

THE ECONOMICS OF A SCREENING INSTRUMENT
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Introduction

At times researchers involved in survey studies must decide between several sampling schemes. These decisions usually involve economic considerations. The classical economic problem states a decision between alternate options when there are limited resources. Survey researchers, such as epidemiologists, are usually interested in the identification of groups of people with a specific characteristic. Once the group is identified factors associated to the characteristic of interest are analyzed. In this type of research an adequate sample size is needed to have acceptable statistical power to detect significant differences. Limited resources require the minimization of costs given the pre-selected sample size. For example, epidemiologic research usually has two main objectives: 1) to establish the prevalence of a diagnosis in a population 2) to analyze the risk factors associated with the target diagnosis. To accomplish these objectives, samples are selected so that they refer to a specific population and at the same time contain a high percentage of respondents who are ultimately classified as positive cases. Screeners

are useful instruments in accomplishing these dual goals.

The estimation of the prevalence rate requires a known probability sample. The analyses of the risk factors requires a sample of positive cases with adequate statistical power. Characteristics which are prevalent in a high percentage of the population pose no major problem because standard sampling procedures such as random sampling can produce a large sample of both positives and negatives.

Surveys can be conducted in one or more phases. Surveys conducted in two phases frequently involve a double sampling scheme. First a screening instrument is given to subjects in larger sample, and based on the results of the screener specific subjects are selected to the second sample. This second subset will have an over-sample of subjects with the characteristic of interest. Several methods for selecting the adequate sampling procedures have been previously discussed (Shrout, 1988; Cochran 1977).

Double sampling can serve the dual purpose of epidemiologic research. The first sample provides a framework form which to weight back the results to a

target population, while the second sample can be designed to meet the statistical power requirements at a minimum cost.

In this paper we discuss the optimal relations that must prevail between the sensitivity and specificity of an instrument and the marginal costs associated to its use in a field study, in order to minimize costs given a pre-determined sample. To address this issue in an economic context, a set of choices must exist. Choices can be expressed as a decision between two different screeners or as a decision between using a screener in a double sampling scheme or no screener in one step sampling.

We base our discussion in the decision between a double sampling scheme selection or not. This approach can also be used to analyze the other option. The decision between two screeners involves the comparison between the costs and screening capacities of the two screeners. While the decision between screener or no screener involves the comparison between the cost and screening capacities of the screener and the costs of the first phase sample, whose screening capacity is equal to the prevalence of the characteristic of interest.

Method

Lets assume: 1) we require a sample size N_2 to be drawn from a population

size N_1 . 2) the estimated prevalence of this characteristic is π . That is, there are $\pi*N$ positive cases in the population 3) only cases classified as positive in the screener will be selected. A screener with perfect sensitivity would be able to select all $\pi*N$ cases. If the sampling rate of positive cases was 1; then $n^+ = \pi*N$, or in the more general case (a sampling rate of $0 \leq k \leq 1$ then $n^+ = k*\pi*N$. In a screener with less than perfect sensitivity ($0 < s < 1$); $n^+_2 = k*\pi*s*N_1$. On the other hand, if only those screened positive are selected the expected number of true negatives would be $n^-_2 = k(1-\pi) (1-sp)N_1$. Where sp is the specificity of the screener, ie. the probability that a true negative is selected among those screened positive ($0 \leq sp \leq 1$).

objective function

$$\text{MIN: Total Costs} = c_1 N_1 + c_2 N_2$$

$$\text{st. } n^+_2 = a$$

$$N_2 = n^+_2 = n^-_2$$

$$n^+_2 = k\pi s N_1$$

$$n^-_2 = k(1-\pi) (1-sp)N_1$$

$$N_2 = [\pi s + (1-\pi) (1-sp)] k N_1 \quad (1)$$

where: c_1 and c_2 are the costs associated with each interviewing procedure N_1 and N_2 are the total size of each sample n^+_2 is

the expected number of positive cases in the second sample n_2^- is the expected number of negative cases in the second sample, then we re-state the problem as follows:

$$\text{MIN. } c_1 N_1 + c_2 [\pi s + (1-\pi)(1-sp)] k N_1$$

$$\text{st. } k \pi s N_1 = a$$

the Langrangean equation to be optimized can be written as follows:

$$L = c_1 N_1 + c_2 [\pi s + (1-\pi)(1-sp)] k N_1 + \tau (a - k \pi s N_1)$$

the optimality conditions can be expressed as:

$$dL/dk = c_2 [\pi s + (1-\pi)(1-sp)] N_1$$

$$- \tau \pi s N_1 = 0 \quad (2)$$

$$dL/d\tau = a - k \pi s N_1 = 0 \quad (3)$$

then solving (2) and (3)

$$k = a(\tau - c_2) / [(1-\pi)(1-sp)c_2 N_1] \quad (4)$$

but the sampling rate of positives (k) can also be re-written as the product of two ratios, the ratio between costs $(\tau - c_2)/c_2$, and the ratios between the required sample of positives and the expected number of true negatives in the second sample based on the predictive capacity of the screener $(a / [(1-\pi)(1-sp)N_1])$

$$k = (\tau - c_2) / c_2 * (a / [(1-\pi)(1-sp)N_1])$$

τ is the marginal total cost of increasing the required number of true positive cases by one (i.e. $dC/da = \tau$). Then $(\tau - c_2)/c_2$ is the relative difference between the costs associated to the second sample (c_2) and the total marginal cost (τ).

$$\lim_{c_2 \rightarrow \tau} k = 0$$

$$c_2 \rightarrow \tau$$

If both interviewing procedures are as costly, then the sampling rate for the second stage must be zero. That is to say a double sampling strategy would not minimize costs.

$$\lim_{\tau \rightarrow 2c_2} k = (a / [(1-\pi)(1-sp)N_1])$$

$$\tau \rightarrow 2c_2$$

If the total marginal cost (τ) equals twice the cost of the second interview procedure (c_2) then the decision on the sampling rate should be based on its screening capacities and not on relative marginal costs.

The screening capacity (for positives) of an instrument can be expressed as follows: $w = \pi s / [\pi s + (1-\pi)(1-sp)]$

substituting from (1) we obtain that the optimal screening capacity of an instrument to minimize costs should be equal to the ratio of the total marginal cost of both procedures (τ) and the second procedure marginal cost (c_2): $w = c_2 / \tau$. The more expensive the second procedure the higher the required screening capacity to justify a double sampling strategy.

Solving (1) and (4) we obtain that the optimal size of the second sample is:

$$N_2 = a(\tau - c_2) / c_2 * [\pi s / [(1-\pi)(1-sp)] + 1]$$

The size of the sub-sample (N_2) will depend on the ratio of the variable costs of the required positives in the first sample $(a(\tau - c_2))$ to the cost in the

second sample (ie. $a(\tau - c_2)/c_2$) plus the costs associated to the relative efficiency of the screener $a(\tau - c_2)/c_2 * [\pi S / (1 - \pi)(1 - sp)]$. The larger the relative efficiency of the screener $(\pi S / (1 - \pi)(1 - sp))$ the larger N_2 . The larger the cost of the second sample in relation to the first, the smaller the second sample.

total marginal costs

Total marginal costs τ at the point where total cost are minimum must be equal to the ratio of the cost of administering the second phase procedure over the predictive capacity of the screening instrument $\tau = c_2/w$. The higher the marginal costs of the second procedure the higher the marginal costs of increasing the number of required positive cases.

Discussion

In this paper we have analyzed a set of issues associated to a survey design. We have concluded that the decision of selecting an adequate sampling strategy must be based not only on the screening capacities of the selected instruments but on the costs associated to its administration as well. The fact that the

association between costs and screening capacities is not linear suggests that to decide the adequate sampling strategy one must analyze cost and efficiency issues simultaneously. An equation has been derived that can help us in evaluating the adequate strategy to follow. For example, the decision to use a screener that would select as many positives as negatives depends solely on costs. The predictive capacity of an instrument, to minimize costs, should be equal to the ratio of the marginal costs (total marginal cost τ , and marginal cost of the second interview c_2).

These associations between costs and screening capacity can help survey researchers in deciding which is the adequate sampling strategy. They can also serve as guidelines to those in grant giving foundations where a survey research is being funded. Pilot studies designed to analyze the predicting capacities of an instrument should also examine the costs associated to its administration.