\hat{I})$, the relative bias is negative in 51 out of 57 months for all three SICs. In 47 out of 57 months the relative bias is negative for $V_7(\hat{I})$. In SIC 2051, the relative bias of $V_3(\hat{I})$ is negative every month, while in the other two SICs it is negative in 5 or 6 of the 19 months.

In two of the SICs, 2711 and 3321, the approximation of the true variance, V, increases over time, however, at a slower rate than the variance estimates. A result of this is a pattern in the relative biases over time. For the variance estimators that underestimate most of the 19 months, i.e. $V_4(\hat{I})$ and $V_7(\hat{I})$ in SIC 2711, the relative bias becomes less negative in time. In SIC 3321 the variance estimators underestimate V in the earlier months with the bias becoming less negative in time, and overestimate V in the later months with the relative bias becoming more positive in time.

The magnitudes of the relative biases range from near zero to as high as near 50 percent in a few months. The relative bias is largest, 30 to 50 percent, in SIC 2051 in the months following the jump in variance in MAY87. In SIC 2711 the magnitude of the relative bias is largest in the earlier months and in SIC 3321, it is largest in the earlier and later months and smallest in the middle months.

The relative mean squared errors of the variance estimators are shown on Table 4. Of the three variance estimators $V_4(\hat{I})$ has the smallest relative MSE for all three SICs and $V_3(\hat{I})$ has the largest.

The relative MSE of the variance estimates shows similar patterns to the relative bias in the changes in magnitude within each SIC. In SIC 2711 the relative MSE decreases in time. The same is true for SIC 2051 except that the level of the relative MSE jumps in MAY87. In SIC 3321 the relative MSE is largest in the earlier months, decreases in the middle months, and then increases again in the later months.

The coverage rates are shown in Table 5 below. $V_3(\hat{I})$ shows the best coverage rates which is expected because $V_3(\hat{I})$ produces the largest estimates of the variance. $V_4(\hat{I})$ has the lowest coverage rates. In SIC 2051 the coverage rates are very low in the first month. The observed variance estimate in that month was near zero which can be attributed to a very small number of items with a change in price. Otherwise, the coverage rates are still not especially high. The rate approaches 95 percent or higher in more than one month for only $V_3(\hat{I})$ in two of the SICs. The coverage rates do not have consistent patterns in time across all three SICs. The rates increase in time only in SIC 3321.

As mentioned in Section 3.D. the variance estimates were also computed with an approximate fpc appropriate for pps sampling. This fpc reduced the sample size 15 variance estimates by 0.60 to 0.65. The resulting relative bias of the variance estimators of all three SICs was negative in 169 of 171 months. $V_4(\hat{I})$ had the largest negative bias and $V_3(\hat{I})$ had the smallest negative bias. The relative MSE was reduced with $V_4(\hat{I})$ having the smallest relative MSE in most months. In SIC 2711, $V_7(\hat{I})$ had the smallest relative MSE in the last eleven months. $V_3(\hat{I})$ had the largest relative MSE in all cases.

5. COMPARISON OF VARIANCE ESTIMATORS FOR ONE MONTH INDEX CHANGE

In addition to publishing monthly industry indexes, BLS also publishes the percent change of the index from the previous month. From the simulation data, monthly percent change of the replicate and complement half-sample indexes and the overall sample indexes was computed. Variance of the percent change was then calculated using $V_3(\hat{I})$, $V_4(\hat{I})$, and $V_7(\hat{I})$ as defined previously in Section 2. Comparison of the estimators was based on the same criteria that were described in Section 3.D.

The variance estimators for the index change compare similarly to the variance estimators for the long term index in the following ways:

a) $V_4(\hat{I})$ always gives the smallest estimate of variance, and $V_3(\hat{I})$ gives the largest estimate of variance.

b) $V_4(\hat{I})$ has the smallest relative MSE and $V_3(\hat{I})$ the largest relative MSE.

c) coverage rates of $V_3(\hat{I})$ are the best and $V_4(\hat{I})$ the worst, with a difference between $V_3(\hat{I})$ and $V_4(\hat{I})$ averaged over the nineteen months of 1% to 3% depending on the SIC.

The variance estimators for the one month index change compare differently from the variance estimators for the long term index in the following ways:

a) There is no increasing trend over time of variance of one month index change. The variance fluctuates from month to month depending on the amount of change of the index.

b) The variance estimators underestimate the approximation of the true variance of the one month change less often than for the long term index. Fifty percent of the months are underestimated as opposed to seventy five percent of the months for the long term index. In two of the SICs, $V_4(\hat{I})$ underestimates the true variance more often than $V_3(\hat{I})$, however the average relative bias of the months underestimated was not that different for the two estimators.

c) Coverage rates for the one month index change fluctuate widely from month to month and are much lower than for the long term index. The one month index change coverage rates are 0.13 to 0.30 lower than the long term index coverage rates depending on the SIC and averaging over all nineteen months. The lower rates correspond to months with lower variance.

d) Relative MSE of the one month index change variance estimators are generally larger than the relative MSE of the long term index variance estimators.

6. CONCLUSIONS

In this simulation study, the variance estimator $V_4(\hat{I})$ performed better in terms of relative mean squared error. However, for the long term index, $V_4(\hat{I})$ underestimated the approximation of the true variance, V, more often and with a larger magnitude than the other estimators. Variance estimator $V_3(\hat{I})$ underestimated less frequently and with a smaller magnitude than the other estimators. Also, and probably the most important criterion for our purposes, $V_3(\hat{I})$ has the best coverage rates. Therefore, of the variance estimators evaluated in this simulation study, $V_3(\hat{I})$ is the preferred estimator of variance for the long term index.

However, for the one month index change, the variance estimates are very unstable and none of the variance estimators behave particularly well in the comparison criteria. The coverage rates for the estimators fluctuate so widely from one month to the next (e.g., 0.97, 0.15, and 0.59 for SIC 3321 in APR87, MAY87, AND JUN87) that even computing variances for the one month index change is questionable.

This study agrees with results reported in a previous study of variance estimation techniques for PPI data. In Collia (1988) most of the long term index variances and coefficients of variation computed for 114 industries increased over time. This trend has been reported for other long term index variance estimates and may be inherent in the index estimation methodology.

The simulation study has identified some problems with using the balanced half-sample methodology. The relative bias of the variance estimates is larger than desired in many months and the coverage rates are lower than desired. Also, as seen in this study, the variance estimators are sensitive to extreme values in the underlying item data.

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EXHIBIT 1
RELATIVE VARIANCE

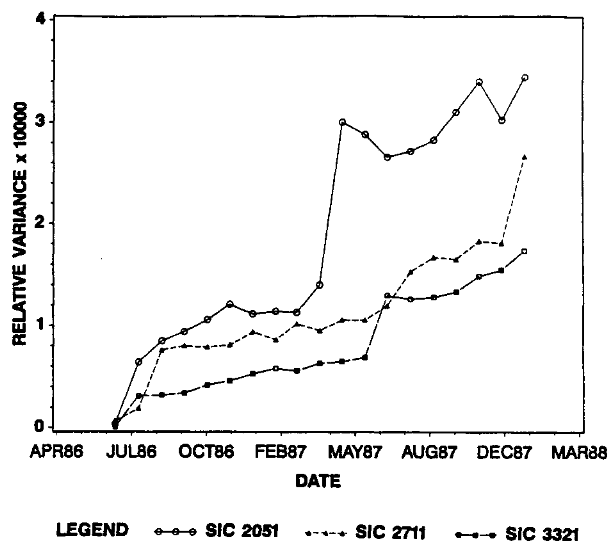

Table 2
Observed Variance Estimates, V, and $\widehat{MSE}(I)$
SIC 2711

Date	$\hat{V}_3(I)$	$\hat{V}_4(I)$	$\hat{V}_7(I)$	V	$\widehat{MSE}(I)$
JUL86	0.06	0.06	0.06	0.05	0.05
AUG86	0.18	0.17	0.18	0.13	0.13
SEP86	0.87	0.71	0.75	1.00	1.00
OCT86	0.91	0.75	0.79	1.04	1.05
NOV86	0.91	0.75	0.79	1.04	1.06
DEC86	0.92	0.76	0.81	1.05	1.06
JAN87	1.12	0.93	0.98	1.16	1.23
FEB87	1.10	0.87	0.92	1.10	1.24
MAR87	1.28	1.04	1.09	1.26	1.44
APR87	1.22	0.97	1.03	1.21	1.40
MAY87	1.32	1.08	1.14	1.27	1.45
JUN87	1.32	1.07	1.13	1.24	1.43
JUL87	1.48	1.23	1.29	1.39	1.52
AUG87	1.86	1.58	1.65	1.67	1.80
SEP87	2.01	1.73	1.80	1.89	2.02
OCT87	2.00	1.72	1.80	1.88	2.01
NOV87	2.29	1.90	1.99	2.27	2.44
DEC87	2.26	1.88	1.97	2.24	2.40
JAN88	3.38	2.94	3.08	3.25	3.29

Table 3
Relative Bias of Variance Estimators

	SIC 2051			SIC 2711			SIC 3321		
Date	$\hat{V}_3(I)$	$\hat{V}_4(I)$	$\hat{V}_7(I)$	$\hat{V}_3(I)$	$\hat{V}_4(I)$	$\hat{V}_7(I)$	$\hat{V}_3(I)$	$\hat{V}_4(I)$	$\hat{V}_7(I)$
JUL86	-0.08	-0.16	-0.12	0.31	0.26	0.29	-0.27	-0.46	-0.42
AUG86	-0.17	-0.30	-0.26	0.40	0.36	0.39	-0.08	-0.23	-0.16
SEP86	-0.17	-0.31	-0.27	-0.13	-0.28	-0.25	-0.08	-0.22	-0.16
OCT86	-0.08	-0.22	-0.18	-0.12	-0.28	-0.24	-0.07	-0.22	-0.15
NOV86	-0.06	-0.22	-0.17	-0.13	-0.28	-0.24	-0.04	-0.20	-0.13
DEC86	-0.04	-0.20	-0.15	-0.12	-0.28	-0.24	-0.03	-0.18	-0.11
JAN87	-0.01	-0.19	-0.13	-0.03	-0.20	-0.16	0.02	-0.15	-0.06
FEB87	-0.07	-0.21	-0.15	0.00	-0.21	-0.16	0.08	-0.12	-0.04
MAR87	-0.07	-0.21	-0.15	0.02	-0.18	-0.13	0.10	-0.11	-0.02
APR87	-0.06	-0.21	-0.15	0.01	-0.19	-0.15	0.11	-0.09	0.00
MAY87	-0.32	-0.47	-0.44	0.04	-0.15	-0.10	0.05	-0.14	-0.06
JUN87	-0.32	-0.47	-0.44	0.06	-0.14	-0.09	0.04	-0.17	-0.09
JUL87	-0.33	-0.49	-0.46	0.07	-0.12	-0.07	0.14	-0.02	0.05
AUG87	-0.33	-0.49	-0.46	0.12	-0.05	-0.01	0.14	-0.02	0.05
SEP87	-0.32	-0.48	-0.45	0.06	-0.08	-0.05	0.13	-0.03	0.04
OCT87	-0.26	-0.43	-0.39	0.06	-0.08	-0.05	0.17	0.00	0.08
NOV87	-0.28	-0.45	-0.42	0.01	-0.16	-0.12	0.46	0.22	0.31
DEC87	-0.28	-0.44	-0.40	0.01	-0.16	-0.12	0.44	0.21	0.31
JAN88	-0.22	-0.38	-0.34	0.04	-0.09	-0.05	0.33	0.14	0.22

Table 4
Relative Mean Squared Error of Variance Estimators

	SIC 2051			SIC 2711			SIC 3321		
Date	$\hat{V}_3(I)$	$\hat{V}_4(I)$	$\hat{V}_7(I)$	$\hat{V}_3(I)$	$\hat{V}_4(I)$	$\hat{V}_7(I)$	$\hat{V}_3(I)$	$\hat{V}_4(I)$	$\hat{V}_7(I)$
JUL86	1.89	1.64	1.74	4.57	4.21	4.46	0.95	0.74	0.78
AUG86	3.06	2.38	2.53	3.58	3.41	3.54	3.64	2.73	3.02
SEP86	2.13	1.67	1.76	2.60	1.42	1.53	3.66	2.75	3.04
OCT86	2.22	1.78	1.89	2.36	1.30	1.41	3.53	2.65	2.93
NOV86	1.80	1.44	1.52	2.21	1.23	1.33	2.51	1.84	2.05
DEC86	1.52	1.23	1.31	2.15	1.19	1.29	2.34	1.76	1.95
JAN87	1.70	1.35	1.45	1.04	0.68	0.74	2.38	1.81	2.03
FEB87	1.63	1.34	1.42	0.86	0.60	0.65	1.74	1.30	1.44
MAR87	1.67	1.38	1.47	0.69	0.50	0.53	1.92	1.43	1.58
APR87	1.50	1.14	1.22	0.69	0.49	0.52	1.78	1.34	1.51
MAY87	2.50	1.46	1.52	0.71	0.50	0.53	1.53	1.13	1.25
JUN87	2.58	1.51	1.57	0.73	0.51	0.54	1.30	0.98	1.08
JUL87	2.80	1.62	1.69	0.74	0.56	0.59	2.48	1.97	2.15
AUG87	2.67	1.56	1.62	0.63	0.48	0.51	2.64	2.10	2.28
SEP87	2.36	1.46	1.52	0.58	0.46	0.49	2.53	2.02	2.19
OCT87	2.21	1.36	1.41	0.61	0.48	0.51	2.63	2.08	2.26
NOV87	1.70	1.13	1.16	0.58	0.42	0.43	4.79	3.94	4.22
DEC87	1.89	1.19	1.23	0.60	0.43	0.44	4.34	3.54	3.83
JAN88	1.65	1.03	1.06	0.48	0.36	0.37	3.01	2.44	2.66

Table 5
Coverage Rates of the Variance Estimators

	SIC 2051			SIC 2711			SIC 3321		
Date	$\hat{V}_3(I)$	$\hat{V}_4(I)$	$\hat{V}_7(I)$	$\hat{V}_3(I)$	$\hat{V}_4(I)$	$\hat{V}_7(I)$	$\hat{V}_3(I)$	$\hat{V}_4(I)$	$\hat{V}_7(I)$
JUL86	0.47	0.46	0.46	0.72	0.71	0.72	0.55	0.52	0.52
AUG86	0.96	0.91	0.91	0.87	0.87	0.87	0.61	0.58	0.58
SEP86	0.96	0.92	0.93	0.80	0.80	0.80	0.60	0.57	0.58
OCT86	0.97	0.95	0.95	0.83	0.82	0.82	0.64	0.60	0.61
NOV86	0.96	0.91	0.92	0.84	0.83	0.84	0.80	0.76	0.77
DEC86	0.93	0.88	0.90	0.82	0.82	0.82	0.84	0.81	0.82
JAN87	0.94	0.90	0.90	0.95	0.93	0.94	0.82	0.78	0.80
FEB87	0.94	0.91	0.92	0.94	0.91	0.91	0.81	0.78	0.80
MAR87	0.96	0.93	0.94	0.95	0.92	0.93	0.82	0.80	0.81
APR87	0.95	0.91	0.93	0.95	0.92	0.93	0.86	0.83	0.85
MAY87	0.92	0.87	0.89	0.96	0.93	0.94	0.80	0.76	0.78
JUN87	0.89	0.84	0.86	0.96	0.93	0.94	0.78	0.74	0.76
JUL87	0.91	0.86	0.88	0.95	0.92	0.92	0.88	0.83	0.85
AUG87	0.89	0.84	0.86	0.96	0.93	0.93	0.88	0.83	0.85
SEP87	0.95	0.90	0.92	0.95	0.93	0.93	0.90	0.86	0.87
OCT87	0.94	0.90	0.91	0.96	0.93	0.94	0.92	0.88	0.90
NOV87	0.97	0.92	0.93	0.95	0.93	0.94	0.90	0.85	0.88
DEC87	0.93	0.87	0.89	0.95	0.93	0.93	0.92	0.87	0.89
JAN88	0.93	0.90	0.91	0.92	0.91	0.91	0.94	0.90	0.92