MULTIVARIATE TESTING IN STRATIFIED SIMPLE RANDOM SAMPLE SURVEYS: HOG & PIG ESTIMATES AS AFFECTED BY CHANGING SURVEY REFERENCE DATES

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I. Introduction

The National Agricultural Statistics Service (NASS) conducts quarterly probability surveys in the 48 conterminious states in order to produce national, regional and state estimates of hog and pig totals. These data are collected through multiframe sampling techniques (area and list frame sampling). Stratified simple random sampling procedures are used to sample from NASS's list frame of farmer operators. Historically, data collection for these quarterly surveys began 10 days prior to the first of the month, and continued for 10 to 14 days. Farm operators were asked to report their hog and pig inventory as of the time of the interview. New survey procedures have recently outlined changing the data collection period to begin the first day of the month, continuing for the 10 to 14 day period. However, farmers would have to recall back to, and report as of, the first of the month. The same questions would be used except for minor modifications to the reference dates. The swine industry was concerned because the traditional data series would be interupted. One question of interest was whether the estimated total inventory would be affected by the change in survey questions and dates.

Altering the dates of data collection would increase or decrease hog and pig inventory depending on where in the hog & pig cycle the survey falls. In addition, if the questionnaire is altered in any way the estimates may also change. Studies dealing with the effects of change in survey questionnaires are numerous. In NASS related literature, several authors have looked at question ordering and wording effects [1] [2] [3] [4] [5] [6].

This paper discusses statistical tests and other comparisons used to evaluate the effect of changing the survey reference dates. Specifically, test statistics are shown that take into account the survey design, results are presented, and procedures for calculating power are given. This is instructive since power is often ignored in published reports.

II. Study Design

Two independent surveys were conducted in May-June 1986 to evaluate these changes in the survey procedures. The first survey, called the Hog & Pig Survey was the operational survey with data collection dates from May 20 - June 1. The "Bridge Survey," or new procedure was conducted from June 1 - June 15, and asked farmers to recall their inventory as of the first of the month. The variables of interest were estimates of pig crop, market hogs, expected 6-month farrowings, expected 3-month farrowings and total hogs and pigs.

The sample size for the Bridge Survey was approximately 2,500 sample units from nine quarterly hog multiple-frame states. These states generally account for over 80 percent of the national estimate. The Bridge Survey sample size was was one-fourth that of the Hog & Pig Survey. Samples were selected through rotation of replicates. No overlap existed in replicates between the Bridge and Hog & Pig Surveys.

The same questionnaires were used in both surveys, with only minor modifications to reflect the reference period change from "time of interview" to "first of the month." Also, the states involved in this study were directed to follow the same survey procedures in both surveys. This included the same mailing and telephoning routine, the same followup procedures, the same field and office edit of the data, and the same keypunch procedures. The only difference between the Bridge and Hog & Pig Surveys, then, was to be due to the 2- to 4- week difference in dates of data collection. Due dates were met, and the data were collected in a timely fashion.

III. Estimators -- Total and Variance of Total

Let $\hat{B}$ and $\hat{O}$ represent the estimated total for some variable for the Bridge and Hog & Pig Surveys, respectively.

Then, $\hat{B} = \sum_{h=1}^{L} \sum_{i=1}^{n_h} N_h/n_{hb}\, b_{hi}$, and

$\hat{O} = \sum_{h=1}^{L} \sum_{i=1}^{n_h} N_h/n_{ho}\, o_{hi}$ where,

N_h = population count in stratum h,

n_{ho} = usable responses in stratum h (sample size minus refusals and inaccessibles) for the Hog & Pig Survey,

n_{hb} = useble responses in stratum h (sample size minus refusals, and inaccessibles) for the Bridge Survey,

$f_{hb} = n_{hb}/N_h$ = sampling fraction in stratum h for the Bridge Survey,

$f_{ho} = n_{ho}/N_h$ = sampling fraction in stratum h for the Hog & Pig Survey,

b_{hi} = list adjustment factor times Bridge Survey variable value for stratum h, sampling unit i, and

o_{hi} = list adjustment factor times Hog & Pig survey variable value for stratum h, sampling unit i.

Also,

$$\text{Var}(\hat{B}) = \sum_{h=1}^{L} N_h^2 (1 - f_{hb}) S^2_{B_h} / n_{hb}$$

$$\text{Var}(\hat{O}) = \sum_{h=1}^{L} N_h^2 (1 - f_{ho}) S^2_{O_h} / n_{ho}$$

where,

$$S^2_{0h} = \sum_{i=1}^{n_h} (o_{hi} - \bar{o}_h)^2 / n_{ho}$$

$$S^2_{Bh} = \sum_{i=1}^{n_h} (b_{hi} - \bar{b}_h)^2 / n_{hb}$$

$$\bar{o}_h = \sum_{i=1}^{n_h} o_{hi} / n_{ho}$$

$$\bar{b}_h = \sum_{i=1}^{n_h} b_{hi} / n_{hb}$$

These parameter and variance estimators are design based for stratified simple random sampling.

Let, $\hat{\mathbf{B}}$ and $\hat{\mathbf{O}}$ represent vectors of population totals for the 5 variables for the Bridge and Hog & Pig Surveys, respectively.

That is, $\hat{\mathbf{B}}' = (\hat{B}_1, \hat{B}_2, \ldots, \hat{B}_5)$, and

$\hat{\mathbf{O}}' = (\hat{O}_1, \hat{O}_2, \ldots, \hat{O}_5)$.

For the Bridge Survey, for example, the vector of totals were:

B_1 = market hogs
B_2 = pig crop
B_3 = expected 6-month farrowings (June-Dec.)
B_4 = expected 3-month farrowings (June-Aug.)
B_5 = total hogs and pigs

Then, $\mathbf{V}(\hat{\mathbf{B}})$, and $\mathbf{V}(\hat{\mathbf{O}})$ are the estimated variance-covariance matrices

for $\hat{\mathbf{B}}$ and $\hat{\mathbf{O}}$, respectively.

$\mathbf{V}(\hat{\mathbf{B}})$, for example, was:

$$\text{Var} (\hat{\mathbf{B}}) = \sum_{h=1}^{L} N^2_h (1 - f_{hb}) \mathbf{S}^2_{Bh} / n_{hb}$$

where,

$$\mathbf{S}^2_{Bh} = \sum_{i=1}^{n_h} (\mathbf{b}_{hi} - \bar{\mathbf{b}}_h)(\mathbf{b}_{hi} - \bar{\mathbf{b}}_h)' / n_{hb} - 1$$

and,

$\mathbf{b}_{hi}$ = matrix of observations of size n_{hb} by 5

$$\bar{\mathbf{b}}_h = 1/n_{hb} \sum_{i=1}^{n_{hb}} \mathbf{b}_{hi}$$

IV. Tests of Hypotheses

The hypothesis of no difference in each of the five variables between the two survey estimates can be formally expressed as:

H_o : $(\mathbf{B} - \mathbf{O}) = 0$

H_a : $(\mathbf{B} - \mathbf{O})$ at least one difference not equal 0

The appropriate test statistic, called Hotellings T^2, has the following form:

$$T^2 = (\hat{\mathbf{B}} - \hat{\mathbf{O}})' [(\mathbf{V}(\hat{\mathbf{B}}) + \mathbf{V}(\hat{\mathbf{O}}))]^{-1} (\hat{\mathbf{B}} - \hat{\mathbf{O}}) ,$$

The exact distribution for T^2 under the assumption of heteroscadastic variance-covariance matrices is unknown. [8] However, T^2 goes to a chi-square distribution for large samples, with p degrees of freedom [8].

V. Simultaneous Confidence Intervals

Simultaneous confidence intervals were constructed to look at which variable(s) was(were) causing the rejection of the multivariate null hypothesis. Confidence intervals were constructed if this null hypothesis was rejected (that is, differences existed in the five variables between the two surveys).

The form of the confidence interval was:

$$c'(\hat{\mathbf{B}} - \hat{\mathbf{O}}) \sqrt{X^2(\alpha)} \sqrt{c'[(\mathbf{V}(\hat{\mathbf{B}}) + \mathbf{V}(\hat{\mathbf{O}}))]c}$$

where, $X^2(\alpha)$ = chi-square critical value, α=0.10.

c' = a contrast vector for any one of the following contrasts:

{1 0 0 0 0}	{0 0 0 1 0}	{0 0 1 0 0}
{0 1 0 0 0}	{0 0 0 0 1}	

VI. Power Calculations - Univariate Case

Power calculations were made only for univariate tests of differences in estimated totals. Specifically, the difference between the operational and Bridge Survey indications that would be declared significant with 80-percent probability were calculated from the data. This information will be of value in deciding how much confidence to put on the test of significance results.

We assume, $\hat{B} \sim N(B,V(B))$, and $\hat{O} \sim N(O,V(O))$

$\hat{B} - \hat{O} \sim N(O,V(B-O))$

$V(B-O) = V(O) + V(B)$, since B and O are independent samples

Then, $P(\text{rejecting } H_o/H_1 \text{ is true})$ =

$$\left| \frac{(\hat{B} - \hat{O}) - (B - O)}{\sqrt{V(\hat{O}) + V(\hat{B})}} > Z_{\alpha/2} \right| +$$

$$P\left| \frac{(\hat{B} - \hat{O}) - (B - O)}{\sqrt{\hat{V}(O) + \hat{V}(B)}} < -Z_{\alpha/2} \right|$$

Let,

$$\delta_{O-B} = B - O, \quad \text{and} \quad Z = \frac{(\hat{B} - \hat{O}) - (B - O)}{\sqrt{\hat{V}(O) + \hat{V}(B)}}$$

Then, the power of the test as given in (1) can be rewritten as:

Power = P(rejecting H_o/H_1 is true) =

$$\left| Z > Z_{\alpha/2} - \frac{\delta_{O-B}}{\sqrt{\hat{V}(O) + \hat{V}(B)}} \right|$$

$$P\left| Z < -Z_{\alpha/2} - \frac{\delta_{O-B}}{\sqrt{\hat{V}(O) + \hat{V}(B)}} \right|$$

For fixed power ($1 - \beta = 0.80$), fixed alpha level ($\alpha = 0.10$), and estimated V(O) and V(B) we iteratively solve for δ_{O-B}. This represents the minimum difference in the estimated totals that can be detected with 80 percent power (probability of detecting H_a when H_a is true) and 10 percent chance of making an error of the first kind (probability of rejecting H_o when H_o is true).

VII. Results

Table 1 provides estimates and test statistic results for the nine-state aggregate level for the five hog and pig variables (list frame contribution only). Table 2 gives these same calculations for one state--North Carolina. In these tables, columns 2, 3, and 5 give the difference in the estimated totals, the standard error of the difference, and the difference expressed as a percentage of the Hog & Pig Survey estimate, respectively. Column 4 gives the difference in the Hog & Pig and Bridge Surveys estimated totals that can be detected with power of 0.80. That is, for the given sample sizes (Bridge Survey sample sizes were one-fourth that of the Hog & Pig Survey), and under the given variance structure of the observations, we could expect to have an 80 percent chance of detecting a real difference (alpha = 0.10 level) of this magnitude. In order to detect smaller differences one must either increase the sample size or be willing to increase the chances of rejecting the null hypothesis of no difference when it is really true (e.g., increase the alpha level above 0.10).

Column 6 gives the detectable difference expressed as a percent of the Hog & Pig Survey estimate. Finally, the seventh column presents the simultaneous confidence interval based on the results of the multivariate test of hypothesis. Only when the multivariate test was significant (alpha = 0.10) were the intervals estimated. The result of the multivariate test is given at the bottom of column 7.

We see from table 1 that the multivariable test could not detect differences in the 5 hog and pig related variables (P=.20). The individual variable differences ranged between 2 - 7 percent (column 5). However, as the univariate power tests show the survey procedures would not have detected differences with any degree of certainty if there was a difference of 8-11 percent or less between the two survey estimates (column 6).

In table 2 one can see that the multivariate test detected differences in the hog and pig variables (p=.01). Normally one would expect at least one confidence interval to exclude the hypothesized parameter value (ie., zero in this case). However, all the confidence intervals included the value of zero. This unusual result can occur with a change in sign of the differences (column 2). Kramer [9] illustrates this phenomenum for the two variable case.

VIII. Conclusions

This paper presents some very practical methods for testing hypotheses on estimated totals, and for constructing estimates of power for these tests. These methods take into account the survey design effects for stratified sampling. Also, the distribution theory assumes large samples, which is not often a problem for state, or national government surveys.

Ideally, it would be nice to construct multivariate power tests, in addition to the univariate power statistics presented in this paper. This was outside the scope of the basic research done for the NASS research report. Also, to be more confident of statements dealing with the differences in the two survey procedures, additional controls would be needed in the survey design, and the surveys would need to be replicated to include the three other quarterly surveys. This study involved the quarterly survey done in June. The additional controls needed would include, for example, controlling the modes of data collection more carefully. While not mentioned in any detail, interviewing for the Hog & Pig and Bridge Surveys were conducted by mail, telephone and personal interviews. In some states, the distribution of responses by mode of data collection was different between the two surveys. Ideally, the same mode of data collection is needed for both test procedures.

Table 1. Estimates and test statistics for the list frame contribution of the Hog & Pig (O) and Bridge (B) Survey estimates for the nine multiple-frame States.

Nine-States	Diff. (B-O)	Std. Error of diff.	Detect. Diff.1/	Percent of Hog&Pig Survey: Diff.	Percent of Hog&Pig Survey: Detect. diff.	90% C.I.2/
	1,000 Head			Percent		1,000 Head
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Market Hogs	883	635	1,530	4	8	--
Pig Crop	667	404	951	7	10	--
Expected 6-MO. Farrowings	56	98	253	2	10	--
Expected 3-MO. Farrowings	34	55	146	3	11	--
Total Hogs & Pig	972	707	1,710	4	8	--
Multivariate test on five variables (P-value)						.20

1/ Difference in the Hog & Pig and Bridge Surveys estimated totals which can be detected with power of 0.80.
2/ 90% Simultaneous confidence interval; calculated if the multivariate test was significant (alpha=.05).

Table 2. Estimates and test statistics for the list frame contribution of the Hog & Pig (O) and Bridge (B) Survey estimates for North Carolina.

North Carolina	Diff. (B-O) d	Std. Error of Diff.	Detect. diff.1/	Percent of Hog&Pig Survey: Diff.	Percent of Hog&Pig Survey: Detect. diff.	90% C.I.2/
	1,000 Head			Percent		1,000 Head
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Market Hogs	48	82	173	6	20	(-201,296)
Pig Crop	-69	44	10	-14	22	(-202,65)
Expected 6-MO. Farrowings	-23	11	29	-18	22	(-56,10)
Expected 3-MO. Farrowings	-11	6	15	-17	23	(-28,6)
Total Hogs&Pigs	24	93	196	2	19	(-253,302)
Multivariate test on five variables				(P-value)		.01*

1/ Difference in the Hog & Pig and Bridge Surveys estimated totals which can be detected with power of 0.80.
2/ 90% Simultaneous confidence interval; calculated if the multivariate test was significant (alpha=.05).
*Significant at alpha=0.10.

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