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## 1. INTRODUCTION

In 1969 the Bureau of the Census began to publish a single family price index [SFPI] based on data collected through the Survey of Construction (SOC). The SOC is a monthly survey that collects data on new housing units started, completed and sold. SOC has a multistage survey design. At the first stage land areas consisting of counties, independent cities or townships are selected. Within these first stage units, permit offices and land areas not covered by permits are selected in the second stage. Monthly, interviewers list and select permits for residential construction from these permit offices and list new construction in the land areas not covered by permits. The interviewers contact the owners of the construction to find out if they have been started, completed or sold, and to obtain characteristics of the construction. A single family unit is included in the price index when it has been sold, that is, a signing of a sales contract or the acceptance of a deposit. The agreed upon sales price is obtained at that time.

The single family price index (SFPI) measures the shift in housing prices by estimating the sales price of a fixed house at each time period. The index is the ratio of the estimated price in the current time period to the price in the base time period. This kind of index is called a Laspeyres index. The technique used for the single family price index has been called the "hedonic", "characteristics", or "regression" method because the sales price of a house is linearly regressed on its major characteristics to determine the relative importance of these characteristics in explaining the variability in house prices. These characteristics are the floor area and the size of the lot, the number of stories, the number of bathrooms, the presence of central air-conditioning, the type of parking facility, the type of foundation, the presence of a fireplace, the geographic location, and the metropolitan location. From these regression coefficents, the predicted price of an average base year [1977) house is estimated. The average, base year house is defined as having the mean of the characteristics of all houses sold during the base year. Thus the predicted price of the average, base year house in year $t$ is

$$
\sum_{i=1}^{p} \hat{b}_{t i} \bar{x}_{t_{0} i}+\hat{b}_{t 0}
$$

where $\hat{b}_{\mathrm{ti}}$ are the estimated coefficients from the cur-
Figure 1: Quarterly SFPI

rent time period and $\overline{\mathrm{x}}_{\mathrm{t}_{0} \mathrm{i}}$ are the average characteristics of the base year houses. Minor adjustments to the predicted price are made to reflect editing of the data and removal of outliers.

Two index series are published by the Bureau of the Census: a quarterly and an annual series based on the sales price including lot value of houses sold. For the quarterly index, the predicted sales price is based on all houses sold in the quarter; similarly, for the annual index, the predicted sales price is based on all houses sold during the year. Since the indexes are regression based, the amual index is not the average of the four quarterly indexes.

Another related set of index series is based on the sales price of the houses without the lot value included. Along with the quarterly and annual index series, a monthly and a moving three-month index series are also produced. The moving three-month series is similar to the quarterly series but uses a moving quarter. The monthly and moving three-month series are provided to the Bureau of Economic Analysis to be used as construction deflators but they are not used by the Bureau of the Census. Because this monthly series is too noisy, Census constructs a monthly series from the quarterly series by trending it using benchmarking techniques to the Engineering News Record - Building (ENR-B) index series. The trended index series is used by Census to deflate the value put in place for residential construction and, in conjunction with other index series, to deflate private nonresidential construction and military facilities. Figures 1 and 2 show the quarterly and monthly single family price index series, the trended series and the ENR-B for 1977 through 1986.

This paper investigates using optimal filtering to smooth the actual monthiy single family price index without the lot value included (called SFPI in the remainder of this paper) to obtain a "better" estimate of the underlying trend than the monthly trended SFPI. An observed time series $y_{t}$, which is assumed to follow an autoregressive intergrated moving average (ARIMA) process, can be decomposed into signal $\left[s_{t}\right.$ ] plus noise $\left(u_{t}\right)$. The signal can be estimated by a two-sided filter applied to the observed series (see Box, Hillmer and Tiao 1978). When

Figure 2: Monthly Index Series

filtered estimates are required for the end of the time series, such as the most recent month, unknown future values of the observed series can be replaced by forecasts.

The decomposition is much easier if the noise $u_{t}$ can reasonably be assumed to be white noise. For the single family price index, the noise can arise from the regression modeling or the survey design and estimation. Since the monthly samples of the single family units sold do not overlap, the errors from the regression modeling will be uncorrelated over time as long as there is no misspecification of the model. From the SOC survey design, autocorrelations can arise from the three stages of sampling. Since the monthly samples of permits are independent of each other given the selection at the first two stages, they will not induce an autocorrelation. In the observed series, the effects from the selections in the first two stages will appear as nearly fixed because these units do not change except during SOC redesigns.

In Section 2, the basic results on filtering, revisions and month-to-month change are presented. These results have been extracted from a paper by Maravall (1986). In Section 3, the time series models for the single family price index are presented. ARIMA models and ARMA model with a special differencing operator intermediate between a first and second difference are used to model the monthly single family index series. Transfer models involving the SFPI as the output and the ENR-B as the input were found to give unacceptable models. The results are not presented in this version of this paper. In Section 4, the filters for these models are developed and the theoretical results for revisions and month-to-month change are presented. The filters are compared in terms of the average revision, the average absolute revision and the average absolute month-to-month change in Section 5.

## 2. OPTIMAL FILTERING, REVISIONS AND MONTH-TO-MONTH CHANGE

We wish to decompose the observed monthly price index series $y_{t}$ into $y_{t}=s_{t}+u_{t}$ where $s_{t}$ is the signal and $u_{t}$ is the noise. As discussed in the previous section, the noise for the single family price index can reasonably be assumed to be white noise. Let the model for $y_{t}$ be

$$
\Phi(B) y_{t} \stackrel{t}{=} \theta(B) a_{t}
$$

where $\Phi(B)$ includes the differencing operators and the stationary $A R$ operator $\phi(B)$. The variance of $a_{t}$ is $\sigma_{a}^{2}$ and the variance of $u_{t}$ is $\sigma_{u}^{2}$.
Let $r=\sigma_{\mathrm{a}}^{2} / \sigma_{\mathrm{a}}^{2}$. The minimum mean square linear estimator of $s_{t}$ is $\hat{s}_{t}=v[B] y_{t}$ where $v(B)$ is
the two-sided filter

$$
\begin{equation*}
v(B]=1-r \frac{\Phi(B) \Phi[F]}{\theta(B] \theta(F)} \tag{2.1}
\end{equation*}
$$

where $F=B^{-1}$. This filter depends on $\sigma_{u}^{2}$ through $r$. $0_{u}^{2}$ can either be estimated from the data or the canonical decomposition can be used. The canonical decomposition uses the maximum value of $\sigma_{u}^{2}$ consistent with the model for $y_{t}$. The canonical decomposition, which yields the smoothest signal, will be used in this paper because the price index tries to measure the underlying inflation or deflation and hence should not be too sensitive to short term fluctua-
tions.
The filter $v(B)$ is centered at $t$ and symmetric, hence $\hat{s}_{t}$ depends on observations posterior to $t$. Let $\hat{s}_{t}$ be the final estimate of $s_{t}$ when all observations are available and let $\hat{s}_{t}^{0}$ be the preliminary (concurrent) estimate found by replacing the future values $\left[y_{t+j}\right]$ with their forecasts from time $t$,
i.e. $\hat{\mathrm{y}}_{\mathrm{t}}[\mathrm{j}]$. The ultimate revision from the preliminary estimate to the final estimate is $d_{t}^{0}=\hat{s}_{t}-\hat{s}_{t}^{0}$ which follows the ARMA(q,h-1) process

$$
\theta(F) d_{t}^{0}=-r[\Phi(F)-\theta[F)] a_{t}
$$

and $h=\max (p, q)$. The variance of the ultimate revision is the constant term in the series expansion of the autocovariance generating function

$$
r^{2} \frac{(\Phi(B)-\theta(B)\}\{\Phi(F)-\theta(F))}{\theta(B] \theta(F)} \sigma_{a}^{2}
$$

Maravall develops similar results for each revised estimated and shows that each revision follows an Ma process.

If $\Phi(B)$ can be written as $(1-B) \phi(B)$ where $\phi(B)$ is a stationary autoregressive operator, then the variance of month-to-month change will be finite. The variance of month-to-month change in the original series is the constant in the covariance generating function for the first difference:

$$
\frac{\theta(B) \theta(F)}{\phi(B) \phi(F)} \sigma_{a}^{2}
$$

The first difference of the final estimate of $s_{t}$ has the following model:

$$
\nabla \hat{s}_{t}=v(B) \frac{\theta[B]}{\phi(B)} a_{t}=\left(\frac{\theta[B)}{\phi(B]}-r(1-B) \frac{\Phi(F)}{\theta(F)}\right) a_{t} .
$$

The variance in month-to-month change of the final estimate is the constant in the covariance generating function
$\left(\frac{\theta(B) \theta[F)}{\phi(B) \phi(F)}+r^{2}[1-B][1-F] \frac{\Phi[B] \Phi(F)}{\theta(B) \theta(F)}-2 r[1-B](1-F)\right) \sigma_{a}^{2}$.

## 3. TIME SERIES MODELS FOR THE MONTHLY SINGLE FAMILY PRICE INDEX

Figures 1 and 3 plot the monthly price index series and the first difference. The series is nonstationary in the mean and requires differencing. It shows no seasonality. Figure 3 shows a sudden increase in the variance beginning in 1980 and an increase in variance as the level of the series rises. This increase in variance can at least partially be explained by two factors. Beginning in 1980, the housing market plunged into a recession which dramatically reduced the Housing Sales sample size as shown in Table 1.

Table l: HOUSING SALES SAMPLE SIZE IN 000'S
$\begin{array}{llllllllll}77 & 78 & 79 & 80 & 81 & 82 & 83 & 84 & 85 & 86\end{array}$
$\begin{array}{llllllllll}15 & 15 & 13 & 9 & 6 & 7 & 10 & 10 & 11 & 11\end{array}$
In 1983, the housing market recovered from the recession and the SOC introduced a change in methodology to keep the sample size more nearly constant. Thus the sample size did not increase after 1983 to the level that it had in 1977-9. Secondly, interest rates were deregulated in 1979 thus introducing additional instability in the market and making the kinds and prices of the houses built much more interest sensitive.

This paper analyses the filters based on the untransformed single family price index series. The portion of the series prior to 1980 was discarded
when fitting the models but was used when evaluating the filters. It was also used to establish the prior state for the Kalman filters. The log of the price index series was briefly looked at but no results significantly different from those for the untransformed series were obtained.

Table 2 shows the mean, standard error of the mean, the variance of the series and the autocorrelations for no difference, a first difference and a second difference and the partial autocorrelation for a first differnce.
Table 2: AUTOCORRELATION FUNCTION
AND PARTIAL AUTOCORRELATION

The table shows that a first difference is satisfactory and that a constant may need to be included in the model. Two possible models are suggested by the autocorrelations and the partial autocorrelations, an IMA(1,1) or an ARI(2,1). In an MA(1) process, the admissible range for the first autocorrelation $\rho_{1}$ is -.5 to .5 . The estimated first autocorrelation is near but falls outside of this range. An ARI(2,1) model can easily fit this data since the estimated autocorrelations for the first differnce fall into the admissible range. The ARI(2,1) model was fit over three spans of time 1980-84, 1981-85 and 1980-85 to see the effect of perturbing the data. The results for 1980-85 without a constant are shown under Model A and with a constant under Model B in Table 3.

|  | Table 3: | ARI(2,1) MODELS |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\phi_{1}$ | $\phi_{2}$ | const | Res. Var. |
| MODEL A | -.6155 | -.2743 | - | 18.245 |
| S.e. | $[.1187]$ | $[.1209]$ |  | 1.1948 |
| MODEL | -6684 | -.3269 | 17.100 |  |
| S.e. | $[.1171]$ | $[.1192]$ | $[.5101]$ |  |

To check the MA model, I fitted an IMA(1,1] with a constant to the 1980-85 time span. The estimated MA parameter $\theta_{1}$ was 0.6538 (.102) and the residual variance was 17.783 . Model $B$ had the smallest residual variance.

I chose simple values for $\phi_{1}$ and $\phi_{2}$ in Models A and $B$ and the constant in Model $B$ consistent with the fitted models. I chose $\phi_{1}=-2 / 3, \phi_{2}=-1 / 3$ and the constant equal to $6 / 5$. These are appealing because the forecast equation fo Model $B$ is

$$
\hat{y}_{t+1}=\left\{y_{t}+y_{t-1}+y_{t-2}\right\} / 3+6 / 5
$$

A series like the single family price index will not be stationary because it is measuring inflation so a first or second difference should be required. A first difference with a constant may not be
adequate because the inflation rate, as indicated by the constant, should change with time. However, a second difference was found to be worse for this series than a first difference. A special difference operator, intermediate between a first and second difference, was developed.

Let us consider a series $x_{t}$ which does not have mean zero and may not be stationary. To achieve stationarity and to remove the mean, we usually take a first difference, $x_{t}-x_{t-1}=(1-B) x_{t}$, or a second difference. If the nonzero mean is the only problem, we usually remove the mean $x_{t}-\bar{x}=x_{t}-\sum_{k=1}^{N} x_{k} / N$. As an alternative to removing the mean or taking a first difference, we could replace the mean by a moving average of the previous $n$ points in the series.
$x_{t}-\bar{x}_{t, n}=x_{t}-\sum_{k=1}^{n} x_{t-k} / n=\left(1-U_{n}(B) B / n\right] x_{t}$
where $U_{n}[B]=1+B+B^{2}+\cdots+B^{n-1}$.
This alternative has the flavor of both removing the mean and taking a first difference. When the series $x_{t}$ has already been first differenced, i.e. $x_{t}=$ $(1-B) y_{t}$, then this differencing operator becomes

$$
V_{n}[B]=1-[n+1] B / n+B^{n+1} / n
$$

by substituting (1-B) $y_{t}$ for $x_{t}$ in (3.1). When $\mathrm{n}=1, \mathrm{~V}_{1}[B]$ is equivalent to a second difference and as $n \rightarrow \infty$ and $N \rightarrow \infty, V_{n}[B]$ approaches a first difference with a mean correction because the series has been assumed to be ergodic.

Model C, an $\operatorname{AR}[2]$ with the differencing operator $V_{n}(B)$, was fit for values of $n$ from 1 to 12 and compared using an analogue of the AIC. The "AIC" used is equal to $-2 l(\hat{\theta}]+2 \mathrm{k}$. The loglikelihood $l[\hat{\theta}]$ was calculated using a Kalman filter as described in Gersch and Kitigawa (1983). The number of parameters was estimated as suggested by Gersch and Kitigawa. The data from 1977-9 were used to establish the initial state by assuming an uninformative prior state for December 1976 and evaluating the Kalman filter through December 1979. This last state was taken to be the initial state for evaluating the Kalman filter on the 1980-85 data. Table 4 shows that the smallest AIC was at $n=7$. The fitted values for $\phi_{1}$ and $\phi_{2}$ are consistent with those used in Models A and B.

Table 4: MODEL C, AR[2] WITH DIFFERENCING OPERATOR $\mathrm{V}_{\mathrm{n}}(\mathrm{B})$

| n | $\phi_{1}$ | $\phi_{2}$ <br> Variance | Residual | AIC |
| :---: | :---: | :---: | :---: | :---: |
| 1 | -1.0739 | -.5910 | 26.446 | 450.1334 |
| 2 | -.7403 | -.5497 | 23.480 | 443.5693 |
| 3 | -.6460 | -.3458 | 22.608 | 442.8444 |
| 4 | -.6774 | -.3409 | 21.525 | 441.3093 |
| 5 | -.6284 | -.3664 | 20.966 | 441.4164 |
| 6 | -.6494 | -.3530 | 20.513 | 441.8447 |
| 7 | -.6549 | -.3415 | 19.467 | 440.0758 |
| 8 | -.6512 | -.3610 | 19.119 | 440.7770 |
| 9 | -.6651 | -.3717 | 19.522 | 444.2774 |
| 10 | -.6439 | -.3617 | 18.709 | 443.2169 |
| 11 | -.6674 | -.3574 | 18.638 | 444.9428 |
| 12 | -.6590 | -.3545 | 18.647 | 446.9764 |

The Model C is

$$
\left(1+\frac{2}{3} B+\frac{1}{3} B\right)\left(1-\frac{8}{7} B+\frac{1}{7} B^{8}\right) y_{t}=a_{t}
$$

with $\sigma_{a}^{2}=19.467$.

## 4. FILTERS, REVISIONS AND MONTH-TO-MONTH CHANCE

### 4.1 FILTERS

Model $A$ is $\Phi(B) y_{t}=a_{t}$, where

$$
\Phi(B)=\left(1+\frac{2}{3} B+\frac{1}{3} B^{2}\right)(1-B), 0_{a}^{2}=18.245 .
$$

The symmetric filter is

$$
V_{A}[B]=1-r \Phi(B) \Phi(F)
$$

$=1-\Gamma\left(\frac{4}{3}-\frac{1}{9}(B+F)-\frac{2}{9}\left(B^{2}+F^{2}\right)-\frac{1}{3}\left(B^{3}+F^{3}\right)\right)$.
For the canonical decomposition

$$
r=\min _{w} \frac{1}{\Phi\left(e^{-i w}\right) \Phi\left(e^{i w}\right)}=0.4589615
$$

Again for simplicity, I will set $r$ to be equal to $9 / 20$. The filters for Model B are the same except for the inclusion of a constant.

Model $C$ is $\Phi_{7}(B) y_{t}=a_{t}$ where

$$
\Phi_{7}(B)=\left(1+\frac{2}{3} B+\frac{1}{3} B\right)\left(1-\frac{8}{7} B+\frac{1}{7} B^{8}\right) .
$$

The symmetric filter is $\mathrm{V}_{\mathrm{f}, \mathrm{C}}(\mathrm{B})=1-\mathrm{r} \Phi_{7}(\mathrm{~B}) \Phi_{7}(\mathrm{~F})$ For the canonical decomposition,

$$
r=\min _{w} 1 / \Phi_{7}\left[e^{-i w}\right) \Phi_{7}\left[e^{i w}\right)=.35303 .
$$

Again, $r$ will be simplified to $7 / 20$.
The nonsymmetric filters can be found by replacing an unobserved point by its forecast. Table 5 shows the filter weights for the preliminary and final filters for Models A, B, and C. The final filters are symmetric around $\mathrm{v}_{0}$ and only one side of the filters are shown.

## Table 5: FILTER WEIGHTS

 MODELS A \& B MODEL C|  | Final | Preliminary | Final | Preliminary |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{v}_{0}$ | 0.40 | 0.55 | 0.444 | 0.650 |
| $\mathrm{v}_{-1}$ | 0.05 | 0.15 | 0.033 | 0.166 |
| $\mathrm{v}_{-2}$ | 0.10 | 0.15 | 0.084 | 0.150 |
| $\mathrm{v}_{-3}$ | 0.15 | 0.15 | 0.133 | 0.133 |
| $\mathrm{v}_{-4}$ |  |  | 0.000 | 0.000 |
| $\mathrm{v}_{-5}$ |  |  | 0.019 | 0.000 |
| $\mathrm{v}_{-6}$ |  |  | 0.034 | 0.000 |
| $\mathrm{v}_{-7}$ |  |  | 0.044 | 0.000 |
| $\mathrm{v}_{-8}$ |  |  | -0.027 | -0.050 |
| $\mathrm{v}_{-9}$ |  |  | -0.025 | -0.033 |
| $\mathrm{~V}_{-10}$ |  |  | -0.016 | -0.016 |
| const.(B) | 0.00 | 0.54 |  |  |

### 4.2 REVISIONS AND MONTH-TO-MONTH CHANGE

Section 2 presented expressions for the variance of the revisions and of the month-to-month change. The results for Models A, B, and C are shown in Table 6.

Table 6: REVISION AND MONTH-TO-MONTH VARIANCE REVISION

|  | Model A | Model B | Model |  |
| :---: | :---: | :---: | :---: | :---: |
| Prelim. to Final | 1.232 | 1.142 | 1.40 |  |
| First Revision | 0.411 | 0.381 | 0.54 |  |
| First to Final | 0.821 | 0.761 | 0.86 |  |
| MONTH-T0-MONTH CHANCE |  |  |  |  |
| Model A | Model B | Model C |  |  |
| Original | 27.368 | 27.368 | 27.368 |  |
| Final | 5.200 | 4.820 | ND |  |

## 5. EMPIRICAL ANALYSIS

The filters constructed from Models A, B and C are compared in this section based on mean absolute month-to-month change, mean revision and mean absolute revision. Figures 4 and 5 graph the final series for Models A, B and C.

Table 8 shows the mean absolute month-to-month change for the original single family price index series, and the preliminary and final series for the models. For Model C, the third revision is also shown because this estimate is obtained at the same time as the final estimates for the other models and because third revised estimates are available for all of 1986. Figures 7 and 7 show the graphs of the month-to-month change for the final series. The filters substantially reduce the month-to-month change. Model C has larger month-to-month variation than these other models. Month-to-month change for the third revised estimates and the final estimates for Model C are nearly identical.

Tables 9 and 10 show the average revisions and the average absolute revisions for the first revision [preliminary to first revised estimate], first revised to final estimate, and the total revision (preliminary to final estimate). The total revisions are graphed in Figures 8-10. Model A shows a significant bias in the preliminary and first revised estimates because the constant is not included in the model. Model C shows negligible bias in the preliminary and revised estimates and was able to adapt to the inflation rate outside of the period in which the model was fit. Even though the constant worked fairly well overall for Model B, it was not able to adapt to changes in the underlying inflation rate and thus showed substantially larger (though not statistically significant] average revisions from the preliminary and first revised estimates. Figures 9 and 10 show that in 1982, when there was a brief period of deflation, initially Model C more seriously overstated the inflation than Model B did and generated a larger revision during the transition but it adapted to the change. Similarly, when the inflation picked up in 1983, Model C initially understated the inflation and had larger revisions during the transition. By using a constant that was fit over a period that included these two years, Model B did not suffer the extremes in revisions as Model C, but Model B can not adapt when ther are changes in the underlying inflation rate and thus preliminary and revised estimates from Model B could do poorly when used beyond 1980-1985. This is evidenced by the generally positive revisions in 1978 and 1979 for Model B but the small average revision for Model C.

Table 10 shows the average absolute revisions for these series. Model A shows larger revisions because of the biases in the preliminary and revised estimates. The other series show comparable results though Model C has slightly smaller absolute revisions. The final estimates of Model C show larger month-to-month change (a less smooth signal) but smaller revisions. The final estimates from Model C may be less smooth than those from the other models, but it may be the price we want to pay for adaptability to changes in the underlying inflation rate.

## 6. CONCLUDING REMARKS

This paper has looked at filters for the monthly single family price index based on ARIMA models and an ARMA model with a special differencing operator. Model $\mathbb{C}$ smoothed the monthly single family price
index nearly as well as the other models and provided protection against changes in the underlying inflation rate. The final index series for all the models follow the general pattern of the original index series unlike the trended series [see Figure 3) which shows an undulating pattern induced by trending the quarterly series to form a monthly series. Of the filters developed in this paper, the filter constructed form Model C is the best.

## FOOTNOTE

${ }^{1}$ This paper reports the general results of research undertaken by Census Bureau Staff. The views expressed are attributable to the author and do not necessarily reflect those of the Census Bureau.
${ }^{2}$ Building Permit Survey is a survey of about 8000 permit offices which report monthly to the Bureau of
the Census on the number of permits issued.

## REFERENCES

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MARAVALL, A. [1986], "Revisions in ARIMA Signal Extraction," Journal of the American Statistical Association, 81, 736-740.

Table 8 - Average Absolute Month-to-Month Change


Table 10 - Average Absolute Revisions

|  | 1978 | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 | OVERALL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Prelim. to First Revised |  |  |  |  |  |  |  |  |  |  |
| Model B | 0.260 | 0.302 | 0.458 | 0.320 | 0.670 | 0.299 | 0.487 | 0.748 | 0.671 | 0.468 |
| Model C | 0.268 | 0.268 | 0.536 | 0.343 | 0.792 | 0.346 | 0.608 | 0.907 | 0.767 | 0.537 |
| Model A | 0.776 | 0.765 | 0.732 | 0.617 | 0.903 | 0.393 | 0.742 | 0.891 | 1.036 | 0.762 |
| Model B | 0.416 | 0.413 | 0.528 | 0.460 | 0.864 | 0.404 | 0.626 | 0.981 | 0.976 | 0.630 |
| Model C - Third | 0.308 | 0.261 | 0.511 | 0.383 | 0.984 | 0.378 | 0.652 | 1.102 | 0.960 | 0.615 |
| Model A | 1.138 | 1.128 | 1.038 | 0.934 | 1.027 | 0.419 | 0.972 | 1.229 | 1.299 | 1.020 |
| Model B | 0.598 | 0.620 | 0.753 | 0.517 | 1.207 | 0.426 | 0.769 | 1.319 | 1.231 | 0.827 |
| Model C - Third | 0.347 0.370 | 0.421 0.470 | 0.663 0.588 | 0.421 0.505 | 1.216 | 0.424 | 0.816 | 1.523 | 1.386 | 0.802 |
| * First Five Months of 1986 |  |  |  |  |  |  |  |  |  |  |

Figure 3: Monthly SFPI


Figure 5: Model C


Figure 7: Model $C$


Figure 9: Model B
Tobal Revision


Figure 4: Models A \& B


Figure 6: Models A \&c B


Figure 8: Model A


Figure 10: Model C


