FINITE POPULATION CORRECTION FOR REPLICATION ESTIMATES OF VARIANCE

Paul A. Flyer, Westat, Inc.
1650 Research Blvd., Rockville, MD 20850

Introduction

In multi-stage samples the use of balanced repeated replication (BRR) for variance estimation has become common. The use of this technique carries the implicit assumption that either sampling is performed with replacement at the first stage, the sampling rate at the first stage is minimal or that the "between" first stage unit variance is not a large proportion of total variance. Essentially, BRR ignores the finite population correction for the first stage units and overestimates variance (usually by a slight amount). In McCarthy's (1969) description of BRR, a procedure was introduced for incorporating a finite population correction in single stage, equal probability sampling. Essentially, the approach suggested by McCarthy consists of modifying the weights used for combining the separate stratum estimates. For single stage, unequal probability sampling, Wolter (1985) suggests modifying the replicate estimate for each stratum to reproduce the Yates-Grundy estimate of variance. In both of these approaches the weighting used for the replicates departs from that used for the entire sample estimate. In practice, these adjustments are not often made and it is usually assumed that the resulting bias is negligible.

In the Institutional Population Component (IPC) of the National Medical Expenditure Survey (NMES) a two stage sample design was used with a high sampling rate for some first stage units (sampling rate of 30-60%). The NMES was designed to provide an assessment of the health care utilization, costs, sources of payment of both the U.S. civilian non-institutional population, and the institutionalized population using nursing homes and facilities for the mentally retarded (IPC). The survey was co-sponsored by the National Center for Health Services Research and Health Care Technology Assessment (NCHSR) and the Health Care Finance Administration (HCFA). In the IPC, it was expected that the between first stage unit contribution to variance is a significant component of total variance. The substantial departures from the usual assumptions were likely to lead to a sizeable overstatement of variance. A technique was developed to avoid this situation. The following contains a description of a procedure to incorporate finite population corrections for both the first stage and second stage units and produce unbiased estimates of variance for linear estimates. The proposed procedure is based upon modifying the formation of replicates and not the weights for the strata, which means that variance can be estimated using standard BRR computer programs.

Sample Design and Notation

The sample design considered is a stratified two-stage design in which two primary selections (PSU's) are made within each of L strata. The two primary selections are made without replacement, with equal probabilities of selection (i.e. using simple random sampling). Subsampling within the selected PSU's is also assumed to be made by simple random sampling. For each stratum, there are assumed to be M_h first stage units, each with M_{hi} second stage units. From each selected PSU, a sample of m_{hi} second stage units is selected. It is also assumed that sampling is conducted independently for the L strata.

The following definitions will be used:

$$Y_{hi} = \sum_{j=1}^{M_{hi}} y_{hij},\ \bar{Y}_{hi} = \frac{Y_{hi}}{M_{hi}} \text{ and } \hat{Y}_{hi} = M_{hi}\frac{\sum_{j=1}^{m_{hi}} y_{hij}}{m_{hi}} = M_{hi}\bar{y}_{hi},$$

where $i=1,\dots,M_h$, $h=1,\dots,L$ and $j=1,\dots,M_{hi}$ or m_{hi}. For this design the usual estimate of a population total

$$Y = \sum_{h=1}^{L}\sum_{i=1}^{M_h} Y_{hi} = \sum_{h=1}^{L} Y_h \text{ is } \hat{Y} = \sum_{h=1}^{L} M_h \frac{\sum_{i=1}^{2}\hat{Y}_{hi}}{2} = \sum_{h=1}^{L}\hat{Y}_h .$$

The variability among the PSU's for the h-th stratum is

$$S_{bh}^2 = \sum_{i=1}^{M_h} \frac{(Y_{hi} - \bar{Y}_h)^2}{M_h - 1}$$

which is estimated by

$$s_{bh}^2 = \sum_{i=1}^{2} (M_{hi}\bar{y}_{hi} - \frac{\hat{Y}_h}{M_h})^2,$$

where $\bar{Y}_h = \frac{Y_h}{M_h}$. Also, define for the i-th PSU in the h-th stratum the following:

$$\sigma_{\hat{Y}_{hi}}^2 = V(\hat{Y}_{hi}) = M_{hi}^2 (1 - f_{hi}) \frac{S_{hi}^2}{m_{hi}}$$

$$\text{where } S_{hi}^2 = \frac{\sum_{j=1}^{M_{hi}} (y_{hij} - \bar{Y}_{hi})^2}{M_{hi} - 1} \text{ and } f_{hi} = \frac{m_{hi}}{M_{hi}}.$$

Unbiased estimates for these quantities are

$$\hat{\sigma}_{\hat{Y}_{hi}}^2 = M_{hi}^2 (1 - f_{hi}) \frac{s_{hi}^2}{m_{hi}} \text{ and } s_{hi}^2 = \frac{\sum_{j=1}^{m_{hi}} (y_{hij} - \bar{y}_{hi})^2}{m_{hi} - 1}.$$

Usual Estimate of Variance and BRR Estimate

The variance of the estimated total, $\hat{Y}$, is

$$V(\hat{Y}) = \sum_{h=1}^{L} M_h^2 (1-f_h) \frac{S_{bh}^2}{2} + \sum_{h=1}^{L} \frac{M_h}{2} \sum_{i=1}^{2} \sigma_{\hat{Y}_{hi}}^2$$

where $f_h = \frac{2}{M_h}$. The usual estimate of this variance (given the sample design) is:

$$\hat{V}(\hat{Y}) = \sum_{h=1}^{L} M_h^2 (1-f_h) \frac{s_{bh}^2}{2} + \sum_{h=1}^{L} \frac{M_h}{2} \sum_{i=1}^{2} \hat{\sigma}_{\hat{Y}_{hi}}^2 .$$

The BRR approach to estimating $V(\hat{Y})$ is to create G balanced half-samples or replicates. These half-samples are formed by using for estimation only one of the two sampled PSU's in each stratum. The half-sample estimate for the h-th stratum is $\hat{Y}_{hr} = M_h\hat{Y}_{hi}$ or $M_h\hat{Y}_{hj}$. The quantity Y is then estimated for each replicate by summing these half-sample estimates over the strata. The replicate estimate of variance for the r-th replicate is calculated using $\hat{V}_r(\hat{Y}) = (\hat{Y}_r - \hat{Y})^2$. The overall estimate of variance is

$$\hat{V}(\hat{Y}) = \frac{\sum_{r=1}^{G} \hat{V}_r(\hat{Y})}{G} .$$

Expectation of the BRR Variance Estimate

The expected value of the r-th replicate estimate of variance can be broken down into an expected value within each of the L strata:

$$E[\hat{V}_r(\hat{Y})] = E(\hat{Y}_r - \hat{Y})^2 = E[\sum_{h=1}^{L} (\hat{Y}_{hr} - \hat{Y}_h)]^2$$

$$= \sum_{h=1}^{L} E(\hat{Y}_{hr} - \hat{Y}_h)^2 + \sum_{h=1}^{L} \sum_{h' \neq h}^{L} E[(\hat{Y}_{hr} - \hat{Y}_h)(\hat{Y}_{h'r} - \hat{Y}_{h'})]$$

$$= \sum_{h=1}^{L} E(\hat{Y}_{hr} - \hat{Y}_h)^2 \quad \text{because} \quad E[(\hat{Y}_{hr} - \hat{Y}_h)(\hat{Y}_{h'r} - \hat{Y}_{h'})] = 0$$

$$= \sum_{h=1}^{L} E[\hat{V}_r(\hat{Y}_h)] \quad \text{due to the independent sampling in the strata.}$$

To show that $\hat{V}_r(\hat{Y})$ is a biased estimate of $V(\hat{Y})$ it will be sufficient to show that $\hat{V}_r(\hat{Y}_h)$ is a biased estimate of $V(\hat{Y}_h)$.

Because $\hat{Y}_{hr}$ is equal to either the i-th or j-th PSU the following is true:

$$\hat{V}_r(\hat{Y}_h) = (\hat{Y}_{hr} - \hat{Y}_h)^2 = M_h^2 (\frac{\hat{Y}_{hi}}{2} - \frac{\hat{Y}_{hj}}{2})^2 .$$

For fixed i and j the expected value of the squared difference is:

$$E_2(\frac{\hat{Y}_{hi}}{2} - \frac{\hat{Y}_{hj}}{2})^2 = (\frac{Y_{hi}}{2} - \frac{Y_{hj}}{2})^2 + (\frac{\sigma^2_{\hat{Y}_{hi}}}{4} + \frac{\sigma^2_{\hat{Y}_{hj}}}{4}),$$

where E_2 denotes the expectation conditional on the choice of first stage units. Taking the expectation over first stage units yields

$$E_1[E_2(\frac{\hat{Y}_{hi}}{2} - \frac{\hat{Y}_{hj}}{2})^2] = \frac{S^2_{bh}}{2} + \frac{1}{2}\frac{\sum_{i=1}^{M_h} \sigma^2_{\hat{Y}_{hi}}}{M_h}$$

This demonstrates that the expected value of the squared difference between the replicate estimate of the total and the estimate produced from the entire sample is not equal to the variance of the estimated total.

$$E[\hat{V}_r(\hat{Y}_h)] = M_h^2 \frac{S^2_{bh}}{2} + M_h \frac{\sum_{i=1}^{M_h} \sigma^2_{\hat{Y}_{hi}}}{2}$$

$$\neq V(\hat{Y}_h) = M_h^2 \frac{S^2_{bh}}{2} + M_h \frac{\sum_{i=1}^{M_h} \sigma^2_{\hat{Y}_{hi}}}{2} - M_h^2 f_h \frac{S^2_{bh}}{2}$$

The form and direction of the bias in the BRR estimate suggests that to make the BRR procedure unbiased it will be necessary to reduce the between PSU contribution to variance by an amount proportionate to the first stage sampling rate.

Modified BRR

To introduce the appropriate finite population correction and produce an unbiased estimate of variance the following scheme can be used in creating the replicates:

(1) With probability $1-f_h$, the h-th stratum's replicate will be made up of either the first PSU or the second PSU, i.e., use the usual BRR approach.

(2) With probability f_h, the h-th stratum's replicate will be made up of a $\frac{100}{2-f_{h1}}$% subsample from the m_{h1} second stage units available from the first PSU and a $\frac{100}{2-f_{h2}}$% sample from the second PSU, i.e, treat the PSUs as if they were selected with certainty, but take more than a 50 percent subsample from each PSU.

Durbin-Brewer-Hanif Estimate of Variance

The modified approach to BRR was motivated by a technique introduced by Durbin (1967), which allows unbiased estimates of variance to be created without requiring that the "within" variance be calculated for each stratum. Brewer and Hanif (1970) formulated Durbin's method in a convenient way for analysis and computation.

The Durbin–Brewer–Hanif procedure uses for each stratum:

$$\hat{V}_{DBH}(\hat{Y}_h) = \begin{cases} M_h^2 \frac{S^2_{bh}}{2} & \text{with probability } 1 - \frac{2}{M_h} \\ [\frac{M_h}{2}]^2 \sum_{i=1}^{2} \hat{\sigma}^2_{\hat{Y}_{hi}} & \text{with probability } \frac{2}{M_h} \end{cases} .$$

This can be rewritten as

$$\hat{V}_{DBH}(\hat{Y}_h) = \alpha_h M_h^2 \frac{S^2_{bh}}{2} + (1-\alpha)[\frac{M_h}{2}]^2 \sum_{i=1}^{2} \hat{\sigma}^2_{\hat{Y}_{hi}}$$

$$\text{where } \alpha_h = \begin{cases} 1 & \text{with probability } 1 - \frac{2}{M_h} \\ 0 & \text{with probability } \frac{2}{M_h} \end{cases} .$$

It is then easy to see that the expectation of this estimate over α is

$$E_\alpha[\hat{V}_{DBH}(\hat{Y}_h)] = (1-f_h) M_h^2 \frac{S^2_{bh}}{2} + \frac{M_h}{2} \sum_{i=1}^{2} \hat{\sigma}^2_{\hat{Y}_{hi}}$$

$$= \text{usual unbiased estimate.}$$

Expectation of the Modified Replication Variance

The following paragraphs will show that the modified approach to BRR produces unbiased estimates of variance for linear statistics. In the modified approach, with probability $1-\frac{2}{N_h}$, the r-th replicate in the h-th stratum will be made up of one of the two selected PSU's. Otherwise, with probability $\frac{2}{N_h}$, the r-th replicate in the h-th stratum will be made up of a $\frac{100}{2-f_{hi}}$% sample from PSU i and a $\frac{100}{2-f_{hj}}$% sample from PSU j. The r–th replicate estimate of variance can be written (considering only the hth stratum):

$$\hat{V}_r(\hat{Y}_h) = \alpha_h M_h^2\left(\frac{\hat{Y}_{hi}}{2} - \frac{\hat{Y}_{hj}}{2}\right)^2 + (1-\alpha_h)(\hat{Y}'_h - \hat{Y}_h)^2 ,$$

where $\alpha_h=1$ with probability $1-\frac{2}{N_h}$, $\alpha_h=0$ with probability $\frac{2}{N_h}$ and $\hat{Y}'_h$ is the estimate for the r-th replicate based upon subsamples of n_{hi} and n_{hj} from the i-th and j-th PSU, respectively. Taking the expectation over α yields:

$$E_\alpha[\hat{V}_r(\hat{Y}_h)] = (1-f_h) M_h^2\left(\frac{\hat{Y}_{hi}}{2} - \frac{\hat{Y}_{hj}}{2}\right)^2 + f_h(\hat{Y}'_h - \hat{Y}_h)^2 .$$

Using the result derived earlier for the usual BRR approach, it can be seen that the expectation for the first term is:

$$(1-f_h)\left[M_h^2 \frac{S_{bh}^2}{2} + M_h \frac{\sum_{i=1}^{M_h} \sigma^2_{\hat{Y}_{hi}}}{2}\right] .$$

The expectation of the second term can be found by noting that it can be written as

$$(\hat{Y}'_h - \hat{Y}_h)^2 = \left[M_h \frac{\sum_{i=1}^{2} \hat{Y}'_{hi}}{2} - M_h \frac{\sum_{i=1}^{2} \hat{Y}_{hi}}{2}\right]^2$$

$$= \left[M_h \frac{\sum_{i=1}^{2} \hat{Y}'_{hi}}{2} - \frac{M_h}{2}\sum_{i=1}^{2} \frac{n_{hi}\hat{Y}'_{hi} + (m_{hi}-n_{hi})\hat{Y}''_{hi}}{m_{hi}}\right]^2$$

$$= \left[\frac{M_h}{2}\sum_{i=1}^{2}\left(1-\frac{n_{hi}}{m_{hi}}\right)(\hat{Y}'_{hi} - \hat{Y}''_{hi})\right]^2 ,$$

where $\hat{Y}'_{hi}$ is based upon a subsample of size n_{hi} from m_{hi} and $\hat{Y}''_{hi}$ is based upon the remainder. Conditional on the choice of first stage units this quantity has expected value

$$E_2\left[\frac{M_h}{2}\sum_{i=1}^{2}\left(1-\frac{n_{hi}}{m_{hi}}\right)(\hat{Y}'_{hi} - \hat{Y}''_{hi})\right]^2$$

$$= \frac{M_h}{2}\sum_{i=1}^{2}\left(1-\frac{n_{hi}}{m_{hi}}\right)^2 E_2(\hat{Y}'_{hi} - \hat{Y}''_{hi})^2 ,$$

because the second stage sampling is conducted independently for different first stage units. The conditional expectation of the squared difference between the mean of the subsampled units and the mean of their complement for the i-th selected first stage unit in the h-th stratum is

$$E_2(\bar{y}'_{hi} - \bar{y}''_{hi})^2 = E_2\left((\bar{y}'_{hi} - \bar{Y}_{hi}) - (\bar{y}''_{hi} - \bar{Y}_{hi})\right)^2$$

$$= V(\bar{y}'_{hi}) + V(\bar{y}''_{hi}) - 2\mathrm{cov}(\bar{y}'_{hi}, \bar{y}''_{hi}) .$$

This can be simplified by noting that the covariance reduces to a very simple form

$$\mathrm{cov}(\bar{y}'_{hi}, \bar{y}''_{hi}) = -\frac{S^2_{hi}}{M_{hi}} .$$

The derivation of this result is contained in the appendix. It can be seen that the covariance of sample means, calculated from a random split of a simple random sample is not a function of the proportion of the initial sample placed in each subsample. It is also true that the covariance does not depend on the size of the initial simple random sample selected.

Combining this result with the variance of the subsample means leads to the following conditional result:

$$E_2(\bar{y}'_{hi} - \bar{y}''_{hi})^2 = \left(1-\frac{n_{hi}}{M_{hi}}\right)\frac{S^2_{hi}}{n_{hi}} + \left(1-\frac{m_{hi}-n_{hi}}{M_{hi}}\right)\frac{S^2_{hi}}{m_{hi}-n_{hi}} + \frac{2S^2_{hi}}{M_{hi}}$$

$$= S^2_{hi}\frac{m_{hi}}{(m_{hi}-n_{hi})\,n_{hi}} .$$

To relate this conditional expectation back to the squared difference between the replicate total (based on the PSU subsamples) and the overall sample total note that $M^2_{hi}(\bar{y}'_{hi} - \bar{y}''_{hi})^2 = (\hat{Y}'_{hi} - \hat{Y}''_{hi})^2$. Hence,

$$\left(1-\frac{n_{hi}}{m_{hi}}\right)^2 E_2(\hat{Y}'_{hi} - \hat{Y}''_{hi})^2 = \left(1-\frac{n_{hi}}{m_{hi}}\right)^2 M^2_{hi} S^2_{hi}\frac{m_{hi}}{(m_{hi}-n_{hi})n_{hi}}$$

$$= M^2_{hi}\frac{m_{hi}-n_{hi}}{m_{hi}\,n_{hi}} S^2_{hi} .$$

When n_{hi} is set to $m_{hi}\left(\frac{1}{2-f_{hi}}\right)$ this reduces to

$$M^2_{hi}(1-f_{hi})\frac{S^2_{hi}}{m_{hi}} = \sigma^2_{\hat{Y}_{hi}} .$$

The conditional expectation of the difference between the replicate estimate and the overall estimate has now been shown to be

$$E_2(\hat{Y}'_h - \hat{Y}_h)^2 = E_2\left[\frac{M_h}{2}\sum_{i=1}^{2}\left(1-\frac{n_{hi}}{m_{hi}}\right)(\hat{Y}'_{hi} - \hat{Y}''_{hi})\right]^2$$

$$= \frac{M_h^2}{2}\,\frac{\sum_{i=1}^{2}\sigma^2_{\hat{Y}_{hi}}}{2}$$

It is now apparent that when the expectation is taken over first stage units the following results

$$E_1E_2\left[\hat{Y}'_h - \hat{Y}_h\right]^2 = \frac{M_h}{2}\sum_{i=1}^{M_h}\sigma^2_{\hat{Y}_{hi}} .$$

Combining the previous results yields:

$$E_1E_2E_\alpha[\hat{V}_r(\hat{Y}_h)] = E_1E_2\Big[(1-f_h)M_h^2\Big(\frac{\hat{Y}_{hi}}{2}-\frac{\hat{Y}_{hj}}{2}\Big)^2 + f_h(\hat{Y}'_h-\hat{Y}_h)^2\Big]$$

$$= (1-f_h)\Big[M_h^2\frac{S_{bh}^2}{2} + M_h\frac{\sum_{i=1}^{M_h}\sigma^2_{\hat{Y}_{hi}}}{2}\Big] + f_h\frac{M_h}{2}\sum_{i=1}^{M_h}\sigma^2_{\hat{Y}_{hi}}$$

$$= (1-f_h)M_h^2\frac{S_{bh}^2}{2} + \frac{M_h}{2}\sum_{i=1}^{M_h}\sigma^2_{\hat{Y}_{hi}}$$

$$= V(\hat{Y}_h)$$

Discussion

The proposed modification to balanced repeated replication allows unbiased estimates of variance, for linear statistics, to be generated using existing computer programs. This is the case because it is based upon modifying the formation of replicates and does not require that the replicate estimates be calculated using a technique different from that used for the entire sample.

The procedure described assumes a somewhat restrictive design: 2 first stage units selected with equal probability per stratum without replacement and a simple random sample within the selected first stage units. It is also be possible to derive a similar procedure for 2 unit per stratum designs in which Durbin's approach to setting unit and joint probabilities of selection is used. Further work should also allow fpc's to be incorporated into a jackknife estimate of variance. One concern involving the described procedure is the variance of the variance estimate. Because of the way the estimate is produced it will be less stable than the "textbook" estimate of variance for linear statistics. The extent of this instability is unknown at this time and will be investigated. Related to this, is the degree of dependence introduced into the replicates because of the overlap introduced by the sampling process. Additionally, the performance of the modified technique for non-linear estimates also needs to be investigated.

Appendix

The covariance of the mean of the subsample from the i-th PSU in the h-th stratum with its complement can be shown to be $-\frac{S_{hi}^2}{M_{hi}}$ by writing the variance of the full sample from the hi-th PSU as a function of the variance of the subsample and its complement and their covariance.

$$V(\bar{y}_{hi}) = V\Big(\frac{n_{hi}}{m_{hi}}\bar{y}'_{hi} + \frac{(m_{hi}-n_{hi})}{m_{hi}}\bar{y}''_{hi}\Big)$$

$$= \frac{n_{hi}^2}{m_{hi}^2}V(\bar{y}'_{hi}) + \Big(\frac{m_{hi}-n_{hi}}{m_{hi}}\Big)^2V(\bar{y}''_{hi}) + 2\frac{n_{hi}(m_{hi}-n_{hi})}{m_{hi}^2}\operatorname{cov}(\bar{y}'_{hi}, \bar{y}''_{hi})$$

$$= \frac{n_{hi}^2}{m_{hi}^2}\Big(\frac{1}{n_{hi}}-\frac{1}{M_{hi}}\Big)S_{hi}^2 + \frac{(m_{hi}-n_{hi})^2}{m_{hi}^2}\Big(\frac{1}{(m_{hi}-n_{hi})}-\frac{1}{M_{hi}}\Big)S_{hi}^2 + 2\frac{n_{hi}(m_{hi}-n_{hi})}{m_{hi}^2}\operatorname{cov}(\bar{y}'_{hi}, \bar{y}''_{hi}) .$$

But this is known to equal $\left(\frac{1}{m_{hi}}-\frac{1}{M_{hi}}\right)S_{hi}^2$. This implies that

$$S_{hi}^2\Big[\Big(\frac{1}{m_{hi}}-\frac{1}{M_{hi}}\Big) - \frac{n_{hi}^2}{m_{hi}^2}\Big(\frac{1}{n_{hi}}-\frac{1}{M_{hi}}\Big) - \frac{(m_{hi}-n_{hi})^2}{m_{hi}^2}\Big(\frac{1}{(m_{hi}-n_{hi})}-\frac{1}{M_{hi}}\Big)\Big]$$

$$= 2\frac{n_{hi}(m_{hi}-n_{hi})}{m_{hi}^2}\operatorname{cov}(\bar{y}'_{hi}, \bar{y}''_{hi}) .$$

This can be rewritten as

$$S_{hi}^2\Big[\frac{1}{m_{hi}}-\frac{1}{M_{hi}}-\frac{n_{hi}}{m_{hi}^2}+\frac{n_{hi}^2}{m_{hi}^2M_{hi}}-\frac{(m_{hi}-n_{hi})}{m_{hi}^2}+\frac{(m_{hi}-n_{hi})^2}{m_{hi}^2}\frac{1}{M_{hi}}\Big]$$

$$= S_{hi}^2\Big[-\frac{1}{M_{hi}}+\frac{n_{hi}^2}{m_{hi}^2M_{hi}}+\frac{(m_{hi}-n_{hi})^2}{m_{hi}^2}\frac{1}{M_{hi}}\Big]$$

$$= S_{hi}^2\Big[-\frac{1}{M_{hi}}+\frac{n_{hi}^2}{m_{hi}^2M_{hi}}+\frac{1}{M_{hi}}-\frac{2m_{hi}n_{hi}}{m_{hi}^2M_{hi}}+\frac{n_{hi}^2}{m_{hi}^2M_{hi}}\Big]$$

$$= S_{hi}^2\Big[-\frac{2n_{hi}(m_{hi}-n_{hi})}{m_{hi}^2M_{hi}}\Big] = \frac{2n_{hi}(m_{hi}-n_{hi})}{m_{hi}^2}\operatorname{cov}(\bar{y}'_{hi}, \bar{y}''_{hi}) .$$

Which shows that

$$\operatorname{cov}(\bar{y}'_{hi}, \bar{y}''_{hi}) = -\frac{S_{hi}^2}{M_{hi}} .$$

Acknowledgement

The author wishes to thank Steve Cohen, Ph.D. of NCHSR for reviewing this manuscript.

References

Brewer, K.R.W. and Hanif, M. (1970). Durbin's New Multi-stage Variance Estimator. *The Journal of the Royal Statistical Society*, Ser B 32, 302-311.

Durbin, J. (1967). Design of multistage surveys for the estimation of sampling errors. *App. Statist.*, 16, 152-164.

McCarthy, P.J. (1969). Pseudo-replication: half-samples. *International Statistical Review* 37, 239-264.

Wolter, K.M. (1985). *Introduction to Variance Estimation*. New York: Springer-Verlag.