GENERALIZATIONS OF THE MITOFSKY-WAKSBERG TECHNIQUE FOR RANDOM DIGIT DIALING: SOME ADDED TOPICS

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1. INTRODUCTION

A recent article (Potthoff 1987), hereafter referred to as (*), proposed a class of two-stage sampling techniques for random digit dialing that generalizes the Mitofsky-Waksberg technique (Waksberg 1978) in two ways. First, every phone number is categorized as either auspicious or inauspicious. The definition of auspicious is up to the user, but every qualifying residential number must be auspicious. Second, c phone numbers are drawn from each primary sampling unit (PSU) in the first stage. The Mitofsky-Waksberg technique constitutes the special case in which auspicious numbers are defined to be the same as residential numbers and $c = 1$.

Briefly, sampling is done as follows. Each phone number drawn for the first stage is dialed, and is classified as either auspicious or inauspicious. If all c numbers in a PSU are inauspicious, the PSU is discarded. If exactly one number is auspicious, the PSU is retained and is called a Type I PSU. If two or more numbers are auspicious, the PSU is retained and is called a Type II PSU.

A value k is chosen. The second-stage sample includes exactly $n = kc$ phone numbers from each Type II PSU. For each Type I PSU, though, it has a nonsequential segment and a sequential segment. The nonsequential segment from each Type I PSU consists of exactly $k(c - 1)$ phone numbers. The sequential segment is not as straightforward, and consists of drawing and dialing as many phone numbers as necessary until the first k auspicious numbers have been obtained.

This paper deals with some topics that it was not possible to cover in (*). Section 2 expands on Section 1.1 of (*) and further examines cons and pros of previously available telephone sampling techniques. Some details of the practical use of the proposed new technique are covered in Section 3. Section 4 discusses the handling of exhausted Type I PSU's. Cost considerations and cost comparisons are dealt with in Section 5. Section 6 presents a few results from recent experience that Valley Forge Information Service has had with the new methodology. Finally, Section 7 provides proofs that were omitted from (*).

2. OTHER TECHNIQUES

The deficiencies of available telephone sampling techniques provided the motivation for developing the methodology of (*). Section 1.1 of (*) described the main available techniques, identifying them as Methods 1 through 4. They will now be examined further.

Under Method 1, one obtains a phone number for the sample by drawing a residential number from a directory and then replacing its last one or two digits with new digit(s). Waksberg (1978, p. 40) dismissed this method with a brief footnote describing it and noting its statistical shortcoming. It appears to be widely used, probably because it is easy to administer and inexpensive, and produces a sample with a high proportion of residential phone numbers. However, many of its users may not be fully aware of its statistical drawbacks.

Different phone numbers have different selection probabilities, which are not even known. Bias can thus result. Suppose for illustration that no nonlisted residential numbers exist and the East Gopher exchange has the same number of residential numbers as the West Gopher exchange. Within active banks of phone numbers, though, suppose that 80 out of each 100 numbers in East Gopher are residential whereas only 40 of each 100 in West Gopher are residential. (This implies that West Gopher has twice as many active banks as East Gopher.) If the last two digits of each number drawn from the directory are replaced, then the new number will be residential 80% of the time for an East Gopher number but only 40% of the time for a West Gopher number. Although the population is evenly split between the two exchanges, the residential phone numbers in the sample will be 2/3 from East Gopher and 1/3 from West Gopher.

A second example will illustrate how Method 1 underrepresents numbers that are not listed. Suppose that, within active banks of phone numbers, 75 numbers out of each 100 are residential in both North Daffodil and South Daffodil, but in North Daffodil 50 of the 75 are listed and 25 are not listed whereas in South Daffodil all 75 are listed. Suppose also that the total number of residential numbers is the same in North Daffodil as in South Daffodil. Then the sample, instead of being split evenly, will have only two residential numbers in North Daffodil for every three in South Daffodil. Although the proportion of numbers not listed is 1/6 in the population, it will be only $1/3 \times 2/5$, or 2/15, in the sample.

Under Method 2, one can sample in various ways by using "working bank" information (commercially prepared data intended to show the number of listed residential numbers in every group of 100 phone numbers having the same first eight digits). This information is derived from phone directories but is available on computer tapes. Phone numbers that are left out of a directory by a household's choice, phone numbers that have been assigned or discontinued since a directory was published, and different types of clerical errors all contribute to reducing the usefulness of the data for sampling purposes. These discrepancies adversely affect both the representativeness and the efficiency of the telephone sampling. For recent assessments that are related to the working-bank information, see Landenberger, Groves, and Lepkowski (1984) and Whitmore, Mason, and Hartwell (1985).

One way of using the working-bank information to obtain a sample is to draw banks of 100 numbers with probability for any bank proportional to its number of listed residential numbers. Then a phone number can be chosen at random from the 100 numbers in each selected bank. Such a procedure is the computerized equivalent of Method 1. It thus yields a high proportion of residential numbers but also is vulnerable because of bias. A second procedure, which yields a lower proportion of residential numbers but is better statistically, is the same as the first except that the banks are drawn with equal probability from all banks that show any listed residential numbers. Costs may suffer with this procedure if there are many banks that really have no listed residential numbers but have one or more shown because of clerical error. Of course, representativeness suffers to the extent that banks that really do have residential numbers are excluded from being drawn because they show no listed residential numbers.

Method 3 consists of simple random sampling from all phone numbers with a valid combination of area code and prefix. It is not complicated and has no statistical deficiencies. Its drawback, however, is not a minor one: Less than a quarter of the phone numbers drawn nationwide will be residential. Not only will the large amount of interviewer time spent on unwanted phone numbers have an adverse effect on cost, but also there may be an impairment of interviewer morale that can reduce the quality of the interviewing. Nevertheless, Method 3 may deserve more use than it has received. In particular, its drawback has less relative impact and is less serious if the interview is a long one; if the percentage of the population that qualifies for the survey is high; if the number of completed interviews to be obtained is low; and if the geographic areas covered by the survey have a lower-than-average percentage of unwanted phone numbers (as will usually be the case in heavily populated areas).

Concerning Method 4, the Mitofsky-Waksberg technique, Waksberg (1985, p. 91) recently suggested a modification to it that reduces delays. One specifies a fixed total number of phone numbers per PSU in the second stage rather than a fixed number of residential numbers. Differential weighting is required. In addition, some difficulty apparently exists if the second-stage phone numbers for a PSU include none that are residential.

3. PRACTICAL CONSIDERATIONS

Certain practical matters pertaining to the techniques proposed in (*) were covered incompletely or not at all in that article. One of them, which concerns exhausted Type I PSU's, is the topic of Section 4. Remaining ones are as follows:

(i) In the first-stage dialing, it is best if the c phone numbers within each PSU are all dialed at different times and by different interviewers.

(ii) If interviewing is done in the first stage, it can involve the same survey and same questionnaire as in the second stage, a pretest for the same survey, or a completely different survey. However, scheduling constraints or other matters may make it awkward to interview in the first stage, especially if the survey and questionnaire are the same as in the second stage.

(iii) If $c > 2$ and no interview is being formally conducted along with the first-stage dialing, then in principle one need not dial all c numbers in a PSU if two auspicious numbers are found before all c numbers are dialed. (This is because the PSU would end up as Type II in any event.) Dialing all c numbers in every PSU may nevertheless be the best policy, though, because there can be more checks and controls.

(iv) Accurate classification of the first-stage phone numbers is essential. After the first-stage dialing is completed, the recorded results should be checked in any way that is appropriate.

(v) In order for sampling probabilities to be correct, interviews in either stage are acceptable only at residential numbers. One should therefore determine whether the number that was dialed to reach a respondent is residential. A phone number that is for both home and business is considered residential. Not considered residential is a nonworking number whose dialing results in the ringing of a residential number, a number different from the one dialed (see Groves and Kahn 1979, pp. 47-48); to detect such situations, which are not uncommon in rural areas, one asks if the number reached is what was dialed.

(vi) Restricting k to an integer can result in undue inflexibility for the value of $n = kc$. There is a way to avoid this inflexibility, and even avoid deciding the value of k in advance, without any serious violation of the probability model. Let the sampling in the second stage proceed by replicates. Each replicate consists of one phone number from each Type II PSU, one phone number from each Type I PSU that falls in its nonsequential segment, and as many phone numbers as are necessary to obtain one auspicious number in each Type I PSU that falls in its sequential segment. The Type I PSU's are to be appropriately divided into c groups of about equal size so that the first group has its sequential segment in the first replicate and every c-th replicate thereafter, the second group has its sequential segment in the second replicate and every c-th replicate thereafter, and so forth. This procedure effectively provides for fractional values for k. One can achieve an even finer graduation for k by breaking up the replicates into subreplicates.

(vii) The set of PSU's that is obtained after the first stage can be used for more than one survey. This will result in obvious cost savings. For some related discussion and development, see Waksberg (1978, pp. 43-44).

4. EXHAUSTED PSU'S

In practice the second-stage sample is drawn without replacement from each retained PSU, even though sampling with replacement is assumed for theoretical purposes. Since an uncertain number of phone numbers is needed in a Type I PSU for the sequential segment, under sampling without replacement one could dial all 100 phone numbers in the PSU (thus exhausting it) and still need more numbers to dial. This creates a bit of a predicament.

The frequency of exhausted Type I PSU's among retained PSU's will be affected by the value of c, by the definition of an auspicious phone number, and, obviously, by the value of k. The last two columns of Table 2 in (*) provide some indication of the effects of both c and the definition.

As c becomes greater, higher values of $n = kc$ can be tolerated, in the sense that one can raise kc and keep the frequency of exhausted Type I PSU's as low as before.

If one should go to the extreme of sampling every phone number in Type II PSU's, then almost all Type I PSU's will become exhausted before sampling is completed.

There are different ways to handle exhausted Type I PSU's, and different situations to be considered. None of the ways are ideal.

Consider first the simple situation where the sample is being used for just one survey. If a Type I PSU becomes exhausted, one can obtain the additional phone numbers needed by starting through the numbers in the PSU a second time, in a predetermined random sequence (perhaps the same one that was used the first time). Of course, one can even start through the PSU more than twice if necessary. Even though phone numbers are being drawn twice or more, there is no need to dial any number after the first drawing. This is because one can record the first time whether each number in a Type I PSU is auspicious or inauspicious, and then use this information, if the PSU later becomes exhausted, to determine how far down the list to go in reusing numbers.

Any completed interview thus receives a weight equal to the number of times that its phone number was drawn. All these weights will be integers. No two weights within the same PSU will differ from each other by more than one. (One can devise other weighting schemes that give the same weight--generally not an integer--to every interview in a PSU. Two of these will be mentioned later.)

Now consider the situation where the sample is being used for more than one survey. Suppose first that one finds it acceptable in Type I PSU's to try to interview more than once at the same phone number, for different surveys. The potential interviewee would not necessarily be the same household member each time. In addition, unavailability, refusal, or failure to qualify might stop a completed interview from being obtained for one survey but not another.

The handling of an exhausted Type I PSU can be similar to what was described before. If a phone number is redrawn for the same survey, then no further dialing is done but any completed interview obtained earlier has its weight increased by one. If a number is redrawn for a different survey, though, then one redials it and tries to obtain an interview for the new survey unless one found earlier that the number was definitely not residential. The first time through a Type I PSU, one can avoid unnecessary redialing later by not only recording whether each number is auspicious or inauspicious (as suggested before) but also noting those auspicious numbers that are definitely not residential.

The final case is the most complex one. Suppose the sample is being used for more than one survey and one does not want to try to interview at the same phone number more than once. There may be a fear that conducting one interview will cause results of a later interview to differ from what they would have been had the previous interview not occurred. There will be a particular concern if the same questionnaire is being used in two or more waves, or even if the different surveys have any common or related questions at all.

A possible approach here is to break up the sample into groups of phone numbers, one group for each survey. For a given survey, choose two integers, n ($= kc$) and n_1 ($\geq n$). Let the group for the survey consist of n numbers from each Type II PSU (if $c > 1$) and n_1 available numbers from each Type I PSU, where the sum of the n_1's across all the surveys does not exceed the number of phone numbers contained in a PSU (normally 100). Sampling can proceed much as before. In each Type II PSU, exactly n numbers (those in the group) are dialed. The number of phone numbers that will turn out to be needed in any Type I PSU will be $\geq n$, and may be $> n_1$. If it exceeds n_1, then, to the extent necessary, phone numbers are redrawn as before, but <u>only</u> from the n_1 numbers in the group, and weights are obtained as before.

An imperfection occurs if a group of n_1 numbers in a Type I PSU includes none at all that are auspicious. There will be no way to obtain the needed number of auspicious numbers even with redrawing.

Instead of what was just described, one may prefer a weighting scheme that assigns the same weight to every interview for a given survey within a Type I PSU. If the number of phone numbers needed in the PSU is $\leq n_1$, the weight is 1, of course. If not, let n_{*1} denote the number of phone numbers still needed for the nonsequential segment after all n_1 numbers are dialed, and let n_{*2} denote the number of auspicious numbers still needed for the sequential segment. Let x_1 denote the number of auspicious numbers among the n_1 in the group. Then, conditional on n_{*1}, n_{*2}, and x_1, the expected total number of auspicious numbers that would be obtained after drawing and redrawing from the group of n_1 numbers would be x_1 (from the initial drawing) plus n_{*1} times x_1/n_1 (from the unfulfilled part of the nonsequential segment) plus n_{*2} (from the unfulfilled part of the sequential segment). The number of distinct auspicious numbers obtained is just x_1. Thus the weight for each auspicious number (and for each interview) can be taken as the ratio of these two values, namely,

$$\frac{x_1 + n_{*1}(x_1/n_1) + n_{*2}}{x_1} = 1 + \frac{n_{*1}}{n_1} + \frac{n_{*2}}{x_1} . \quad (1)$$

Note that, as before, an imperfection exists if $x_1 = 0$.

Additionally, note that the weighting based on (1) can be used not only if there are two or more surveys, but also if there is just one.

If $c = 1$ and $n_1 = n$, then $n_{*1} = 0$ and $n_{*2} = n - x_1 = n_1 - x_1$. Thus (1) reduces to n_1/x_1, which is equivalent to the weighting suggested by Waksberg (1985, p. 91) for this case.

When $n_1 = n$, a weighting that disregards n_{*1} and n_{*2} is possible in place of (1). The expected total number of auspicious numbers that would be obtained after drawing and redrawing from the n_1 numbers can be figured as $(c - 1)/c$ times n_1 times x_1/n_1 (from the entire nonsequential segment) plus $1/c$ times n_1 (from the entire sequential segment). Dividing this sum by x_1, one obtains $1 + (n_1 - x_1)/cx_1$ as the weight.

The techniques described in this section have been aimed at minimizing bias. Using high weights in the rare Type I PSU that has just a tiny number of auspicious numbers does bring about minimal bias, but at the expense of much greater variance. One might therefore prefer to choose a ceiling value to be imposed for any weight that would otherwise be higher.

5. COST

Section 4 of (*) mentioned four relatively tangible cost elements, identified as (a)-(d), that vary with c and with the definition of an auspicious phone number. It also described five less tangible factors affecting cost, whose influence happens to be uniformly in the direction of favoring higher values of c and favoring DefB over DefR. DefR refers to a residential definition of an auspicious number, and DefB refers to the broader definition described in Section 5 of (*).

Even though one cannot easily measure the impact of the five less tangible factors, it is still instructive to examine more closely the effects of elements (a)-(d). To begin, let the subscript g refer to DefR if g = 1 or DefB if g = 2.

Element (a) is the first-stage cost, and may be expressed as $e_{1g}cm$, where e_{1g} denotes average cost per first-stage number sampled. Since it is harder to classify phone numbers as auspicious or inauspicious under DefR than under DefB, e_{11} will be greater than e_{12}.

Element (b), the second-stage cost of dialing in those unwanted PSU's that have some auspicious numbers but no residential numbers, is zero under DefR. Under DefB it may be written as $e_{22}(L_1 - L_2)mn$, where e_{22} denotes average cost per phone number for the numbers involved. L_g is the value of L under DefR for g = 1 and under DefB for g = 2, where L denotes the fraction of PSU's that have no auspicious numbers.

Element (c) represents special second-stage costs encountered in the sequential segments of Type I PSU's, costs over and above the normal ones that are incurred in Type II PSU's. Element (d) represents the same thing for the nonsequential segments. The special or extra costs include those pertaining to scheduling, delays, controls, administration, and training, as well as to classification of phone numbers, for Type I PSU's.

Among all second-stage phone numbers, let the fraction in Type I PSU's in sequential segments be denoted by a_{3gc}; in nonsequential segments, by a_{4gc}. Estimates of a_{3gc} and a_{4gc} for g = 1, 2 and c = 1 to 6 appear in the fourth-from-last and third-from-last columns of Table 2 of (*).

Elements (c) and (d) can be taken as $e_{3g}a_{3gc}(1 - L_g)mn$ and $e_{4g}a_{4gc}(1 - L_g)mn$, respectively, where e_{3g} and e_{4g} denote average cost per phone number for the phone numbers involved. Because of the complications in the sequential segment, e_{3g} will exceed e_{4g}, perhaps substantially so. Because DefR entails more complexity than DefB, e_{31} will exceed e_{32}. In addition, e_{41} will equal or exceed e_{42}, but both will be small.

In comparing costs for different g and c, it is appropriate to hold m and n = kc constant. With m and n constant, the expected number of residential numbers in the second stage will be the same regardless of what g and c are. In fact, it will be equal to mn times the proportion of residential numbers in the frame, as noted in Section 3.5 of (*).

Let K_{gc} denote the sum of the cost elements (a)-(d). A numerical illustration may be useful. First, from results in (*), one can set $L_1 = .6483$, $L_2 = .4686$, $a_{311} = 1$, $a_{411} = 0$, $a_{312} = .3318$, $a_{412} = .1930$, $a_{321} = 1$, $a_{421} = 0$, $a_{322} = .1795$, and $a_{422} = .1132$.

The cost coefficients can vary widely in different circumstances, but the values $e_{11} = 15$, $e_{12} = 8$, $e_{22} = 10$, $e_{31} = 11$, $e_{32} = 3$, $e_{41} = 1$, and $e_{42} = 1$ do not appear unreasonable in relation to one another and are suitable for an example. For g = 1, 2 and c = 1, 2, the four cost elements (a)-(d) and their total (K_{gc}), all after division by the common factor m, are then given by

$$\frac{1}{m}K_{11} = 15 + 0 + 3.87n + 0 = 15 + 3.87n,$$

$$\frac{1}{m}K_{12} = 30 + 0 + 1.28n + .07n = 30 + 1.35n,$$

$$\frac{1}{m}K_{21} = 8 + 1.80n + 1.59n + 0 = 8 + 3.39n,$$

$$\frac{1}{m}K_{22} = 16 + 1.80n + .29n + .06n = 16 + 2.14n.$$

Several points can be noted. Under DefR, c = 2 is cheaper than c = 1 if n ≥ 6. Under DefB, c = 2 is cheaper than c = 1 for n ≥ 7. With c = 1, DefB is cheaper than DefR for any n. With c = 2, DefR is cheaper than DefB if n ≥ 18. Of course, the results will be different if the cost coefficients are changed. Moreover, K_{gc} disregards the five less tangible factors mentioned earlier, which work in favor of DefB and higher c.

6. RECENT EXPERIENCE

In early 1987, interviewers at Valley Forge Information Service completed the dialing of a first-stage sample of 4,800 phone numbers consisting of c = 4 phone numbers from each of m = 1,200 PSU's. The sample came from the entire United States except Hawaii and Alaska, and was drawn in the same way as described in Section 3.7 of (*). Auspicious numbers were again defined as in Section 5 of (*). Certain results are worth noting:

(i) Of the 4,800 numbers, 290 were classified as inauspicious by virtue of reaching a fast busy signal three times. Of these 290, all but 19 were in PSU's with no auspicious numbers among the four drawn. The 290 numbers were spread among 104 PSU's, all but 14 of which had no auspicious numbers out of four.

(ii) There were 99 numbers, distributed among 46 PSU's, that were classified as inauspicious after reaching "dead air" (lack of any sound) three times. All but 12 of the 99 numbers were in PSU's where no auspicious numbers were found. In all but 11 of the 46 PSU's, there were no auspicious numbers out of four.

One infers from (i) and (ii) that few numbers with a fast busy signal or dead air will be encountered in the second stage. This will be helpful for the second-stage dialing.

(iii) Only 22 phone numbers, spread among 13 PSU's, were classified as inauspicious after reaching a slow busy signal eight times. Of the 22 numbers, 13 were in PSU's where no auspicious numbers were dialed. No auspicious numbers were found in 7 of the 13 PSU's.

(iv) There were 200 numbers, spread among 140 PSU's, that were classified as auspicious upon reaching any of four specific types of recorded messages (which had to mention the last seven digits dialed), or upon reaching an intercept operator who gave similar information. The breakdown of the 200 numbers into the four categories showed 112 disconnected, 9 temporarily disconnected, 69 changed to a specified new number, and 10 changed to a nonlisted or nonpublished number. Of the 200 numbers, 190 reached a recording only, 2 reached an intercept operator only, and 8 reached an intercept operator followed by a recording. The 140 PSU's yielded 307 auspicious numbers in addition to the 200, and 53 inauspicious numbers.

(v) There were 64 numbers, distributed among 33 PSU's, that were classified as auspicious by virtue of yielding tones. The 33 PSU's had 49 auspicious numbers besides these 64, and 19 inauspicious numbers.

(vi) Of the 1,200 PSU's, 583 had no auspicious numbers, 26 had one, 83 had two, 146 had three, and 362 had four. These results are close to those that appeared in Table 1 of (*).

(vii) Had the 200 numbers in (iv) been classified as inauspicious rather than auspicious (by changing the definition of an auspicious number), 17 PSU's would have changed from Type II to Type I, 13 from Type II to discards, and 1 from Type I to a discard. Thus the number of Type I PSU's would have increased more than 60%, from 26 to 42.

(viii) Had the 64 numbers in (v) been classified as inauspicious rather than auspicious (with no change for the 200 numbers just discussed), then 3 PSU's would have changed from Type II to Type I, 13 from Type II to discards, and 0 from Type I to discards.

The results in (vi)-(viii) were based on sampling with replacement within a PSU, so that occasionally the same phone number was counted more than once. The results in (i)-(v), however, were based on the four distinct phone numbers that were drawn from every PSU. Sampling with replacement and sampling without replacement were conducted simultaneously as described in Section 2 of (*).

7. PROOFS

In Sections 3.3 and 3.5 of (*), some proofs were omitted. They will be covered here. The basic notation, of course, is explained in (*). A few equations from (*) are appealed to below; equations (1) of (*), for example, will be referred to as (1*).

Section 3.3 of (*) stated, but did not prove, that $E(MNy/mn) = Y$ and $E(MNu/mn) = U$. It will only be necessary to prove the first of these equations, since the proof of the second is analogous. To begin, note that

$$E(y) = E(\sum_{i=1}^{m} y_i) = mE(y_i) . \tag{2}$$

Then

$$E(y_i) = (1/M) \sum_{I=1}^{M} E(y_i | i \to I), \tag{3}$$

where the notation "$|i \to I$" means "given that the i-th PSU in the sample is the I-th PSU in the population." Next,

$$\begin{aligned} E(y_i | i \to I) = {} & Pr(I; 0)E(y_i | i \to I; 0) \\ & + Pr(I; 1)E(y_i | i \to I; 1) \\ & + Pr(I; 2)E(y_i | i \to I; 2), \end{aligned} \tag{4}$$

where "$|i \to I; 0$" means "given that the i-th PSU in the sample is the I-th PSU in the population and is discarded after the first stage," and "$|i \to I; 1$" and "$|i \to I; 2$" mean the same thing except that "discarded" is replaced by "classified as Type I" and "classified as Type II," respectively. Now

$$E(y_i | i \to I; 0) = nE(y_{ij} | i \to I; 0) = 0,$$

$$E(y_i | i \to I; 2) = n(1/N) \sum_{J=1}^{N} E(y_{ij} | i \to I, j \to J), \quad (5)$$

where "$| i \to I, j \to J$" means "given that the j-th observation in the i-th PSU in the sample is associated with the J-th phone number in the I-th PSU in the population." Also, if $P_I > 0$,

$$E(y_i | i \to I; 1) = k(c - 1)(1/N) \sum_{J=1}^{N} E(y_{ij} | i \to I, j \to J) + k(1/NP_I) \sum_{J \text{ au.}} E(y_{ij} | i \to I, j \to J), \quad (6)$$

where the notation "J au." indicates that the summation is to be confined to those NP_I J-values whose phone numbers are auspicious. However,

$$\sum_{J \text{ au.}} E(y_{ij} | i \to I, j \to J) = \sum_{J=1}^{N} E(y_{ij} | i \to I, j \to J), \quad (7)$$

since the terms for the NQ_I inauspicious phone numbers that were excluded are all 0. From (2*) and (5*) or from (3*) and (6*) one determines that

$$E(y_{ij} | i \to I, j \to J) = Y_{IJ} \quad (8)$$

for a survey of either individuals or households. Upon combining (4) with (1*) and (5)-(8) and then simplifying, one finds that

$$E(y_i | i \to I) = (n/N)Y_I. \quad (9)$$

The desired final result follows at once after combining (2), (3), and (9).

Section 3.5 of (*) omitted the proofs of three statements concerning expected numbers of phone numbers dialed in the second stage. To begin, let z_i denote the total number of phone numbers dialed in the i-th PSU in the sample in the second stage, and define $z = \Sigma_{i=1}^{m} z_i$. Then

$$E(z_i | i \to I) = \Pr(I; 0)\cdot 0 + \Pr(I; 2)\cdot n + \Pr(I; 1)\cdot[k(c - 1) + k(1/P_I)] \quad (10)$$

if $P_I > 0$, and $E(z_i | i \to I) = 0$ if $P_I = 0$. The right side of (10) is simply equal to n. One thus obtains

$$E(z) = mE(z_i) = m(1/M) \sum_{I=1}^{M} E(z_i | i \to I) = m(1/M)[(M - M')n + M'\cdot 0] = (1 - L)mn \quad (11)$$

as the expected value of the total number of phone numbers dialed in the second stage.

To prove that the expected number of auspicious numbers in the second stage is $\bar{P}mn$, define w_i to be the number of auspicious numbers dialed in the i-th PSU in the sample. Then note that, for $P_I > 0$, $E(w_i | i \to I)$ is equal to the right side of (10) times P_I. Hence $E(w_i | i \to I) = P_I n$ for each I, and the proof is completed as in (11).

A similar derivation shows that the expected number of residential numbers in the second stage is $\bar{P}^* mn$. Use w_i^* for number of residential numbers, note that $E(w_i^* | i \to I)$ is equal to the right side of (10) times P_I^* for $P_I > 0$, observe then that $E(w_i^* | i \to I) = P_I^* n$ for each I, and finish the proof as before.

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