1. INTRODUCTION

The dual system estimator (DSE) is used in several contexts for estimating the size of a population. Its applications range from wildlife populations to human populations. DSEs of births and deaths are used at the U.S. Bureau of the Census in the formation of the demographic analysis estimates of the national population. Currently, the Census Bureau intends to use DSEs for measuring coverage error in the 1990 Decennial Census. This paper focuses on the application of the DSE in the census context where the two systems are the original enumeration and a Post Enumeration Survey (PES).

DSEs are subject to several components of nonsa mpling error, in addition to sampling error. We present models of the total error and the components of error in the DSE. The models relate observed indicators of data quality to the first two moments of the components of the error. We then use techniques of propagation of error to estimate the bias and variance of the DSE. In doing so, we assess the total error, i.e. the joint effect of the errors.

The methodology is applied to the 1986 Census of Central Los Angeles County, also known as the 1986 Test of Adjustment Related 0 perations (TAR0) conducted in Los Angeles (Diffendal, 1987). The PES in TARO comprised about 6,000 housing units and over 19,000 people. Estimates of the total error in the TAR 0 DSE are interesting both in and of themselves and for what they suggest for the likely error in the DSEs to be produced for 1990. A sensitivity analysis shows how the component errors interact, which ones cancel, and which ones compound each other. The methods described here to estimate the error in the TARO DSE can be extended to estimate the error in the 1990 DSEs.

Section 2 presents rationale for the TARO DSE and its major components. Our strategy for assessing the component errors and combining them to estimate the total error in the DSE is described next (Section 3). A detailed description of the DSE, with notation, is necessary for precise description of the component errors (Section 4). Following is an assessment of selected errors (Section 5). A synthesis of the component errors leads to estimates of the total error of the DSE (Section 6). Our major conclusions are then presented (Section 7).

## 2. DUAL SYSTEM ESTIMATUR

 requires assuming that there are two lists of the population. The first list is the original census enumeration, and the second is an implicit list of those covered by the sampling frame for the $P$-sample of the PES, whom we will call the P-sample population. The sampling frame itself is not a list of people, but of census blocks.The P-sample is one of the two samples that comprise the PES. The PES is composed of the E-sample, which is a sample of census enumerations, and the P-sample, which is a sample of the population. The E-sample is selected to estimate the number of enumerations that are erroneous. The $P$ sample is selected to estimate the number of people missed by the original enumeration through dual
system estimation.
The dual system estimator is based on the model (Wolter, 1986) that the joint event that the i-th individual in the population of the size $N$ is in the census or not and in the $P$-sample or not is modelled by the probabilities of falling in a cell shown in Table 2.1.

Table 2.1 Probabilities of Falling in a Cell

|  | original enumeration <br> in |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| P-sample | out | total |  |  |
|  | in | $p_{i 11}$ | $p_{i 12}$ | $p_{i 1+}$ |
|  | out | $p_{i 21}$ | $p_{i 22}$ | $p_{i 2+}$ |
|  | total | $p_{i+1}$ | $p_{i+2}$ | $p_{i++}$ |

The true population size in each category are defined in Table 2.2.

Table 2.2 True Population Size in Each Category

|  | original <br> in |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| P-sample | out | total |  |  |
|  | in | $N_{11}$ | $N_{12}$ | $N_{1+}$ |
|  | out | $N_{21}$ | $\left(N_{22}\right)$ | $\left(N_{2+}\right)$ |
|  | total | $N_{+1}$ | $\left(N_{1+}\right)$ | $\left(N_{++}\right)$ |

In the table above, $N_{++}=N$, the total population size. Even if we could observe the $N_{i j}$ 's in the first row and first column, the $\mathrm{N}_{\mathrm{ij}}{ }^{3} \mathrm{~s}$ in parentheses would not be observed directly, but would have to be estimated from the model. The DSE of $N$ then would have the form $N_{1+}{ }^{\star} N_{+1} / N_{11}$, which we will refer to as the ideal DSE.

In estimating population size for measuring census coverage error, the observed N's are replaced by estimates from the original enumeration and two sample surveys, the P-sample and the E-sample. The survey data are weighted by the reciprocal of the selection probabilities. The following definitions are required for the development of the observed DSE and its nonsa mpling error:

| $\hat{N}_{p}=$ | the estimate of the total population |
| ---: | :--- |
|  | from the P-sample. |
| $C E N=$ | the size of the original enumeration |
| $\mathrm{II}_{1}=$ | the number of persons imputed |
| $\mathrm{II}_{2}=$ | the weighted number of census |
|  | enumerations with insufficient |
|  | information for matching |
| $E E=$ | the weighted number of erroneous |
|  | enumerations in the original |
|  | enumeration, based on the E-sample |

```
\(\widehat{E E}=\) the estimate of the number of erroneous enumerations in the original enumeration
```

$\hat{C}=C E N-I_{1}-\mathrm{II}_{2}-\hat{E E}=$ the estimate of the number of distinct people in the original enumeration from the E-sample,
$\hat{M}=$ the estimate of the number of people in the census and the P-sample.
With this notation, $\hat{N}_{p}$ estimates $N_{1+}$ and $\hat{C} / \hat{M}$ estimates $N_{+1} / N_{11}$. Thus, the estimator has the form

$$
\hat{\mathbb{N}}_{++}=\hat{\mathbb{N}}_{p} * \hat{C} / \hat{M} .
$$

The ratio $\hat{C} / \hat{M}$ contains a correction for erroneous enumerations and for cases with insufficient information for matching, $\mathrm{II}_{1}$ and $\mathrm{II}_{2}$, so that cases with no chance of being included in the denominator are also exluded from the numerator.

The DSE is used to estimate the percent net undercount, or the undercount rate, in the original enumeration,

$$
\hat{U}=100^{*}\left(\text { CEN }-\hat{N}_{++}\right) / \hat{N}_{++} .
$$

For the TARO site (i.e. Central Los Angeles County) as a whole, CEN $=355,352, \hat{N}_{\mathrm{p}}=336,707, \hat{\mathrm{C}}=$ $343,567, \hat{M}=298,204$, and $\hat{N}_{++}=388,040$. The estimate of the percent net undercount is 8.42 .
3. STRATEGY FOR ASSESSING TOTAL ERROR

The DSE is subject to various sources of error, including error due to incorrect addresses from the P-sample, error due to missing data (unit and item nonresponse), response errors, intervieiwer errors, correlation bias, sampling error, etc. We wish to estimate the effects of these diverse sources of error on the DSE.

The first step in our strategy is to express the DSE as a function of components. We have constructed the components so that, for the most part, the different sources of error act either independently or perfectly dependently on different components. By isolating the effects of the various errors, we are better able to identify the major distinct sources of error.

Next, we estimate the first two moments of the component errors, one component at a time. In doing so we draw upon the results of the TAR0 evaluations and quality control programs. The way we constructed the components implies that correlation between component errors typically equal either 0 or 1 .

To study the propagation of errors we have used computer simulation methods. A multivariate distribution of the error components, say $F$, was assumed. The specification of F was consistent with the first two moments as estimated in Section 5. Realizations of the component errors were simulated by pseudo-random draws from $F$ and then the DSE was calculated; this procedure was repeated 10,000 times and the resulting empirical distribution of the DSE was used as an estimate of its actual distribution. The first two moments of the latter distribution provide nu merical estimates of the total error of the DSE.

Sensitivity analysis was performed to discover the importance of using one distributional form for $F$
rather than another. The results suggest that the exact distributional form (beyond the first two moments) is relatively unimportant (see Section 6).

We adopted a Bayesian approach in investigating of the error in the DSE. We estimated the first two moments of the distributions for the error components, then we derived the posterior distribution of the undercount rate conditional on the observed values of $\hat{C}, \hat{N}_{p}, \hat{M}$, etc.

## 4. COMPONENTS OF THE DSE

The DSE is subject to sampling errors and nonsampling errors, including failure of assumptions underlying the DSE model. The DSE does have a bias, but the bias in the census context is negligible (Wolter, 1986). Nonsa mpling errors may affect the accuracy of estimation of $N_{+1}, N_{1+}$, and $N_{11}$. Descriptions of the nonsampling error foltow. The nonsampling errors are described in more detail in Mulry and Spencer (1988.)

The nonsampling error in the estimation of $\mathrm{N}_{+1}$, called $c$, arises during the processing of the E-sample when respondents are misclassified as to whether they are correctly or erroneously enumerated in the original enumeration. Therefore, $c$ has three components: $c_{e}$, which occurs during the data collection and processing, $c_{b}$, caused by a PES design that fails to balance estimates of the gross overcount and gross undercount, and $c_{i}$ caused by missing data,

$$
c=c_{e}+c_{b}+c_{i} .
$$

The nonsampling error in the estimation of $\mathrm{N}_{1+}$, called $n_{p}$, arises during the interviewing for the $P$ sample when the P-sample selections are not interviewed. This situation occurs when household members are fabricated or when there is missing data. Therefore, $n_{p}$ has two components: $n_{p f}$, the error due to fabrication and $n_{p i}$, the error due to missing data,

$$
n_{p}=n_{p f}+n_{p i}
$$

The nonsampling error in the estimation of $\mathrm{N}_{11}$, called $m$, has four components: $m_{m}$, which is the error introduced in the matching operation; $m_{a}$, which is the error introduced by respondents giving the wrong Census Day address; $m_{i}$, missing data, and $m_{f}$, fabrication.

The ideal DSE can be written as follows (Mulry and Spencer, 1988):
$N_{1+} * N_{+1} / N_{11}=(\hat{C}-c) *\left(\hat{N}_{p}-n_{p}\right) /(\hat{M}-m)$.
5. COMPONENTS OF PES ERROR

Estimates of the first two moments of the posterior distribution of the undercount rate derive from estimates of the first two moments of the components of PES error.

The components are
correlation bias,
matching error,
accuracy of the reported Census Day address,
fabrication in the P-sample,
measurement of erroneous enumerations,
balancing the estimates of the gross overcount and the gross undercount, missing data,
sampling error.
We next describe the source of three components of PES error, correlation bias, accuracy of reported Census bay address, and missing data. We model the component errors in terms of observable indicators of data quality. We estimate the first two moments of the distributions of the errors for use in the total error model in Section 6. In Mulry and Spencer (1988), we give the models for each of the component errors.

### 5.1 CORRELATIUN BIAS

### 5.1.1. SOURCE OF ERROR

An important concern for dual system estimation is that the estimate of the proportion of the population enumerated in the census, based on the $p$-sample, is accurate. The violation of one of the independence assumptions underlying dual system estimation may cause the estimate of the proportion of the population in the census, and thereby the estimate of the population, to be biased.

Three independence assumptions are made for dual system estimator:

Causality. The event of being included in the census is independent of the event of being included in the PES. That is, the cross-product ratio satisfies

$$
\theta=p_{i 11} p_{i 22} / p_{i 12} p_{i 21}=1, \text { for } i=1, \ldots, N .
$$

Homogeneity. The capture probabilities satisfy $p_{i 1+}=p_{1+\text { or } p_{i+1}}=p_{+1}$ for $i=1, \ldots, N$, within each of the post-strata.

Autonomy. The census and the PES are created as a result of N mutually independent trials.

Next, we model the combined effect of the sources of correlation bias on the DSE.

### 5.1.2. DEFINITION

For insight into the effect of correlation bias, write the true population size as follows:

$$
N=N_{11}+N_{12}+N_{21}+\theta *\left(N_{12} * N_{21} / N_{11}\right)
$$

where $\theta$ is the cross-product ratio defined in Section 5.1.1.

The correlation bias affects only the last term because the other three may be estimated directly. The parameter, $\theta$, represents the effect of the failure of the independence assumptions. When the independence assumptions hold, $\theta=1$.

The correlation bias, arising when $\theta$ does not equal 1 , is the only contributor to $t$, the error due to failure of the model. The population size can be
written as follows:

$$
\begin{aligned}
N & =N_{1+}{ }^{*} N_{+1} / N_{11}+t \\
& =N_{1+} * N_{+1} / N_{11}+(\theta-1) *\left(N_{12} * N_{12} / N_{11}\right) .
\end{aligned}
$$

Therefore, the correlation bias, $t$, satisfies

$$
t=(\theta-1) *\left(N_{12} * N_{21} / N_{11}\right)
$$

### 5.1.3 MEASUREMENT

The parameter $\theta$ may be estimated at the national level for subgroups categorized by age, race, and sex using demographic analysis estimates of the population size. Note, however, that this technique presumes that the demographic analysis estimates are accurate. Even so, this formulation also permits varying $\theta$ to assess the sensitivity of the DSE to the estimate of the effect of the violation of the independence assumptions.

### 5.1.4 ESTIMATION

Estimates for $\theta$ were not made for the 1986 TAR 0 because an alternate source for population estimates did not exist, e.g., no demographic analysis estimates were feasible. However, Ericksen and Kadane (1985) made estimates of $\theta$ for blacks for the 1980 census. They made three estimates of $\theta: 2.1,2.7$, and 3.7. Since the population in the 1986 TARO was predominantly minority (73 percent Hispanic, 12 percent Asian, and 15 percent non-Asian and nonHispanic), the Ericksen and Kadane estimates for 1980 will be used in this paper:

$$
\begin{aligned}
E(\theta) & =2.1,2.7, \text { or } 3.7 . \\
\operatorname{Var}(\theta) & =0 .
\end{aligned}
$$

These estimates of $\theta$ are consistent with the reports of the participant observers in the Los Angeles test site (Childers, et al, 1987).

### 5.2 QUALITY OF THE REPORTED CENSUS DAY ADDRESS

### 5.2.1 SOURCE OF ERROR

Some of the respondents in the P-sample have moved between Census Day and their PES interview. The respondents may misreport whether they have moved. If they have moved, they may not report their previous address accurately, or their previous address may not be geocoded correctly by the staff. Any of these types of errors may cause the matching operation to search the census in an area other than where the respondent was enumerated. These errors may lead to assigning a nonmatch status to respondents who actually were enumerated because the matching operation is unable to locate their enumerations. Inappropriate assignment of the status of nonmatch will cause the estimate of the number of people missed by the census to be biased upward.

Circumstances under which inaccurate reporting of the Census Day address by a PES respondent will not cause a false nonmatch do exist. If the Census Day address is inside the search area for the reported address, and the reported address is geocoded correctly, then the matching operation will find the person.

### 5.2.3. MEASUREMENT

The conditional expected value and variance of $\mathrm{m}_{\mathrm{a}}$
given the observed value $\hat{M}$ are denoted by $E\left(m_{a}\right)$ and $\operatorname{Var}\left(m_{\mathrm{a}}\right)$.

Measurement of $m_{a}$ is based on a follow-up of a sample of P-sample respondents whose enumeration status is "not enumerated." Data from the follow-up are used to estimate the error that arises when people who were enumerated mis-report their Census Day address when they respond to the PES.

An evaluation of the quality of the reporting of the Census Day address was conducted after the 1986 TAR0. A post-production follow-up which reinterviewed a sample of the nonmatches to determine the number of nonmatches caused by misreporting mover status. Another search to match respondents who reported they in fact had moved within the test site was made at the new address.

### 5.2.4. ESTIMATION

The sample cases found to have errors in their reported Census Day address may be used to estimate
$L_{e}=$ the weighted number of people who erroneously report their Census Day address in their Psample interview.

A search of census enumerations at the newly reported addresses produces
$r_{a m}=$ the estimator of the percentage of people with errors in the location of their reported Census Day address who match census enumerations.

Then the expected value of the error $m_{a}$ is estimated by

$$
E\left(m_{a}\right)=-r_{a m} * L_{e} .
$$

The results of the post-production follow-up (Childers, et at, 1987) yielded a misreporting rate of 2.7 percent in the P-sample. A match rate of 32 percent was estimated for those who misreported their Census Day address and moved within the test site. The expected value of $\mathrm{m}_{\mathrm{a}}$ is

$$
E\left(m_{a}\right)=-0.027 * 0.32 * 19,552 * 17=-2871
$$

An estimate of the variance of the error due to mis-reporting has not been made. Our professional judgment is that a conservative estimate of the variance at the PES sample level is 900 . Therefore, the variance at the TAR 0 site level is

$$
\operatorname{Var}\left(\mathrm{m}_{\mathrm{a}}\right)=(17)^{2} * 900=260,100
$$

If no attempt had been made to match the sample cases to census enumerations, or if the variance of such an estimate would be unacceptably large, then $r_{a m}$ can be replaced by the estimator of the final overall match rate for P -sample movers. Then the underlying assumption would be that the movers who report accurate addresses are like movers who give inaccurate addresses.

### 5.3. MISSING DATA

### 5.3.1. SOURCE OF ERROR

Both the E-sample and the $P$-sample have missing data. The E-sample has cases where the information
required to determine whether the person is correctly or erroneously enumerated in the census is not available. The P-sample has cases where the information needed to determine whether the person is enumerated in the census is not available. The enumeration status is imputed statistically to compensate for the inablility to resolve the case.

Missing data occur in more than one way. The interviewer may be unable to obtain an interview during the $P$-sample interviewing or during the PES follow-up. A P-sample or E-sample questionnaire may not have all the demographic and housing information required for the estimation. Even with all the information requested on the questionnaires, the circumstances may be so unclear that the enumeration status can not be resolved.

### 5.3.2. MEASUREMENT

We assess the error in the DSE caused by missing data instead of considering each component $c_{i}, m_{i}$ and $n_{p i}$ separately. Our approach is to perform a sensitivity analysis of reasonable alternative models for compensating for missing data. First a preferred method of imputation for unresolved P-sample and E-sample enumeration statuses is specified prior to the implementation of the PES. Reasonable alternative treatments of the missing data can be suggested by problems that arise during the collection and processing of the PES data. The DSE can be computed under these alternative models for compensating for missing data. The range of the alternative estimates indicates the sensitivity of the DSE to the method of imputation. For example, a narrow range implies that the estimates are robust, and the missing data cause little uncertainty in the estimates.

### 5.3.3. ESTIMATION

The effect of missing data on the estimates from the 1986 TAR 0 was assessed by exa mining the range of estimates obtained when methods of imputation based on reasonable alternative assumptions were used in place of the preferred method. These included alternative treatment of proxy responses, movers, and designation of ficticious enumerations (Schenker, 1987). The alternative treatment of the proxy interviews for P-sample cases classified them as noninterviews and applied the weighting adjustment. This essentially assigned proxy cases the same match rate as nonproxy cases. The alternative treat ment of the $P$-sample movers reclassified them all as unresolved and imputed a match status, instead of imputing for only those who were not resolved. This essentially assigned movers the same match rate as nonmovers. The alternative treatment of fictitious cases resulted from a review of the unresolved E-sample cases by experienced matching personnel who converted some unresolved cases to fictitious. This raised both the observed and imputed rates of erroneous enumeration.

Models 10 and 11 shown in Table 4 of Schenker's paper give the upper and lower bounds of the estimates of undercount rates, respectively. Model 10 has the TARO treatments while Model 11 has all the alternative treatments. Both models differ from TARO in that they have E-sample outmovers as substitutes for $P$-sample inmovers and imputations for their match statuses. In the 1986 TAR 0 the E-sample outmovers and the P-sample inmovers were omitted
from the PES estimation. The omission of the outmovers from estimation essentially assumes that they had the same capture rate in the original enumeration as the included cases. Movers are believed to have a lower capture rate than nonmovers.
6. SYNTHESIS OF TOTAL ERROR

The combined effect of the component errors will be summarized by posterior distibutions for the net undercount rate. The bias in the estimate of net undercount rate, $B(\hat{U})$, is estimated by the difference between $\hat{U}$ and the mean of the posterior distribution. To construct the posterior distribution, we used a simulation method with 10,000 repetitions, generating pseudo-random component errors and adding them to the TARO estimates. Using the formulas in Section 5.1.2, we obtain the following formula:
$N=\left(\hat{N}_{p}-n_{p}\right)+(\hat{C}-c-(\hat{M}-m))$
$+\theta *(\hat{C}-c-(\hat{M}-m)) *\left(\hat{N}_{p}-n_{p}-(\hat{M}-m)\right) /(\hat{M}-m)$
$=(\hat{C}-c) *\left(\hat{N}_{p}-n_{p}\right) /(\hat{M}-m)$
$+(\theta-1) *(\hat{C}-c-(\hat{M}-m)) *\left(\hat{N}_{p}-n_{p}-(\hat{M}-m)\right) /(\hat{M}-m)$
Several different distributions were used to reflect alternative estimates of imputation error, alternative estimates of correlation bias (parameterized by $\theta$ ), and alternative marginal distributional forms for the components -- normal, gam ma, and uniform.

In this study, the percent net undercount estimate for the TARO site is 8.42 with a sampling standard deviation of 0.7. This estimate was selected because estimates of nonsampling error components are available only for the site as a whole. When a DSE is constructed for each post-stratum and then the DSEs are summed to give an estimate for the site, the percent net undercount estimate is 9.02 percent.

Table 6.1 displays the means and standards deviations of the error components for the PES sample. Recall that $\hat{N}_{++}=$
388, 040, $\hat{M}=298,204, \hat{C}=343,567$,
and $\hat{N}_{p}=336,707$ for the TARO site.
The overall sampling weight, 17 , was used consistently throughout all the simulations so that comparisons of the effect of alternative assumptions such as correlation bias parameter values, error distributions, and imputation models are appropriate. The methodology generalizes to other applications where a different sampling weight is used in each stratum.

Table 6.1 Assumed distributions of error estimates

|  | Mean | $\frac{\text { Standard }}{\text { Deviation }}$ |
| :---: | :---: | :---: |
| Net Matching | -782 | 115 |
| Census Address | -2871 | 510 |
| Fabrication | -1751 | 172 |
| Net E-sample | -1390 | 153 |

Table 6.2 displays the effects of the individual errors on the posterior distribution of the undercount when the TARO imputation is used. The net matching,

Census Day address, and fabrication errors are all errors in $\hat{M}$. Therefore, the presence of only one of them alone causes the bias in the estimate of percent net undercount to be positive. The net E-sample error
is an error in $\hat{C}$. The presence of E -sample error alone causes the bias in the estimate of percent net undercount to be negative. The estimate for correlation bias, $\theta$, was chosen to be 2.7 , the median of Ericksen and Kadane's estimates. The presence of only correlation bias causes the bias in the percent net undercount estimate to be negative.

Table 6.2 Individual effects of errors on posterior distribution of net undercount rate

|  | E(U) | Std. Dev. |  | $B(\hat{U})$ |
| :--- | ---: | :---: | :---: | :---: |
|  | 8.11 | 0.03 |  | 0.31 |
| Net Matching | 7.53 | 0.16 |  | 0.89 |
| Census Address | 7.85 | 0.06 |  | 0.57 |
| Fabrication | 8.68 | 0.04 | -0.26 |  |
| Net E-sample | 10.61 | 0.00 | -2.19 |  |

Varying the value of the estimate of $\theta$ for the correlation bias did affect the moments of the posterior distribution of the undercount. The variation appears in the mean and in the standard deviation. Table 6.4 shows the results for the different values of $\theta$, where the distribution for the errors are normal. The case where $\theta=1$ portrays virtually no correlation bias, while for the other sources of error are present. In the cases where $\theta=2.1,2.7$, and 3.7, all the sources of error are taken into account. The distribution of the undercount shifts to the right as the estimate of $\theta$ for the correlation bias increases. The variance also increases as the estimate of $\theta$ increases. For $\theta=2.1$ the bias $B(\hat{U})$ is very close to zero, but positive. For $\theta=2.7$ and 3.7 , the bias is negative.

Table 6.4 Posterior distribution of the net undercount rate for several values of $\theta$.

| $\frac{\theta}{1}$ | $\frac{E(U)}{}$ |  | St. dev. |  |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 7.15 | 0.17 |  | 1.27 |
| 2.1 | 8.35 | 0.22 |  | 0.07 |
| 2.7 | 8.95 | 0.23 |  | -0.53 |
| 3.7 | 10.12 | 0.26 | -1.70 |  |

The simulations were conducted with reasonable alternate models for the imputation for unresolved match status. Although there was some variation in the first two moments of the distribution of the net undercount rate, the estimate of net undercount rate in TARO appears robust to missing data. Table 6.5 illustrates the results of the simulations using models 10 and 11 described in Section 5.7.3. Models 10 and 11 yielded the upper and lower bounds of the undercount estimates under all the reasonable alternative imputation models. The bias in the estimate of the
percent net undercount rate ranges from -0.86 to 1.04. In other words, the bias is between a negative 9 percent and a positive 13 percent of the net undercount rate estimate of 8.42. Varying the imputation model has almost no effect on the standard deviation.

Table 6.5 Posterior distribution of the net undercount rate under reasonable alternative imputation models when $\theta=2.7$

|  | $\frac{E(U)}{}$ |  | St. dev. |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  | $B(\hat{U})$ |  |  |
| TA R 0 | 8.95 | 0.23 |  | -0.53 |
| Model 10 | 9.19 | 0.22 |  | -0.77 |
| Model 11 | 7.31 | 0.22 |  | 1.11 |

The total variance of the estimated net undercount rate may be estimated by the sum of the sampling variance and the nonsampling variance. For the case where $\theta=2.7$, the standard deviation show $n$ in Table 6.5 for both models 10 and 11 is 0.22 which translate to a nonsampling variance of 0.05 when all errors are considered.

The standard randomization theory model for survey sampling is appropriate for estimating the vamiance of the DSE. The coefficient of variation which is the ratio of the square root of the variance of the observed DSE to the mean of the distribution of the DSE provides information on the amount of sampling error in the DSE.

The Taylor series estimator of variance (Moriarty, 1987) yields as standard deviation of 3100.37 for the dual system estimate of 388,040 for the TAR 0 site. The coefficient of variation is 0.008 . This implies the standard deviation for the estimated net undercount rate is 0.70 which translates to a sampling variance of 0.49 .

Therefore, the total variance is 0.54 and standard error is 0.73 . The coefficient of variation of the net undercount rate is 0.083 . The nonsampling variance contributes very little to the total variance relative to the contribution by the sampling variance.

## 7. CONCLUSIONS

When all the sources of error are considered in the posterior distribution of the undercount and the estimate of $\theta$ for correlation bias is 2.7, the estimated net undercount rate for $\operatorname{TAR} 0, \hat{U}=8.42$, has a small bias:

$$
-0.77<B(\hat{U})<1.11
$$

The standard deviation of the posterior distribution for the net undercount rate is 0.73 .

When the post-stratification is used in the estimation, the undercount estimate for TARO is 9.02. The post-stratification increased the net undercount rate estimate by 0.6 , which is less than one standard deviation from the estimate of 8.42.

Although we expect the error in the post-stratified estimate is smaller, the result is consistent with the error analysis.

The DSE appears to be robust to the joint effect of errors arising in the data collection of the 1986 TAR0.

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