Optimal Two Phase Sampling for Estimating the Exploitable Biomass of a Fishery accounting for Nonsampling Errors

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1. INTRODUCTION

The exploitable biomass of a fishery is the total weight in pounds of the portion of the population that is available to a fishery. In fishery research this part of the population is commonly referred to as the portion that is *recruited* into the fishery. In large commercial fisheries, regulated fishing gear catches fish from the harvestable population *selectivly* by age: young fish are too small to be caught with the gear but mature fish are more likely to be landed once "hooked." The proportion of the population that is susceptible to the legal gear at age j is known as *the selectivity coefficient at age j.* This parameter will be denoted by S_j.

Letting W_j denote the average weight at age j for an individual fish and N_{tj} the abundance of age j fish in year t, the exploitable biomass in year t is

$$\beta_t = \sum_{j=1}^{D} S_j W_j N_{tj} \tag{1}$$

where D is the number of age classes in the population. Letting M denote the instantaneous natural mortality rate and C_{tj} the number of fish caught (i.e., the *catch*) at age j in year t, Pope's (1972) cohort approximation for $N_{t+1,j+1}$ is

$$N_{t+1,j+1} = \left(N_{tj} e^{-M/2} - C_{tj}\right) e^{-M/2}. \tag{2}$$

According to (2), the abundance of fish at age j+1 in year t+1 is equal to the survivors in the same cohort from the prevous year that were not caught in the previous year. One referee noted that assuming all exploitation takes place in the middle of the year, (2) may be alternately derived from the catch equations.

Letting C_t denote the total number of fish caught in year t, and Π_{tj} the proportion of age j among the C_t fish caught in year t, Pope's (1972) recursion (2) yields

$$N_{tj} = \begin{cases} N_{t+D-j,D} e^{(D-j)M} + \sum_{k=0}^{D-j-1} C_{t+k} \Pi_{t+k,j+k} e^{(k+\frac{1}{2})M}, \\ C_t \Pi_{tD} / \kappa, \end{cases} \tag{3}$$

for $j = 1, \ldots, D-1$, and and $j = D$, , respectively, where κ is the instantaneous fishing mortality rate determined independently from the survey.

It should be noted that the abundance N_{tj} of the cohort corresponding to j=D is expected to be very small since the species under consideration is assumed to live no more than D years: in the following year this cohort is *extinct* ($N_{t+1,D+1} = 0$) as a result of natural mortality and harvesting. Although (3) may be considered to be a crude approximation for the case when j=D, because abundance at age D is very small the cohort at this age contributes a negligible amount to the total exploitable biomass (1). Hence, the approximation is expected to be adequate.

Substitution of (3) into (1) yields an analytic expression for exploitable biomass in year t:

$$\beta_t = \sum_{j=1}^{D-1} S_j W_j \left[N_{t+D-j,D} e^{(D-j)M} + \sum_{k=0}^{D-j-1} C_{t+k} \Pi_{t+k,j+k} e^{(k+\frac{1}{2})M} \right] + S_D W_D C_t \Pi_{tD} / \kappa. \tag{4}$$

The purpose of this paper is to present statistical methods for determining optimal probability sampling plans that account for important biological considerations in obtaining estimates of exploitable biomass (1). In particular, Pope's approximation (2) is used as the fundamental biological law that governs the dynamics of the fish population. Although this approximation lacks many important biological nuances it captures the spirit of these more complex biological considerations. In contrast, previous researchers (e.g., Quinn, *et. al.* (1983), Schweigert and Sibert (1983), Chester and Waters (1985), and Waters and Chester (1987)) have not accounted for biological considerations in developing an optimal sampling plan.

Also, nonsampling errors that are expected in catch at age data are explicitly taken into account in developing an optimal sampling design. These errors result from incorrect age determinations of the sampled fish and have an untoward effect upon the bias of the resulting estimate of exploitable biomass.

Section 2 sets forth additional assumptions about the dynamics of the population. Further, a general expression is given for the estimate of exploitable biomass when nonsampling errors have been corrected statistically. Section 3 outlines the two phase sampling design which is recommended by current and authoritative literature in fisheries research (e.g., Ricker 1975; Seber 1982). Section 4 presents methods for optimizing the sampling design for the purposes of minimizing the variance of the estimate of exploitable biomass. Also, a computational method is given for determining the optimal allocation of sampling effort between the first and second phase of the two phase design for a fixed survey budget. Section 5 gives an example taken from the Pacific halibut fishery is given to illustrate the methods.

2. THE SAMPLING VARIANCE OF EXPLOITABLE BIOMASS

Inspection of equation (4) shows that estimation of exploitable biomass requires estimates of S_j, W_j, $N_{t+D-j,D}$, C_{tk}, and $\Pi_{t+k,j+k}$.

Both S_j and W_j are estimated from an independent prior survey and will not effect the optimization of the current survey.

The parameters $\{N_{t+D-j,D}\}$, $j = 1, \cdots, D-1$, are the abundance of each living cohort at their last year prior to extinction. The magnitude of these parameters are expected to be very small and, as discussed in Section 1, to effect the total exploitable biomass negligibly. Therefore, it is traditional in fisheries cohort analysis to *assign* small numbers for their values.

The proportions $\{\Pi_{t+k,j+k}\}$ are to be estimated from the sample survey for the purposes of estimating β_t, exploitable biomass.

With respect to estimates of these proportions the variance of the estimate of exploitable biomass may be seen to be

$$V_{\{\hat{\Pi}\}}(\hat{\beta}_t) = \sum_{k=1}^{D} \left[\sum_{j=k}^{D} z_{tjk}^2 V(\hat{\Pi}_{t+k-1,j}) + 2 \sum_{j>l=k}^{D} z_{tjk} z_{tlk} Cov(\hat{\Pi}_{t+k-1,j}, \hat{\Pi}_{t+k-1,l}) \right] \quad (5)$$

where

$z_{tjk} = 0$ when $j < k = 2, \ldots, D$;

$z_{tjk} = S_{j-k+1} W_{j-k+1} C_{t+k-1} e^{(k-\frac{1}{2})M}$ when $1 = k \le j \le D-1$;

$z_{tjk} = 1/\kappa$ when $k = 1, j = D$; and

$z_{tjk} = 0$ when $j = D, k = 2, \ldots, D$.

For given estimates of the selectivity coefficients $\{S_j\}$ and weights at age $\{W_j\}$, (5) depends upon the total catch sizes $\{C_{t+k-1}\}$ and the variance of $\{\hat{\Pi}_{t+k-1,j}\}$, that are determined in the *future*. In order to obtain an objective function for the purpose of optimizing the *current* year's survey it is assumed that a) the total catch sizes will remain at the current legally regulated maximum quota, C_t, and b) the fish population is stable in the sense that the proportions of fish in the catch at each age remain constant over time. Consequently,

$$C_{t+k-1} = C_t$$

and

$$\Pi_{t+k-1,j} = P_{\cdot j}$$

for $k = 1, \ldots, D$. We require the $\{P_{\cdot j}\}$ to be estimated so as to minimize

$$\begin{aligned} V_{\{\hat{P}_{\cdot j}\}}(\hat{\beta}_t) &= \sum_{k=1}^{D} \left[\sum_{j=k}^{D} z_{tjk}^2 V(\hat{P}_{\cdot j}) + 2 \sum_{j>l=k}^{D} z_{tjk} z_{tlk} Cov(\hat{P}_{\cdot j}, \hat{P}_{\cdot l}) \right] \\ &= \sum_{j=1}^{D} b_{jj} V(\hat{P}_{\cdot j}) + 2 \sum_{1 \le k < j \le D} b_{jk} Cov(\hat{P}_{\cdot j}, \hat{P}_{\cdot l}) \end{aligned} \quad (6)$$

with respect to the sampling design for a fixed survey budget, where

$$b_{jk} = \sum_{l=1}^{k} z_{tjl} z_{tkl}.$$

In the case of Pacific halibut an ear bone, known as the saggita otolith, is cut from each sampled fish. The otolilths are subsequently taken to the laboratory where they are examined. The age of a fish is determined by counting the annual rings on its otolith. The number of sampled otoliths at each age are used to estimate the catch's age composition, $P = (P_{\cdot 1}, \ldots, P_{\cdot D})^T$.

However, nonsampling errors are likely to be incurred as otoliths are examined: otoliths are opaque and irregular features on the bone can mislead the biologist examining it. Consequently, some otoliths are likely to be classified incorrectly by age. The distribution that may be estimated from the resulting data is the *classification* distribution. This distribution is different from the age composition since the data used to construct it is marred by "reading" errors.

However, the classification distribution may be adjusted statistically to yield the age composition distribution: Let $Q_{\cdot j}$ denote the probability of *classifying* an otolith as age j and let $Q = (Q_{\cdot 1}, \ldots, Q_{\cdot D})^T$. Then letting ψ_{jk} denote the conditional probability of misclassifying a k year old fish as age j, and $\Psi = \{\psi_{jk}\}$, then

$$P = \Psi^{-1} Q.$$

Given an unbiased estimate of the proportion of otoliths *classified* as age j, $\hat{Q}_{\cdot j}$, an unbiased estimate of the proportion of otoliths at age j is

$$\hat{P}_{\cdot j} = \sum_{v=1}^{D} w_{jv} \hat{Q}_{\cdot j} \quad (7)$$

where $\Psi^{-1} = \{w_{jv}\}$. Also,

$$V(\hat{P}_{\cdot j}) = \sum_{v=1}^{D} w_{jv}^2 V(\hat{Q}_{\cdot v}) + 2 \sum_{v<u}^{D} w_{jv} w_{ju} Cov(\hat{Q}_{\cdot v}, \hat{Q}_{\cdot u}) \quad (8)$$

and

$$Cov(\hat{P}_{\cdot j}, \hat{P}_{\cdot k}) = \sum_{v=1}^{D} w_{jv} w_{kv} V(\hat{Q}_{\cdot v}) + 2 \sum_{v<u}^{D} w_{jv} w_{ku} Cov(\hat{Q}_{\cdot v}, \hat{Q}_{\cdot u}). \quad (9)$$

Equations (8) and (9) may be combined with (6) to yield an expression for the sampling variance of exploitable biomass when there are reading errors.

Alternatively, laboratory procedures may be devised so as to eliminate reading errors. For example, nonsampling errors may be eliminated by reading otoliths twice, independently on each occasion. We assume that a) ages for otoliths for which there is no discrepancy between the independent readings correspond to the true age, and b) the true age of otoliths for which there are discrepant independent readings are determined by subsequent reconciliation.

In actuality, the true age may not be determined for each and every otolith read using this alternative. However, this procedure is in widspread use in fisheries biology and a definition of what the "true" age is would be required in any analysis. Our definition is a useful expedient.

For this alternative lab procedure $\Psi = I$ and the per unit cost of making age determinations is twice that of making single readings and using (7) to adjust the classification distribution statistically. Section 4 compares the merits of these competing lab strategies for Pacific halibut data by evaluating the sampling variances of exploitable biomass for both cases.

3. $V(\hat{\beta}_t)$ UNDER TWO PHASE SAMPLING

Let N denote the total number of fish in the catch and let N_{ij} denote the unknown number of fish in the catch categorized as belonging to otolith weight stratum i and age class j. Then $P_i = N_{i.}/N$ denotes the proportion of fish in the catch belonging to stratum i and $Q_{ij} = N_{ij}/N_{i.}$ denotes the proportion of fish in stratum i categorized as belonging to age class j, $j = 1, \ldots, D$, $i = 1, \ldots, L$.

In the first phase of the two phase design a simple random sample of n' fish are selected from the catch. Otoliths from the sampled fish are weighed and stratified according to weight. The number of fish in the sample belonging to stratum i, n_i' is noted, $i = 1, \ldots, L$.

In the second phase, a random subsample of n_i otoliths ($n_i \leq n_i'$) is selected from the n_i' otoliths belonging to otolith weight stratum i, $i = 1, \ldots, L$. Each subsampled otolith is examined and the number of subsampled fish in stratum i categorized into age class j, n_{ij}, is noted, $j = 1, \ldots, D$.

Letting Q_{ij} denote the proportion of otoliths *categorized* into age class j in weight stratum i, an unbiased estimator of $Q_{\cdot j}$ is

$$\hat{Q}_{\cdot j} = \sum_{i=1}^{L} \hat{Q}_{ij} \hat{P}_i, \tag{9}$$

$j = 1, \ldots, D$ where $\hat{Q}_{ij} = n_{ij}/n_i$ and $\hat{P}_i = n_i'/n'$. Letting N_i' denote the number of fish in the catch belonging to weight stratum i, the associated variance of (9) is

$$\begin{aligned} V(\hat{Q}_{\cdot j}) &= E_{\{\hat{P}_i\}}\left[V_{\{\hat{Q}_{ij}\}|\{\hat{P}_i\}}(\hat{Q}_{\cdot j})\right] + V_{\{\hat{P}_i\}}\left[E_{\{\hat{Q}_{ij}\}|\{\hat{P}_i\}}(\hat{Q}_{\cdot j})\right] \\ &= \sum_{i=1}^{L}\left[\frac{g'}{n'}P_i(1-P_i) + P_i^2\right]\frac{g_i}{n_i}Q_{ij}(1-Q_{ij}) \\ &\quad + \frac{g'}{n'}\sum_{i=1}^{L}P_i(Q_{ij}-Q_{\cdot j})^2 \end{aligned} \tag{10}$$

where $g' = (N-n')/(N-1) \doteq 1$, and $g_i = (N_i'-n_i)/(N_i'-1) \doteq 1$ assuming $N \gg n'$ and $N_i' \gg n_i$.

Also,

$$\begin{aligned} Cov(\hat{Q}_{\cdot j}, \hat{Q}_{\cdot k}) &= E_{\{\hat{P}_i\}}\left[Cov_{\{\hat{Q}_{ij}\}|\{\hat{P}_i\}}(\hat{Q}_{\cdot j}, \hat{Q}_{\cdot k})\right] \\ &\quad + Cov_{\{\hat{P}_i\}}\left[E_{\{\hat{Q}_{ij}\}|\{\hat{Q}_i\}}(\hat{Q}_{\cdot j}), E_{\{\hat{Q}_{ij}\}|\{\hat{P}_i\}}(\hat{Q}_{\cdot k})\right] \\ &= -\sum_{i=1}^{L}\left[\frac{g'}{n'}P_i(1-P_i) + P_i^2\right]\frac{g_i}{n_i}Q_{ij}Q_{ik} \\ &\quad + \frac{g'}{n'}\sum_{i=1}^{L}P_i(Q_{ij}-Q_{\cdot j})(Q_{ik}-Q_{\cdot k}) \end{aligned} \tag{11}$$

Combining (6), (8), (9), (10), and (11) yields the variance of the estimate of exploitable biomass, $V(\hat{\beta}_t|\Psi)$, when nonsampling errors have been corrected statistically. The expression for $V(\hat{\beta}_t|\Psi)$, is referred to as equation (12) and is found in Appendix A.

4. SURVEY DESIGN OPTIMIZATION

This section outlines the manner in which sample sizes and sampling strategies may be specified so as to yield the most precise estimate of exploitable biomass for a given survey budget. This specification depends upon the per unit sampling costs.

In the Annual Market Survey (AMS) conducted by the International Pacific Halibut Commission (IPHC) there are four distinct per unit sampling costs: (1) the cost of cutting otoliths from halibut in the field, (2) the cost of measuring the lengths of otoliths, (3) the cost of examining an otolith once to determine its age, and (4) the cost of weighing otoliths.

We denote the per unit cost of each of these four expenses by c_i, $i = 1, \ldots, 4$. At the IPHC these costs have been estimated to be $c_1 = US\$0.50$, $c_2 = US\$0.22$, $c_3 = US\$0.32$, and $c_4 = US\$0.09$. Although otolith length is not used in estimating age distribution in the IPHC application it is collected each year to maintain the time series. This measurement is required from every otolith that has been aged.

In the two phase sampling design all otoliths must be weighed for the purposes of stratification. The per unit sampling costs for fish in the first phase of the design but not subsampled in the second phase is $c_f = c_1 + c_4 = US\$0.59$. When otoliths are read only once to determine age the per unit sampling cost of fish in the first phase *and* the second phase of the design is $c_s = \sum_{i=1}^{4} c_i = US\$1.13$. However, when otoliths are read twice to eliminate errors this per unit cost is $c_s = US\$1.45$.

4.2 OPTIMIZATION OF THE TWO PHASE SAMPLE DESIGN

For the two phase design the optimal allocation of B between the first and second phase sample minimizes the variance of the estimate of exploitable biomass. To determine the optimal allocation let n_f denote the number of sampled units that are sampled in only the first phase of the design, and not in the second. Also, let n_s denote the total number of sampled units that are subsampled in the second phase. Therefore, $n' = n_f + n_s$ and these values must satisfy $B \geq c_f n_f + c_s n_s$.

For a given value of n_f, $n_s = (B - c_f n_f)/c_s$. With respect to n_f and n_s, the expression for the variance of the estimate of exploitable biomass, (12), may be written more simply as

$$V(n_f, \{n_i\}) = \sum_{i=1}^{L} \frac{1}{n_i} \left[\frac{c_s P_i(1-P_i)}{n_f(c_s - c_f) + B} + P_i^2 \right] \Theta_i + \frac{c_s \Omega}{n_f(c_s - c_f) + B} \quad (13)$$

For a given value of n_f the optimal second phase subsample sizes $\{n_i\}$ minimize

$$\sum_{i=1}^{L} \frac{1}{n_i} \left[\frac{c_s P_i(1-P_i)}{n_f(c_s - c_f) + B} + P_i^2 \right] \Theta_i \quad (14)$$

subject to the constraints

$$1 \leq n_i \leq E(n_i') \quad (15)$$

and

$$\sum_{i=1}^{L} n_i = (B - c_f n_f)/c_s \quad (16)$$

where

$$E(n_i') = \frac{B + n_f(c_s - c_f)}{c_s} P_i.$$

The expression $E(n_i')$ is used in (15) because n_i' is undetermined prior to selection of the sample. A modification of the algorithm given by Rao and Ghangurde (1972) that accounts (15) may be used for computing the optimal subsample sizes. Letting $n_{i,opt(n_f)}$ denote these optimal subsample sizes, the optimal value of n_f minimizes

$$V(n_f, \{n_{i,opt(n_f)}\}) = \sum_{i=1}^{L} \frac{1}{n_{i,opt(n_f)}} \left[\frac{c_s P_i(1-P_i)}{n_f(c_s - c_f) + B} + P_i^2 \right] \Theta_i + \frac{c_s \Omega}{n_f(c_s - c_f) + B} \quad (17)$$

subject to the constraint

$$0 \leq n_f \leq (B - c_s L)/c_f. \quad (18)$$

Because (13) becomes infinite when any subsample size $n_i = 0$, we require that $n_i \geq 1$, $i = 1, \cdots, L$. Therefore, (18) follows from the fact that the maximum survey expense that can be incurred by first phase sampling is $B - c_s L$.

For a given budget B available for sampling expenses, the optimal two phase sampling plan chooses n_f and $\{n_{i,opt(n_f)}\}$ so as to minimize (17) and to satisfy the constraints (15), (16), and (18). The minimum value of (17) will be denoted by $V_{opt}(B; \Psi = I)$ for the lab procedure requiring two independent readings per subsampled otolith, and by $V_{opt}(B; \Psi)$ for the lab procedure requiring only one reading.

5. THE AMS

In the cross-tabulation of age by fish weight stratum although age generally increases with weight, the variability with regard to age within strata is great.

After reviewing more than 80,000 initial and reconciled readings by an IPHC biologist, initial readings were correct 70% of the time. However, 25% of the readings differed by one year from the true age and 5% by more than one year.

Figure 1 gives a plot of the minimum standard deviations

$$V_{opt}^{1/2}(B; \Psi = I), V_{opt}^{1/2}(B; \Psi)$$

over a broad range of possible survey budgets, B. Because the standard deviation is proportional to the width of a confidence interval, Figure 1 illustrates that an interval is fairly broad for small survey budgets but narrows rapidly as larger budgets are considered. As budgets greater than $US\$5000$ are considered, diminishing returns are obtained in terms of narrowing the width of the interval.

For a given survey budget B, the optimal laboratory procedure p, and optimized sampling design d, is associated with

$$V_{d,p}^{1/2} = \min \left\{ V_{opt}^{1/2}(B; \Psi = I), V_{opt}^{1/2}(B; \Psi) \right\}.$$

From Figure 1 it can be seen that except for small survey budgets the minimum standard deviation is obtained by using the two phase sampling design with the lab practice of reading subsampled otoliths twice. However, the simple random sampling design used with the same lab procedure yields an estimate of exploitable biomass with nearly identical precision.

For the Pacific halibut application the relative increase in standard deviation for the simple random sampling design and the "double reading" lab procedure over $V_{d,p}^{1/2}$ is never more than 2.4%. The failure of the two phase design to yield an estimate with much greater precision can be attributed to unfulfilled conditions in the IPHC application that normally would enable the sampling strategy to perform with greater efficacy: if a) the predictive realtionship is strong between the stratification variable and the primary variable of interest, and b) the per unit sampling cost of obtaining measurements from the stratification variable, then the two phase design will generally yield better estimates for a given survey budget than the simple random sampling design. However, in the IPHC application otolith weight is not a powerful predictor of halibut age. For example, for mid-weight strata (where most of the population's exploitable biomass lies) the range of the age distribution is very broad. Consequently, the value in allocating survey budget to determine ololith weight is marginal.

Also, the relative per unit sampling cost of making age determinations is not great enough to justify allocating survey budget for obtaining information on the relatively less expensive variable, otolith weight.

APPENDIX A: $V(\hat{\beta}_t | \Psi)$

The expression for the variance of the estimate of exploitable biomass is

$$V_{tps}(\hat{\beta}_t|\Psi) = \sum_{i=1}^{L}\frac{1}{n_i}\left[\frac{P_i(1-P_i)}{n'}+P_i^2\right] \left\{\sum_{j=1}^{D} b_{jj}\left[\sum_{v=1}^{D} w_{jv}^2 Q_{iv}(1-Q_{iv}) - 2\sum_{v<u}^{D} w_{jv}w_{ju}Q_{iv}Q_{iu}\right] + 2\sum_{1\le k<j\le D} b_{jk}\left[\sum_{v=1}^{D} w_{jv}w_{kv}Q_{iv}(1-Q_{iv}) - 2\sum_{v<u}^{D} w_{jv}w_{ku}Q_{iv}Q_{iu}\right]\right\} + \frac{1}{n'}\sum_{i=1}^{L}P_i\left\{\sum_{j=1}^{D}b_{jj}\left[\sum_{v=1}^{D}w_{jv}^2(Q_{iv}-Q_{\cdot j})^2 + 2\sum_{v<u}^{D}w_{jv}w_{ju}(Q_{iv}-Q_{\cdot v})(Q_{iu}-Q_{\cdot u})\right] + 2\sum_{1\le k<j\le D}b_{jk}\left[\sum_{v=1}^{D}w_{jv}w_{kv}(Q_{iv}-Q_{\cdot v})^2 + 2\sum_{v<u}^{D}w_{jv}w_{ku}(Q_{iv}-Q_{\cdot v})(Q_{iu}-Q_{\cdot u})\right]\right\} \quad (12)$$

REFERENCES

Chester, A.J. and Waters, J.R. (1985). Two stage sampling for age distribution in the Atlantic Menhaden Fishery. *North American Journal of Fisheries Management*, 5, 449-456.

Pope, J.G. (1972). An investigation of the accuracy of virtual population analysis using cohort analysis. *International Commission for the Northwest Atlantic Fisheries Research Bulletin*, No. 9, 65-74.

Quinn, T.J., Best, E.A., Bijsterveld, L., and McGregor, I.R. (1983). *Sampling Pacific Halibut (Hippoglossus stenolepis) Landings for Age Composition: History, Evaluation, and Estimation.* International Pacific Halibut Commission Scientific Report No. 68: Seattle.

Rao, J.N.K. and Ghangurde, P.D. (1972). Bayesian optimization in sampling finite populations. *Journal of the American Statistical Association*, 67, 439-443.

Ricker, W.E. (1975). *Computation and Interpretation of Biological Statistics of Fish Populations.* Department of the Environment, Fisheries and Marine Service: Ottawa.

Schweigert, J.F. and Sibert, J.R. (1983). Optimizing survey design for determining age structure of fish stocks: and example from British Columbia Pacific Herring (*Clupea harengus pallasi*). *Canadian Journal of Fisheries and Aquatic Sciences*, 40, 588-597.

Seber, G.A.F. (1982). *The Estimation of Animal Abundance*, Macmillan: New York.

Waters, J.R. and Chester, A.J. (1987). Optimal allocation in multivariate, two stage sampling designs. *The American Statistician*, 41, 46-49.

Figure 1: Minimum standard deviations of the estimate of exploitable biomass